Predicting the spectral behaviour of colour printers for transparent inks on transparent support[*]

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Abstract

A spectral colour prediction model is proposed and applied to an ink jet printer. We limit the field of investigation to transparent inks printed on a transparent substrate. The proposed method is based on the computation of the transmittance spectra of colour primaries applying Beer's law, and on the calculation of their respective surface coverage using a high resolution grid. The proposed prediction model requires measuring the transmittance spectra of the inks and approximating the density distribution function of a single printed dot of each ink. We hereby obtain accurate spectral predictions of colour patches, while the number of required sample measurements is reduced to a minimum.

Introduction

While developing a new printing device, it is highly desirable to be able to simulate its behaviour. Designers are interested in studying the influence of various parameters on the reproduced colour. Can the lack of mechanical precision induce significant colour deviations? What is the gamut available with only two different chromatic inks? How does the gamut change when other inks are used? These questions and many others can be answered thanks to a colour prediction method.

In existing prediction models, accuracy is reached by increasing the number of colour primaries[1] (where "colour primary" means a uniform ink or ink superposition whose spectrum is known). In most cases, these colour primaries are computed thanks to physical parameters or by measuring a large number of patches. This may imply numerous or sophisticated experiments. The present study aims to accurately predict the spectra and the colours of printed samples, using a reduced number of measurements. In order to limit the number of physical parameters required, the study is done with transparent inks printed on a transparent substrate, thus avoiding all problems related to the diffusion of light and ink in the paper. This restriction guarantees also the validity of Beer's absorption law. Hence, the spectrum of each colour primary, which is a combination of uniformly superposed transparent inks, can be computed using the spectra of the inks.

Our method is a compromise between highly sophisticated physical models, and improvements of the Neugebauer model[4] which are based on a reduced set of colour primaries. In a first approach,[5] we could only predict the CIE-XYZ coordinates of colour samples and the colour primaries were deduced by applying a best fit algorithm on a test set. In our new method, the whole spectrum is computed and only the transmittance spectra of the inks must be measured. This reduces to a minimum the number of experiments to be performed.


A different way of writing Beer's law in relation to the additivity of colours

Assuming that one transparent ink is printed on a transparent substrate, Beer's absorption law[12] gives the transmittance of the printed substrate: $T(\lambda, cd) = \varepsilon(\lambda)^{cd}$ where $\varepsilon(\lambda)$ is a characteristic function of the dye, d the thickness of the ink layer, c the concentration of the dye and λ the light wavelength. In most books, this relation is given for the density $D(\lambda) = -Log_{10}(T(\lambda)) = -cd \cdot Log_{10}(\varepsilon(\lambda)) = cd \cdot D'(\lambda)$, where $D'(\lambda) = -Log_{10}(\varepsilon(\lambda))$ is the reference density of the dye. We use the term "dye" to refer to the light-absorbing chemical substance.

As shown below we can neglect the internal reflectance. From literature[7] we know that the transmittance of a transparent medium is $T(\lambda) = \dfrac{T_{int}(\lambda) \cdot (1-r)^2}{1 - (r \cdot T_{int}(\lambda))^2}$ (see equation (1) in Fig.1) where r is the internal reflection and $T_{int}(\lambda)$ the internal transmittance (see Fig.1). According to Wyszecki,[12] r is small so $T(\lambda)$ can be approximated by $T_{int}(\lambda) \cdot (1-r)^2$. Let $T_W(\lambda) = (1-r)^2 \cdot T_{IntWhite}(\lambda)$ be the transmittance of the clear transparent substrate and let $T_S(\lambda) = (1-r)^2 \cdot T_{Int}(\lambda)$ be the transmittance of a sample. Since our measuring instrument gives us $T_{Meas}(\lambda)$, the transmittance of a sample relative to $T_W(\lambda)$, the coefficient $(1-r)^2$ disappears when the ratio $T_{Meas}(\lambda) = \dfrac{(1-r)^2 \cdot T_{Int}(\lambda)}{(1-r)^2 \cdot T_{IntWhite}(\lambda)}$ is computed.
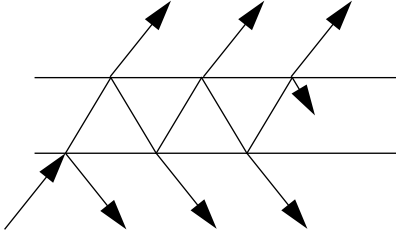
$$T(\lambda) = (1-r)^2 \cdot T_{int}(\lambda) \qquad (1)$$
$$+ (1-r)^2 \cdot T_{int}(\lambda) \cdot (r \cdot T_{int}(\lambda))^2$$
$$+ (1-r)^2 \cdot T_{int}(\lambda) \cdot (r \cdot T_{int}(\lambda))^4 + \ldots$$
$$= \frac{T_{int}(\lambda) \cdot (1-r)^2}{1 - (r \cdot T_{int}(\lambda))^2}$$

Fig.1 Derivation of the transmittance of a transparent substrate

When printing with only one ink, the product cd is never constant. In fact, a wide range of colours based on the same ink are printed side by side.[11] The following mathematical development helps us take this into account.

The classical additivity law of colours printed side by side can be written as a finite weighted sum of n different small patches of colour primaries lying side by side. The resulting predicted transmittance $T_P(\lambda)$ is:

$$T_P(\lambda) = \sum_{i=1}^{n} a_i \cdot T_i(\lambda) \qquad \text{with} \quad \sum_{i=1}^{n} a_i = 1 \qquad (2)$$

where $T_i(\lambda)$ is the transmittance spectra of the ith colour primary and a_i is its respective area coverage. Let us extend this relation to a continuous form by replacing the summation by an integration. The variable $q = cd$ is introduced to simplify notations; q can be interpreted as the amount of dye the light has to go through. The function $a(q)$ replaces the coefficients a_i. By definition, $a(q) \cdot dq$ is the area whose transmittance spectrum is exactly $T(\lambda, q) = \varepsilon(\lambda)^q$. The integral form of equation (2) is:

$$T_P(\lambda) = \int_0^\infty a(q) \cdot T(\lambda, q) dq \qquad \text{with} \quad \int_0^\infty a(q) dq = 1 \qquad (3)$$

Let us denote: $p(\lambda) = D'(\lambda) \ln(10)$.

Now, since:
$$T(\lambda, q) = \varepsilon(\lambda)^q \tag{4}$$
$$= \exp\left(q \ln\left(\varepsilon(\lambda)\right)\right)$$
$$= \exp\left(-q D'(\lambda) \ln\left(10\right)\right)$$

Then equation (3) can be written:
$$T_P(\lambda) = \int_0^\infty a(q) \cdot e^{-qp(\lambda)} dq \tag{5}$$

The Laplace-transform[8] of $a(q)$ is defined as: $L_a(p) = \int_0^\infty a(q) \cdot e^{-qp} dq$. Therefore, the final form of equation (3) is:

$$T_P(\lambda) = L_a(p(\lambda)) = L_a(D'(\lambda) \ln\left(10\right)) \tag{6}$$

This means that $T_P(\lambda)$ is the Laplace-transform of $a(q)$ modulated by the density spectrum of the ink.

Equation (6) can be generalized for several inks. According to Beer's law, when two transparent inks are superposed, the resulting density is the sum of the densities of the two inks: $D(\lambda) = D_1(\lambda) + D_2(\lambda)$. By applying this rule to the above calculations, a similar relation is established between the area coverage, the spectra of the inks and the resulting colour. Here we will just give the equation for three inks (cyan, magenta and yellow):

$$T_P(\lambda) = L_a(D'_c(\lambda) \ln\left(10\right), D'_m(\lambda) \ln\left(10\right), D'_y(\lambda) \ln\left(10\right)) \tag{7}$$

where
$$L_a(p_c, p_m, p_y) = \tag{8}$$
$$\int_0^\infty \int_0^\infty \int_0^\infty a(q_c, q_m, q_y) \cdot e^{-(q_c p_c + q_m p_m + q_y p_y)} dq_c dq_m dq_y$$

$L_a(p_c, p_m, p_y)$ is the three dimensional Laplace-transform[3] of the function $a(q_c, q_m, q_y)$. The expression $a(q_c, q_m, q_y) dq_c dq_m dq_y$ is the fraction of the area where light goes through an amount q_c of cyan, q_m of magenta and q_y of yellow dye.
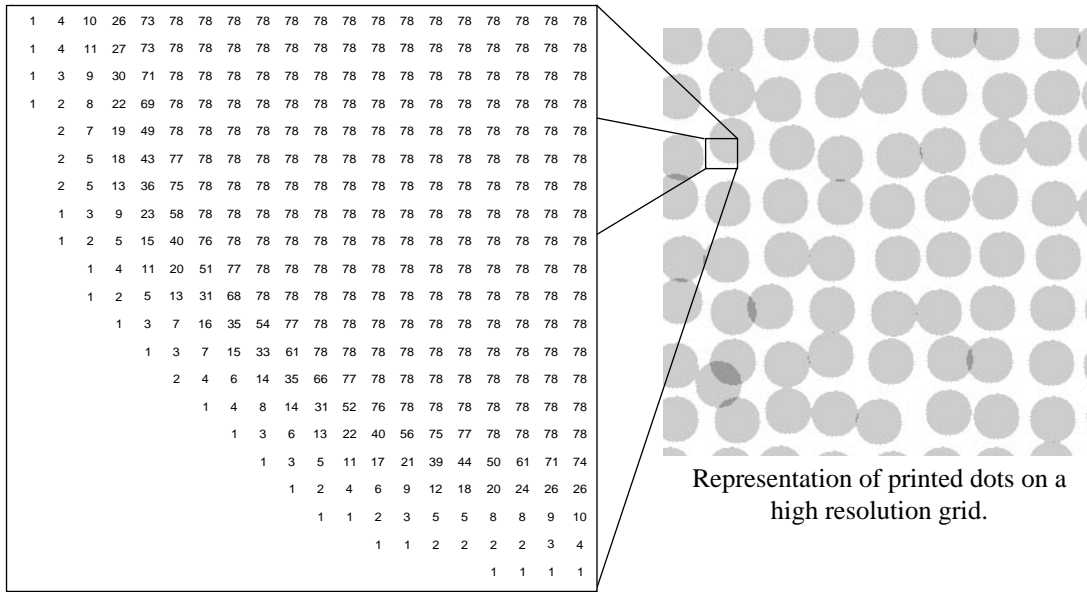$T(\lambda, q_c, q_m, q_y) = e^{-(q_c D'_c(\lambda) \ln\left(10\right) + q_m D'_m(\lambda) \ln\left(10\right) + q_y D'_y(\lambda) \ln\left(10\right))}$ is the transmittance spectrum of the colour primary obtained by superposing an amount q_c of cyan, q_m of magenta and q_y of yellow dye.

In practice, such a mathematical expression is not directly useful to compute the resulting colour but it gives a new insight on the subject. Furthermore, the Laplace-transform is a well known mathematical tool with many interesting properties. This formalism is more general than the traditional weighted sum, and it allows the study of gradually varying ink layers.

The grid-based surface coverage computation method

The key to our method is the function $a(q_c, q_m, q_y) dq_c dq_m dq_y$ which defines the area of a certain colour primary. Earlier approaches of surface coverage computation assumed that all dots were circular with only one density level.[9] In our present approach, this computation can be done for an infinite number of densities. In practice, however, only a finite number of them are taken into account to compute a quantified approximation of $a(q_c, q_m, q_y)$.

We propose the following methodology. At first, the computer needs a model of a single dot. Therefore, the area occupied by a single dot is divided into a grid made of subpixels. When light goes through a subpixel it encounters an amount q of dye which may be different for each subpixel. The value q is stored in a matrix $Q(x, y)$. This dot pattern matrix can be different for each ink.

```
1  4  10 26 73 78 78 78 78 78 78 78 78 78 78 78 78 78 78 78
1  4  11 27 73 78 78 78 78 78 78 78 78 78 78 78 78 78 78 78
1  3  9  30 71 78 78 78 78 78 78 78 78 78 78 78 78 78 78 78
1  2  8  22 69 78 78 78 78 78 78 78 78 78 78 78 78 78 78 78
   2  7  19 49 78 78 78 78 78 78 78 78 78 78 78 78 78 78 78
   2  5  18 43 77 78 78 78 78 78 78 78 78 78 78 78 78 78 78
   2  5  13 36 75 78 78 78 78 78 78 78 78 78 78 78 78 78 78
   1  3  9  23 58 78 78 78 78 78 78 78 78 78 78 78 78 78 78
   1  2  5  15 40 76 78 78 78 78 78 78 78 78 78 78 78 78 78
      1  4  11 20 51 77 78 78 78 78 78 78 78 78 78 78 78 78
      1  2  5  13 31 68 78 78 78 78 78 78 78 78 78 78 78 78
      1  3  7  16 35 54 77 78 78 78 78 78 78 78 78 78 78 78
      1  3  7  15 33 61 78 78 78 78 78 78 78 78 78 78 78 78
      2  4  6  14 35 66 77 78 78 78 78 78 78 78 78 78 78 78
         1  4  8  14 31 52 76 78 78 78 78 78 78 78 78 78 78
         1  3  6  13 22 40 56 75 77 78 78 78 78 78 78 78 78
         1  3  5  11 17 21 39 44 50 61 71 74
            1  2  4  6  9  12 18 20 24 26 26
            1  1  2  3  5  5  8  8  9  10
               1  1  2  2  2  2  3  4
                  1  1  1  1
```

Part of a dot pattern matrix giving the local amounts of dye

Representation of printed dots on a
high resolution grid.

Fig.2 Grid based representation of local amounts of dye in dots

High resolution grids modelize the ink layers, where one grid is used per ink. For each ink layer, the contents of the dot pattern matrix are added to the values in the corresponding high resolution grid wherever a real dot should be printed, according to the halftoning algorithm. For each ink layer, the number in a cell of the resulting grid corresponds to the amount of dye present at that point. Superposing several ink layers means superposing grids. So, for each location, the set of values contained in the cells indicates the amount of each type of dye.

The next step consists of building-up the quantified function $a(q_c, q_m, q_y)$. The value of $a(q_1, q_2, q_3)\Delta q_c \Delta q_m \Delta q_y$ is the number of cells divided by the total number of cells, where $q_c \in [q_1, q_1 + \Delta q_c]$, $q_m \in [q_2, q_2 + \Delta q_m]$ and $q_y \in [q_3, q_3 + \Delta q_y]$. The computer only needs to count the number of cells which satisfy these conditions. The quality of the approximation depends on the quantification step Δq and on the total number of cells.

Results of the spectral and colorimetric predictions

The prediction method we propose needs both the density spectra of the inks and the positions of the printed dots. The reference density spectra of the inks $D'(\lambda)$ are measured on samples obtained by applying the inks uniformly on the transparent substrate. For cyan, magenta and yellow inks, we obtained the following measured transmittance spectra:
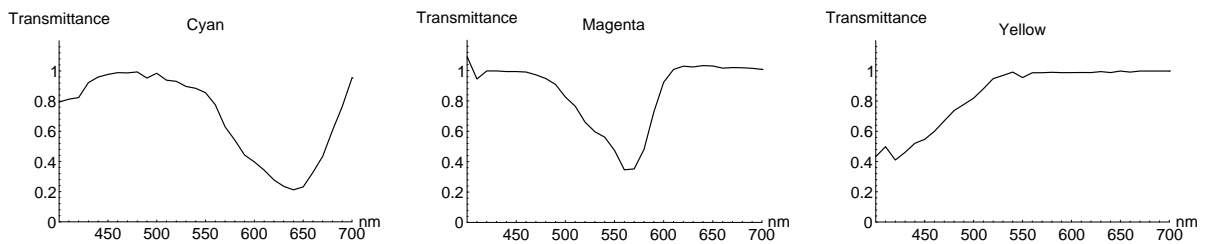

Fig.3 Transmittance spectra of cyan, magenta and yellow ink on transparent substract.

The positions of the printed ink dots depend on the halftoning algorithm. Once the dot positions are known,

we may compute the function $a(q_c, q_m, q_y)$ by using the method described in the previous section. We also need to define the density of a single dot. If a microscope is available, the picture of a single dot enables the dot density to be approximated. Otherwise, we may compute the density as a function of the distance to the dot center on the base of a theoretical model. We use the function plotted in Fig.4b.
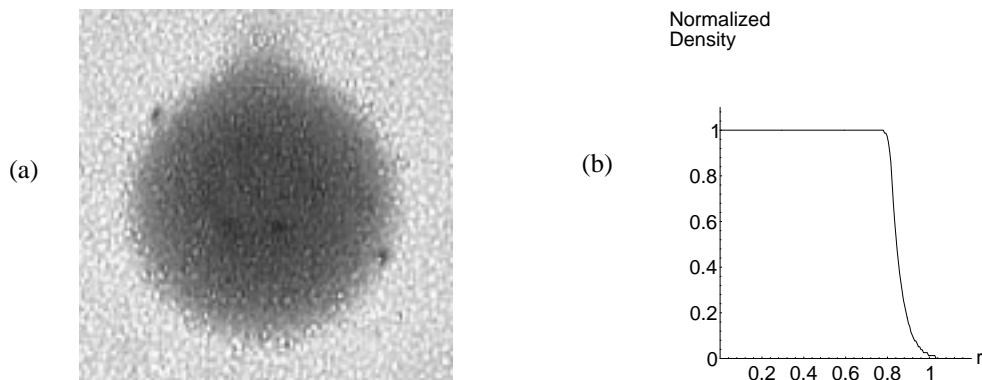

Fig.4 (a) Microscope picture of a single dot and (b) approximation by a radial function

This model was applied to predict the spectra of sample patches for the three following ink combinations: cyan-magenta, cyan-yellow and magenta-yellow. For each pair of inks, 81 samples were produced by combining nine equally spaced intensity levels of the first ink with nine intensity levels of the second ink. These 3x81 patches were printed on transparencies by a HP DJ 560 C ink-jet printer using three different halftoning methods: Bayer's dispersed-dot,[2] a 33 levels clustered dot[6] and the Floyd-Steinberg error diffusion algorithm.[10] For each printed sample both inks were printed dot-on-dot, as far as possible. The spectra of the inks and of the printed samples were measured by transparence using a LightSource Colortron I spectrophotometer with a D50 light source.

The colorimetric deviations between measured and predicted spectra are computed in the CIE-Lab colour space. The results are summarized in Table 1 which gives the mean error, the root mean square error and the maximal deviation. Figure 5 shows measured and predicted spectra of 15 typical samples.

Dithering Method	Ink combination	Mean ΔE	$\sqrt{\dfrac{\sum\limits_{i=1}^{n} (\Delta E)^2}{n}}$	ΔE_{Max}
	Cyan-Magenta	2.36	2.56	4.96
Bayer dispersed dot	Cyan-Yellow	2.64	3.26	8.37
	Magenta-Yellow	1.93	2.22	5.12
	Cyan-Magenta	2.45	2.77	6.28
Clustered dot	Cyan-Yellow	2.84	3.24	6.84
	Magenta-Yellow	1.24	1.42	4.45
	Cyan-Magenta	3.60	4.08	7.32
Floyd-Steinberg error diffusion	Cyan-Yellow	3.45	4.38	9.49
	Magenta-Yellow	2.90	3.31	6.38

Table 1 : Deviations between measured and predicted colours in CIE-Lab

Bayer

Clustered

Floyd-Steinberg

Levels: C=95, M=223 dE=1.11

Levels: C=95, Y=223 dE=2.6

Levels: M=95, Y=223 dE=2.52

Levels: C=159, M=159 dE=2.58

Levels: C=159, Y=159 dE=1.64

Levels: M=159, Y=159 dE=0.83

Levels: C=223, M=95 dE=1.39

Levels: C=223, Y=95 dE=1.38

Levels: M=223, Y=95 dE=0.73

Levels: C=0, M=63 dE=4.76

Levels: C=0, Y=63 dE=5.13

Levels: M=0, Y=63 dE=3.38

Levels: C=63, M=0 dE=3.25

Levels: C=63, Y=0 dE=5.77

Levels: M=63, Y=0 dE=5.04

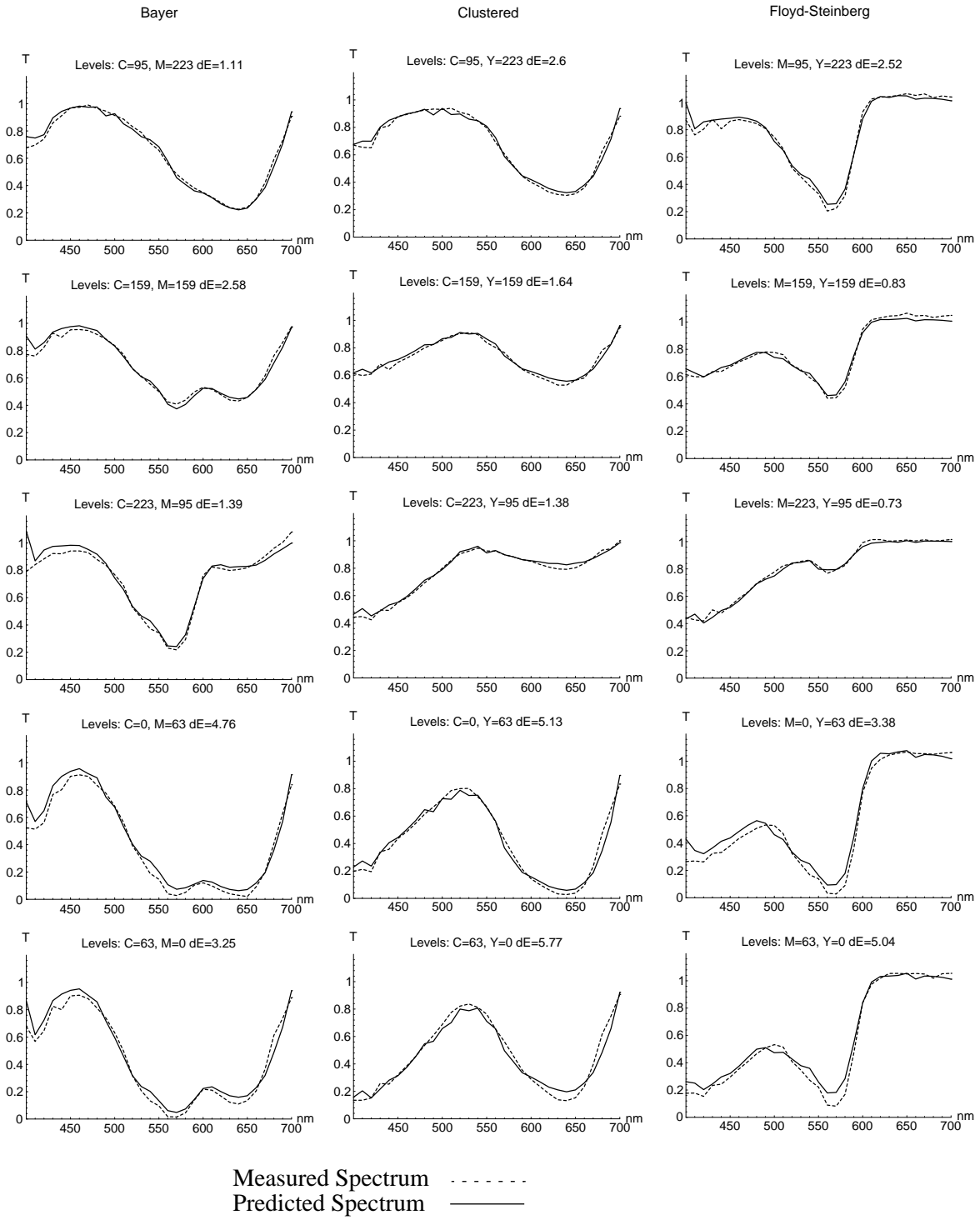Measured Spectrum - - - - - -
Predicted Spectrum ————

Fig.5 Measured and predicted spectra of several samples

Conclusions

Our proposed new spectral prediction method is based on the computation of the transmittance spectra of colour primaries applying Beer's law, and on the calculation of their respective surface coverage using a high resolution grid. This model only requires measuring the transmittance spectra of the inks and approximating the density distribution function of a single dot of each ink.

Using this method we predicted the transmittance spectra of 81 samples for each pair of inks printed with three different dithering methods (Bayer's dispersed-dot,[2] a 33 levels clustered dot[6] and the Floyd-Steinberg error diffusion algorithm[10]). The global average prediction error is about $\Delta E = 2.6$ with a maximum of $\Delta E_{Max} = 9.5$. In contrast to previous prediction methods, we need only one spectrophotometer measurement per ink, and hence it is no longer necessary to measure large sets of patches or to evaluate numerous physical parameters.

The prediction errors can be explained by the violation of Beer's law at certain wavelengths. A more thorough investigation showed that these wavelengths correspond to the emission band of fluorescent substances present in the inks. However, the influence of fluorescence is beyond the scope of the present paper.

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