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A Bilevel Approach to Optimal Pricing

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A combination of several factors, such as increased competition in a global economy, real-time access to a wealth of transparent information, and the rise of a knowledgeable and pragmatic generation of consumers, is currently modifying the role and nature of pricing. Whereas pricing models have often, in the past, been approached from a purely academic standpoint, they are nowadays considered as central financial and operational tools in major industries, and have become one of the most important levers toward profitability.

Indeed, a one percentage point increase in price yields larger gains in operating profit than a similar increase in variables costs, fixed costs, or volumes. Alternatively, inappropriate pricing decisions may result in a tremendous drag on profit. The need to get pricing management right is thus increasingly recognized, especially in the Operations Research community. To be convinced, one need look no further than the last INFORMS annual meeting (www.informs.org/Conf/NO2005), where a search on "pricing" or "revenue" yields some 300 presentations!

Price optimization involves three main agents: a firm looking for a revenue maximizing pricing strategy, its competitors, and the customers. It is well suited to a game theoretic approach, where each agent seeks to maximize its own utility, taking into account the behaviour of the other players. In the context of price optimization, firms want to maximize revenue, while customers seek to satisfy their needs at minimal cost. In the MITACS project entitled "Optimal Pricing via mathematical and bilevel programming", we address the pricing issue through a leader-follower game, where one player integrates within her decision process the reaction of the follower to her course of action. In this framework, the decisions of the leader firm are contingent on the reaction of the customers to the products and price schedules set by all firms. Leader-follower games have a long history, dating back to the concept introduced by von Stackelberg for a group of agents competing over a market. Although they are now a part of mainstream economics, leader-follower games have not, with few exceptions, been applied to real-life situations, mainly due to the lack of algorithms able to tackle large problem instances.

While economists have focused on the qualitative properties of leader-follower games, applied mathematicians have tried to develop efficient algorithms for solving the closely related bilevel programs, where the upper level is associated with the leader, and the lower level with the follower. This concept has been generalized to include lower levels that describe the equilibrium state of physical or social systems. For instance, before implementing a toll policy designed to alleviate congestion, the mayor of a large city (the leader, or "upper level") would be well advised to predict the group reaction of private transportation users (the followers, or "lower level"), most of whom are also voters!

The above framework provides a powerful modeling paradigm and has been the topic of many theoretical and algorithmic studies over the past twenty years. At first glance, the situation looks rather bleak. Indeed, bilevel programs are generically difficult to solve, as they combine the intricacies associated with continuous and combinatorial optimization problems. They are actually theoretically and numerically "intractable", and one has to take into account the inner structure of the application in order to develop efficient solution procedure.

Over the past few years, our MITACS team has

developed powerful algorithms for solving variants of the basic model that applies to commercial airlines, passenger rail, and the telecommunication industries. In particular, we have developed a bilevel model that jointly optimizes and automates the pricing and allocation processes of an airline, within a dynamic, network-based environment.

One approach that our team pursues takes explicitly into account the network structure, as well as the aircrafts' capacities. In contrast with standard practice, demand is not the result of an econometric model. Rather, customers are assigned to "products" according to their own preferences, expressed in terms of choice criteria such as fare, number of transfers, flight duration or quality of service. Once implemented within a dynamic environment, our management tool allows a firm to adapt, in a rational fashion, its fare and allocation policies to a shifting environment influenced by its competitors, as well as the behaviour of its partners in strategic alliances. While the methods developed within this project pertain to airline or telecommunication firms, they apply to a wide range of industries that share features such as perishable inventory, fluctuating demand and rigid capacity. An alternative modeling approach consists in adapting the theory of real options to revenue management issues, taking into account demand variability as well as the effect of competition on the pricing-allocation process. In both cases, the quality of the solutions must be validated using state-of the art simulation models.

Does all this sound too good to be true? It is a fact that bilevel models, although intuitive in most applications, are difficult to solve for an optimal or near-optimal solution. However, given the recent interest of MITACS industrial partners in this mathematical technology, the goal is worth pursuing. In conclusion, and notwithstanding the financial rewards to strategic industries, there remains the core motivation: bilevel programs provide a rich source of challenging issues, and thus constitute a magnificent playground for mathematicians, operations researchers and computer scientists alike!

References

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