Supplementary material for Median of 3 Permutations, 3-Cycles and 3-Hitting Set Problem

May 9, 2018

Here we present results on real and randomly generated data as supplementary material for the "Median of 3 Permutations, 3-Cycles and 3 Hitting Set Problem" article. Tables 1 and 2 show results on real data from PrefLib.org [3]. Tables 4 and 5 show results on randomly uniformly generated data. In all cases, Conjecture 2 is holding.

A clear distinction can be seen between those two sources of data: real life data permutations have much more similarity than the randomly uniformly generated ones. Therefore, our approach has a stronger impact on real life data. It is also easier to compute the median permutation for real life data.

For real life data from PrefLib (Tables 1 and 2), we have that, in average, 87.7% of all the pairs of elements were ordered by the 3-cycle theorem. Also, in average, only 3.2% of the maximal number of 3-cycles were present pointing to the efficiency in term of time complexity of our new approach. Table 2 shows effectively that using the 3-cycle theorem constraints with the Integer Linear Programming (ILP) model is, on average, 3.7 time faster then solving the original ILP model and solving the 3-Hitting Set ILP model is, on average, 187.1 time faster then solving the original ILP model.

For uniformly generated random sets (Tables 4 and 5), we have that, in average, 48.5% of all the pairs of elements were ordered by the 3-cycle theorem. Also, in average, 20.4% of the possible 3-cycles were present, which is more than 6 times the quantity we had for real data. This affect our gain in term of time complexity. Table 5 shows effectively that using the 3-cycle theorem constraints with the Integer Linear Programming (ILP) model on random data is, on average, only 1.6 time faster then solving the original ILP model and solving the 3-Hitting Set ILP model is, on average, 19.3 time faster then solving the original ILP model.

The sets of permutations used for to compute all the statistics of Table 1 to Table 5 can be all found at http://www.iro.umontreal.ca/∼hamelsyl/M3P_3Cycles_3HS.html.

		A	B		C	D				A	B		C	D	
ID	n	# 3-cycles	max # 3-cycles	A/B	nb constr.	max nb constr.	C/D	ID	n	# 3-cycles	max # 3-cycles	A/B	nb constr.	max nb constr.	C/D
ex1	29	78	1015	7.7%	302	406	74.4%	ex36	44	255	3542	7.2%	675	946	71.4%
ex2	20	2	330	0.6%	185	190	97.4%	ex37	18	0	240	0%	153	153	100%
ex3	44	0	3542	0%	946	946	100%	ex38	17	23	204	11.3%	98	136	72.1%
ex4	64	124	10912	1.1%	1848	2016	91.7%	ex39	40	0	2660	0%	780	780	100%
ex5	24	0	572	0%	276	276	100%	ex40	73	203	16206	1.3%	2378	2628	90.5%
ex6	67	1116	12529	8.9%	1457	2211	65.9%	ex41	67	1203	12529	9.6%	1403	2211	63.5%
ex7	23	1	506	0.2%	250	253	98.8%	ex42	12	2	70	2.9%	61	66	92.4%
ex8	42	173	3080	5.6%	662	861	76.9%	ex43	18	6	240	2.5%	141	153	92.2%
ex9	28	36	910	4%	326	378	86.2%	ex44	45	14	3795	0.4%	962	990	97.2%
ex10	11	9	55	16.4%	39	55	70.9%	ex45	82	1444	22960	6.3%	2282	3321	68.7%
ex11	70	92	14280	0.6%	2283	2415	94.5%	ex46	26	9	728	1.2%	306	325	94.2%
ex12	67	873	12529	6.7%	1524	2211	68.9%	ex47	36	157	1938	8.1%	449	630	71.3%
ex13	63	63	10416	0.6%	1856	1953	95%	ex48	70	23	14280	0.2%	2380	2415	98.6%
ex14	23	0	506	0%	253	253	100%	ex49	81	1317	22140	6%	2303	3240	71.1%
ex15	43	4	3311	0.1%	894	903	99%	ex50	52	0	5850	0%	1326	1326	100%
ex16	21	10	385	2.6%	187	210	89%	ex51	89	1555	29370	5.3%	2796	3916	71.4%
ex17	14	0	112	0%	91	91	100%	ex52	18	0	240	0%	153	153	100%
ex18	23	0	506	0%	253	253	100%	ex53	68	1293	13090	9.9%	1423	2278	62.5%
ex19	40	0	2660	0%	780	780	100%	ex54	37	3	2109	0.1%	659	666	98.9%
ex20	52	314	5850	5.4%	1046	1326	78.9%	ex55	23	8	506	1.6%	236	253	93.3%
ex21	17	7	204	3.4%	122	136	89.7%	ex56	87	2064	27434	7.5%	2340	3741	62.6%
ex22	23	1	506	0.2%	250	253	98.8%	ex57	32	35	1360	2.6%	433	496	87.3%
ex23	72	6	15540	0.04%	2544	2556	99.5%	ex58	60	669	8990	7.4%	1207	1770	68.2%
ex24	20	6	330	1.8%	176	190	92.6%	ex59	41	94	2870	3.3%	697	820	85%
ex25	14	0	112	0%	91	91	100%	ex60	23	0	506	0%	253	253	100%
ex26	55	600	6930	8.7%	960	1485	64.6%	ex61	38	0	2280	0%	703	703	100%
ex27	77	1319	19019	6.9%	1978	2926	67.6%	ex62	30	0	1120	0%	435	435	100%
ex28	56	542	7308	7.4%	1032	1540	67%	ex63	19	0	285	0%	171	171	100%
ex29	32	8	1360	0.6%	481	496	97%	ex64	19	2	285	0.7%	166	171	97.1%
ex30	71	554	14910	3.7%	1963	2485	79%	ex65	20	1	330	0.3%	187	190	98.4%
ex31	41	162	2870	5.6%	636	820	77.6%	ex66	18	3	240	1.3%	145	153	94.8%
ex32	24	0	572	0%	276	276	100%	ex67	55	415	6930	6%	1088	1485	73.3%
ex33	18	10	240	4.2%	133	153	86.9%	ex68	21	22	385	5.7%	175	210	83.3%
ex34	69	0	13685	0%	2346	2346	100%	ex69	20	0	330	0%	190	190	100%
ex35	24	2	572	0.4%	271	276	98.2%	ex70	30	96	1120	8.6%	319	435	73.3%

Table 1: Statistics on real data from PrefLib.org[3]. Table 3 gives the correspondance between each example ID and the original file name of this example on the site PrefLib.org, as well as the number of permutations in that example. For each of them, 3 permutations were randomly drawn to compute the statistics of this table. For each example, n represents the length of the permutations, column A gives the number of 3-cycles found in this example, column B gives the maximal number of 3-cycles that could be present in an example of size n (see [1] for the formulas for odd and even length cases), column A/B gives the proportion, in %, of 3-cycles found compared to the maximum number of 3-cycles, column C gives the number of constraints $i \prec j$, $1 \leq i \neq j \leq n$ found, column D gives the maximal number of constraints to be found for permutations of size n and finally column C/D gives the % of constraints found.

		A	B		C				A	B		C	
ID	n	time ILP normal	time ILP 3 cyc.thm.	acc. factor B/A	time 3HS	acc. factor C/A	ID	n	time ILP normal	time ILP 3 cyc.thm.	acc. factor B/A	time 3HS	acc. factor C/A
ex1	29	57.033	23.992	2.4	1.507	37.8	ex36	44	204.593	82.487	2.5	4.353	47.0
ex2	20	17.791	4.582	3.9	0.285	62.4	ex37	18	12.574	2.312	5.4	0.041	304.0
ex3	44	192.875	35.615	5.4	0.241	799.8	ex38	17	10.927	4.135	2.6	0.534	20.5
ex4	64	622.724	192.885	3.2	4.915	126.7	ex39	40	145.282	26.675	5.5	0.199	728.9
ex5	24	30.349	5.614	5.4	0.072	419.6	ex40	73	931.125	269.027	3.5	6.509	143.1
ex6	67	995.763	388.635	2.6	28.514	34.9	ex41	67	854.930	425.523	2.0	46.834	18.3
ex7	23	27.296	7.602	3.6	0.369	74.0	ex42	12	3.659	0.956	3.8	0.107	34.1
ex8	42	179.002	64.705	2.8	3.311	54.1	ex43	18	12.958	3.310	3.9	0.261	49.7
ex9	28	50.616	15.729	3.2	0.999	50.7	ex44	45	212.428	57.061	3.7	1.573	135.1
ex10	11	2.792	1.120	2.5	0.142	19.7	ex45	82	1662.284	868.206	1.9	30.926	53.8
ex11	70	816.276	217.777	3.8	6.523	125.1	ex46	26	39.927	10.785	3.7	0.517	77.3
ex12	67	2525.575	551.535	4.6	67.437	37.5	ex47	36	113.438	46.559	2.4	2.693	42.1
ex13	63	594.110	158.109	3.8	5.285	112.4	ex48	70	813.056	202.603	4.0	4.308	188.7
ex14	23	26.629	4.925	5.4	0.067	400.2	ex49	81	1536.425	619.233	2.5	31.926	48.1
ex15	43	185.888	45.939	4.1	1.331	139.7	ex50	52	319.479	59.188	5.4	0.337	948.2
ex16	21	20.769	7.250	2.9	0.355	58.5	ex51	89	1781.754	725.566	2.5	27.735	64.2
ex17	14	5.808	1.061	5.5	0.026	225.5	ex52	18	12.571	2.311	5.4	0.041	304.0
ex18	23	26.614	4.925	5.4	0.067	400.0	ex53	68	823.985	409.850	2.0	23.489	35.1
ex19	40	145.265	26.685	5.4	0.199	728.8	ex54	37	118.024	29.106	4.1	0.974	121.2
ex20	52	570.203	162.058	3.5	6.082	93.8	ex55	23	27.455	7.360	3.7	0.428	64.2
ex21	17	10.856	4.129	2.6	0.294	36.9	ex56	87	5203.084	873.588	6.0	41.691	124.8
ex22	23	27.306	6.977	3.9	0.369	74.1	ex57	32	76.113	23.250	3.3	1.162	65.5
ex23	72	882.952	218.870	4.0	3.876	227.8	ex58	60	539.151	237.575	2.3	10.432	51.7
ex24	20	17.831	5.519	3.2	0.308	58.0	ex59	41	164.004	51.911	3.2	2.498	65.7
ex25	14	5.825	1.063	5.5	0.026	226.2	ex60	23	26.613	4.916	5.4	0.067	400.0
ex26	55	410.391	185.020	2.2	9.666	42.5	ex61	38	124.144	22.813	5.4	0.180	690.0
ex27	77	1154.598	499.389	2.3	22.834	50.6	ex62	30	60.214	11.105	5.4	0.112	535.8
ex28	56	445.740	192.977	2.3	9.181	48.6	ex63	19	14.818	2.734	5.4	0.046	323.0
ex29	32	75.622	19.030	4.0	0.796	95.0	ex64	19	15.238	4.489	3.4	0.258	59.0
ex30	71	865.348	319.743	2.7	11.106	77.9	ex65	20	17.816	4.976	3.6	0.279	64.0
ex31	41	166.500	72.933	2.3	4.831	34.5	ex66	18	12.928	3.963	3.3	0.237	54.6
ex32	24	30.347	5.614	5.4	0.072	419.6	ex67	55	405.842	161.717	2.5	6.858	59.2
ex33	18	12.944	4.682	2.8	0.346	37.4	ex68	21	22.442	8.917	2.5	0.553	40.6
ex34	69	748.569	139.640	5.4	0.594	1260.0	ex69	20	17.346	3.205	5.4	0.051	342.3
ex35	24	31.121	7.994	3.9	0.409	76.1	ex70	30	97.063	36.056	2.7	3.651	26.6

Table 2: Deterministic (CPLEX's ticks) time to solve each example of Table 1. For each example, n represents the length of the permutations, column A gives the time required to solve the Integer Linear Programming (ILP) model of this example, column B gives the time required to solve the ILP model with constraints added from the 3-cycle theorem, column B/A gives the acceleration factor of ILP + 3-cycles with respect to ILP, column C gives the time required to solve the ILP model of the 3-Hitting Set Problem related to this example and finally, column C/A gives the acceleration factor of the 3-Hitting Set Problem with respect to the ILP model.

ID	PrefLib.org file name	m	ID	PrefLib.org file name	m
ex1	ED-00015-00000063.soc	4	ex36	ED-00015-00000056.soc	4
ex2	ED-00006-00000044.soc	9	ex37	ED-00006-00000035.soc	9
ex3	ED-00015-00000076.soc	4	ex38	ED-00015-00000071.soc	4
ex4	ED-00015-00000030.soc	4	ex39	ED-00015-00000046.soc	4
ex5	ED-00006-00000048.soc	9	ex40	ED-00015-00000057.soc	4
ex6	ED-00015-00000031.soc	4	ex41	ED-00015-00000037.soc	4
ex7	ED-00006-00000033.soc	9	ex42	ED-00015-00000078.soc	4
ex8	ED-00015-00000075.soc	4	ex43	ED-00006-00000036.soc	9
ex9	ED-00015-00000047.soc	4	ex44	ED-00015-00000044.soc	4
ex10	ED-00012-00000001.soc	30	ex45	ED-00015-00000026.soc	4
ex11	ED-00015-00000016.soc	4	ex46	ED-00015-00000050.soc	4
ex12	ED-00015-00000070.soc	4	ex47	ED-00015-00000073.soc	4
ex13	ED-00015-00000011.soc	4	ex48	ED-00015-00000041.soc	4
ex14	ED-00006-00000034.soc	9	ex49	ED-00015-00000069.soc	4
ex15	ED-00015-00000064.soc	4	ex50	ED-00015-00000066.soc	4
ex16	ED-00015-00000058.soc	4	ex51	ED-00015-00000039.soc	4
ex17	ED-00006-00000003.soc	9	ex52	ED-00006-00000021.soc	7
ex18	ED-00006-00000008.soc	9	ex53	ED-00015-00000035.soc	4
ex19	ED-00015-00000065.soc	4	ex54	ED-00015-00000062.soc	4
ex20	ED-00015-00000055.soc	4	ex55	ED-00015-00000053.soc	4
ex21	ED-00015-00000072.soc	4	ex56	ED-00015-00000019.soc	4
ex22	ED-00006-00000007.soc	9	ex57	ED-00015-00000045.soc	4
ex23	ED-00015-00000060.soc	4	ex58	ED-00015-00000054.soc	4
ex24	ED-00015-00000074.soc	4	ex59	ED-00015-00000061.soc	4
ex25	ED-00006-00000004.soc	9	ex60	ED-00006-00000032.soc	9
ex26	ED-00015-00000034.soc	4	ex61	ED-00015-00000049.soc	4
ex27	ED-00015-00000051.soc	4	ex62	ED-00006-00000046.soc	7
ex28	ED-00015-00000077.soc	4	ex63	ED-00006-00000037.soc	9
ex29	ED-00015-00000068.soc	4	ex64	ED-00006-00000029.soc	9
ex30	ED-00015-00000010.soc	4	ex65	ED-00006-00000011.soc	9
ex31	ED-00015-00000079.soc	4	ex66	ED-00006-00000022.soc	7
ex32	ED-00006-00000018.soc	9	ex67	ED-00015-00000059.soc	4
ex33	ED-00015-00000043.soc	4	ex68	ED-00015-00000052.soc	4
ex34	ED-00015-00000015.soc	4	ex69	ED-00006-00000012.soc	9
ex35	ED-00006-00000028.soc	9	ex70	ED-00015-00000067.soc	4

Table 3: Correspondance between the IDs of the examples in Table 1 and 2 and their PrefLib.org file name. The column m gives, for each example, its original number of permutations, from which 3 permutations were randomly chosen to compute the data of Table 1 and 2.

		A	B		C	D				A	B		C	D	
ID	n	# 3-cycles	max # 3-cycles	A/B	nb constr.	max nb constr.	C/D	ID	n	# 3-cycles	max # 3-cycles	A/B	nb constr.	max nb constr.	C/D
ex1	29	123	1015	12.1%	260	406	64%	ex36	44	833	3542	23.5%	357	946	37.7%
ex2	20	58	330	17.6%	105	190	55.3%	ex37	18	40	240	16.7%	96	153	62.8%
ex3	44	773	3542	21.8%	430	946	45.5%	ex38	17	25	204	12.3%	94	136	69.1%
ex4	64	2356	10912	21.6%	840	2016	41.7%	ex39	40	443	2660	16.7%	381	780	48.9%
ex5	24	139	572	24.3%	126	276	45.7%	ex40	73	4549	16206	28.1%	768	2628	29.2%
ex6	67	1839	12529	14.7%	1043	2211	47.2%	ex41	67	2844	12529	22.7%	750	2211	33.9%
ex7	23	97	506	19.2%	140	253	55.3%	ex42	12	6	70	8.6%	53	66	80.3%
ex8	42	566	3080	18.4%	402	861	46.7%	ex43	18	29	240	12.1%	106	153	69.3%
ex9	28	145	910	15.9%	219	378	57.9%	ex44	45	954	3795	25.1%	368	990	37.2%
ex10	11	7	55	12.7%	42	55	76.4%	ex45	82	5938	22960	25.9%	1143	3321	34.4%
ex11	70	3331	14280	23.3%	824	2415	34.1%	ex46	26	165	728	22.7%	142	325	43.7%
ex12	67	2846	12529	22.7%	833	2211	37.7%	ex47	36	379	1938	19.6%	307	630	48.7%
ex13	63	1769	10416	17%	908	1953	46.5%	ex48	70	2767	14280	19.4%	966	2415	40%
ex14	23	155	506	30.6%	103	253	40.7%	ex49	81	5358	22140	24.2%	1150	3240	35.5%
ex15	43	566	3311	17.1%	449	903	49.7%	ex50	52	905	5850	15.5%	640	1326	48.3%
ex16	21	100	385	26%	100	210	47.6%	ex51	89	8193	29370	27.9%	1190	3916	30.4%
ex17	14	17	112	15.2%	61	91	67%	ex52	18	72	240	30%	73	153	47.7%
ex18	23	122	506	24.1%	119	253	47%	ex53	68	2575	13090	19.7%	944	2278	41.4%
ex19	40	507	2660	19.1%	374	780	48%	ex54	37	475	2109	22.5%	305	666	45.8%
ex20	52	832	5850	14.2%	689	1326	52%	ex55	23	110	506	21.7%	133	253	52.6%
ex21	17	38	204	18.6%	84	136	61.8%	ex56	87	5744	27434	20.9%	1379	3741	36.9%
ex22	23	85	506	16.8%	155	253	61.3%	ex57	32	296	1360	21.8%	220	496	44.4%
ex23	72	3398	15540	21.9%	1021	2556	40%	ex58	60	1035	8990	11.5%	955	1770	54%
ex24	20	32	330	9.7%	138	190	72.6%	ex59	41	560	2870	19.5%	394	820	48.1%
ex25	14	17	112	15.2%	59	91	64.8%	ex60	23	109	506	21.5%	119	253	47%
ex26	55	2112	6930	30.5%	492	1485	33.1%	ex61	38	453	2280	19.9%	344	703	48.9%
ex27	77	5584	19019	29.4%	805	2926	27.5%	ex62	30	224	1120	20%	206	435	47.4%
ex28	56	1575	7308	21.6%	600	1540	39%	ex63	19	57	285	20%	81	171	47.4%
ex29	32	317	1360	23.3%	228	496	46%	ex64	19	59	285	20.7%	95	171	55.6%
ex30	71	3342	14910	22.4%	889	2485	35.8%	ex65	20	64	330	19.4%	112	190	59%
ex31	41	514	2870	17.9%	397	820	48.4%	ex66	18	31	240	12.9%	101	153	66%
ex32	24	159	572	27.8%	108	276	39.1%	ex67	55	1818	6930	26.2%	533	1485	35.9%
ex33	18	33	240	13.8%	99	153	64.7%	ex68	21	82	385	21.3%	103	210	49.1%
ex34	69	2835	13685	20.7%	984	2346	41.9%	ex69	20	79	330	23.9%	97	190	51.1%
ex35	24	231	572	40.4%	89	276	32.3%	ex70	30	197	1120	17.6%	231	435	53.1%

Table 4: Statistics on randomly uniformly generated data. The size n of each example corresponds to the size of the examples on real data of Table 1. For this n, three permutations were randomly and uniformly generated using the Fisher-Yates shuffle [2]. Column A gives the number of 3-cycles found in this example, column B gives the maximal number of 3-cycles that could be present in an example of size n (see [1] for the formulas for odd and even length cases), column A/B gives the proportion, in %, of 3-cycles found compared to the maximum number of 3-cycles, column C gives the number of constraints $i \prec j$, $1 \leq i \neq j \leq n$ found, column D gives the maximal number of constraints to be found for permutations of size n and finally column C/D gives the % of constraints found.

		A	B		C				A	B		C	
ID	n	time ILP normal	time ILP 3 cyc.thm.	A/B	time 3HS	A/C	ID	n	time ILP normal	time ILP 3 cyc.thm.	A/B	time 3HS	A/C
ex1	29	57.309	29.520	1.9	1.99	28.8	ex36	44	258.971	189.261	1.4	15.877	16.3
ex2	20	18.123	11.007	1.7	0.819	22.1	ex37	18	13.032	6.505	2.0	0.649	20.1
ex3	44	247.767	159.238	1.6	13.751	18.0	ex38	17	10.836	4.743	2.3	0.389	27.9
ex4	64	1187.455	904.222	1.3	141.036	8.4	ex39	40	172.935	107.624	1.6	6.861	25.2
ex5	24	31.999	24.004	1.3	1.940	16.5	ex40	73	10639.121	8172.505	1.3	866.353	12.3
ex6	67	926.143	571.386	1.6	31.726	29.2	ex41	67	1269.947	866.333	1.5	76.922	16.5
ex7	23	27.796	19.054	1.5	1.348	20.6	ex42	12	3.658	1.171	3.1	0.132	27.8
ex8	42	208.341	132.143	1.6	9.284	22.4	ex43	18	13.053	5.759	2.3	0.562	23.2
ex9	28	52.716	30.045	1.8	2.426	21.7	ex44	45	295.712	247.429	1.2	53.682	5.5
ex10	11	2.806	0.942	3.0	0.184	15.3	ex45	82	46019.661	55961.999	0.8	4206.247	10.9
ex11	70	3889.091	1407.340	2.8	415.431	9.4	ex46	26	41.819	30.889	1.4	2.260	18.5
ex12	67	1144.591	929.713	1.2	75.092	15.2	ex47	36	124.236	80.875	1.5	5.575	22.3
ex13	63	883.163	589.764	1.5	48.775	18.1	ex48	70	2657.183	1048.878	2.5	83.453	31.8
ex14	23	28.464	23.320	1.2	2.012	14.2	ex49	81	4548.700	4060.120	1.1	953.759	4.8
ex15	43	403.926	194.758	2.1	17.374	23.3	ex50	52	372.485	244.676	1.5	15.148	24.6
ex16	21	21.823	16.694	1.3	1.411	15.5	ex51	89	702800.102	461614.250	1.5	69214.169	10.2
ex17	14	5.995	2.715	2.2	0.319	18.8	ex52	18	13.507	8.975	1.5	1.057	12.8
ex18	23	28.289	19.712	1.4	1.700	16.6	ex53	68	1124.715	797.670	1.4	66.016	17.0
ex19	40	165.408	104.434	1.6	7.104	23.3	ex54	37	129.996	85.142	1.5	5.832	22.3
ex20	52	747.845	222.887	3.4	24.697	30.3	ex55	23	28.334	19.386	1.5	1.569	18.1
ex21	17	11.039	5.470	2.0	0.637	17.3	ex56	87	4591.207	3709.316	1.2	522.010	8.8
ex22	23	27.823	17.738	1.6	1.204	23.1	ex57	32	80.648	65.864	1.2	4.131	19.5
ex23	72	1863.480	1519.209	1.2	236.237	7.9	ex58	60	792.506	375.007	2.1	160.510	4.9
ex24	20	17.922	7.123	2.5	0.607	29.5	ex59	41	515.166	276.698	1.9	8.181	63.0
ex25	14	5.950	2.626	2.3	0.240	24.8	ex60	23	28.381	20.756	1.4	1.651	17.2
ex26	55	1447.574	580.477	2.5	49.691	29.1	ex61	38	136.756	90.964	1.5	5.884	23.2
ex27	77	4475.162	4385.677	1.0	1059.166	4.2	ex62	30	65.561	44.190	1.5	3.031	21.6
ex28	56	578.007	799.169	0.7	30.233	19.1	ex63	19	15.367	10.678	1.4	0.803	19.1
ex29	32	88.696	60.937	1.5	5.039	17.6	ex64	19	16.054	9.492	1.7	0.915	17.6
ex30	71	1880.580	1539.885	1.2	216.318	8.7	ex65	20	18.177	9.526	1.9	0.995	18.3
ex31	41	306.660	173.185	1.8	7.959	38.5	ex66	18	12.980	6.390	2.0	0.521	25.0
ex32	24	31.513	25.869	1.2	2.249	14.0	ex67	55	645.411	488.770	1.3	36.521	17.7
ex33	18	13.028	6.195	2.1	0.540	24.1	ex68	21	21.029	14.649	1.4	1.214	17.3
ex34	69	1359.295	1082.704	1.3	141.539	9.6	ex69	20	18.476	12.420	1.5	1.291	14.3
ex35	24	39.692	32.967	1.2	3.024	13.1	ex70	30	67.788	38.599	1.8	2.983	22.7

Table 5: Deterministic (CPLEX's ticks) time to solve each example of Table 4. For each example, n represents the length of the permutations, column A gives the time required to solve the Integer Linear Programming (ILP) model of this example, column B gives the time required to solve the ILP model with constraints added from the 3-cycle theorem, column B/A gives the acceleration factor of ILP + 3-cycles with respect to ILP, column C gives the time required to solve the ILP model of the 3-Hitting Set Problem related to this example and finally, column C/A gives the acceleration factor of the 3-Hitting Set Problem with respect to the ILP model.

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