Towards a Biologically Plausible Replacement for Back-Prop: Target-Prop

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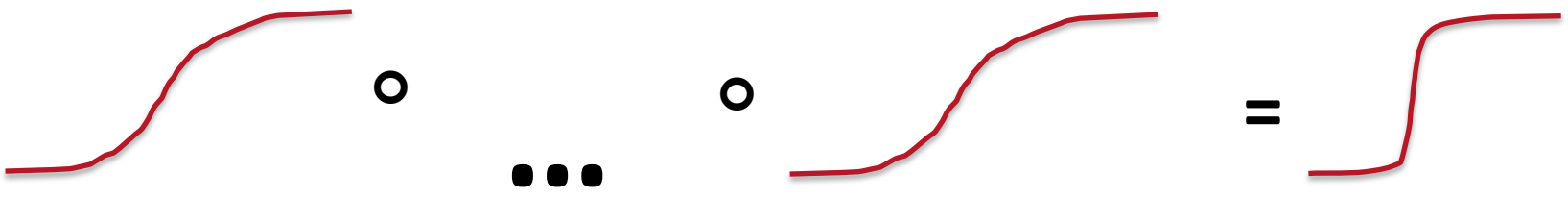
PLUG: **Deep Learning**, MIT Press book in preparation, draft chapters online for feedback

Issues with Back-Prop

- Over very deep nets or recurrent nets with many steps, non-linearities compose and yield sharp non-linearity → gradients vanish or explode
- Training deeper nets: harder optimization
- In the extreme of non-linearity: discrete functions, can't use back-prop
- ***Biological plausibility?***



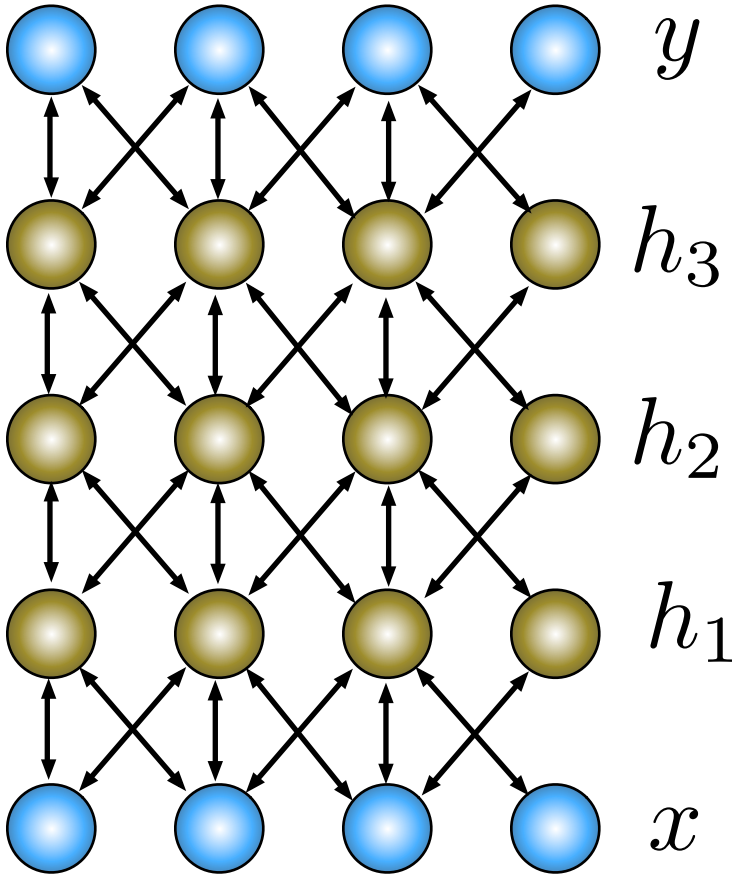
Biological Implausibility of Backprop vs Targetprop

- Not quite local, and not quite neural backprop operations:
- ✔ Purely linear operation
- ✔ Needs precise knowledge of derivative of fprop nonlinearity
- ✔ Spikes?
- ✔ Non-local requirement: "weight transport"
  - The w_{ij} used in fprop network must match the w_{ij} used in the bprop (feedback) network, i.e., symmetric weights → Auto-encoders?
- ✔ Where is the target coming from? → Other modalities & rewards
- ✔ Needs a clock to alternate between fprop and bprop
- BPTT is even worse (storing sequence of all activations and running gradients backwards in time) → Future work…

Previous work: Boltzmann machine

(Hinton et al, Salakhutdinov et al)

- Not yet successful but biologically reasonable algorithm
  - Clamp *x* and *y*
  - Stochastic relaxation
  - Measure and add pre *x* post
  - Release *y* (or both *x* and *y*)
  - Stochastic relaxation
  - Measure and subtract pre *x* post
- Sleep phase: both *x* and *y* released.
- Needs symmetric weights



Issues with Undirected Graphical Models & Boltzmann Machines

- Sampling from the MCMC of the model is required in the inner loop of training
- As the model gets sharper, mixing between well-separated modes stalls

REINFORCE & Stochastic Perturbation

(Williams 1992, "Simple Statistical Gradient-Following Algorithms Connectionist Reinforcement Learning")
(Fiete & Seung 2006, "Gradient learning in spiking neural networks by dynamic perturbations of conductances")

- Correlating reward (or log-likelihood) with stochastic perturbations gives rise to a gradient estimator and a learning algorithm that can be applied to spiking neural networks

- Very inefficient (even with variance reduction) and does not scale well with size of the network, whereas backprop does

Deterministic Relaxation

(Xie & Seung 2003, "Equivalence of backpropagation and contrastive Hebbian learning in a layered network")

- **Contrastive Hebbian learning**
  - Similar to deep Boltzmann machine but deterministic
  - Also needs symmetric weights

- Equivalence to back-prop shown in the case where a fixed point of the relaxation is reached (for both cases of free Y and clamped Y) and the feedback weights are weak (to obtain derivatives by Taylor expansion)

- Is a single step or a few steps enough?

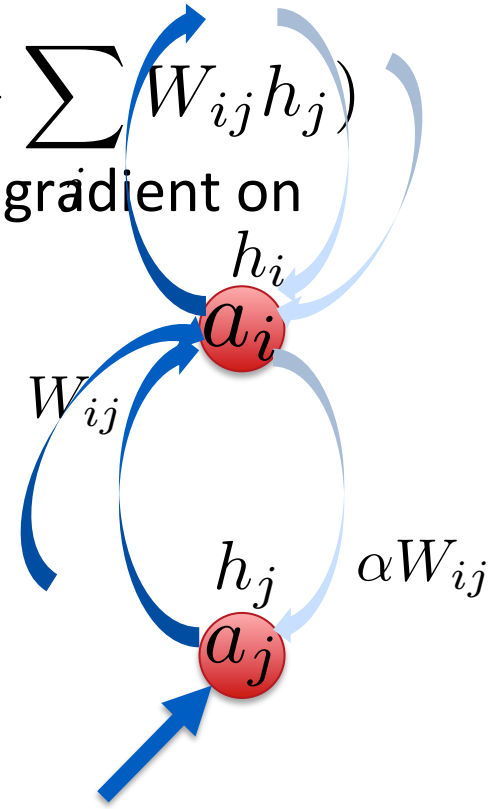
Temporal Derivative = Loss Gradient

(Hinton, NIPS DL Workshop 2007 talk, "How to do backpropagation in a brain")

- Weak **feedback weights are symmetric** and are used to propagate gradients
- Non-linear activation: $h_i = \sigma(a_i) = \sigma(b_i + \sum W_{ij} h_j)$
- Temporal derivative of spiking rate represents gradient on potential: $\frac{\partial h_i}{\partial t} = \frac{\partial C}{\partial a_i}$
- **Weak feedback** influence goes through σ

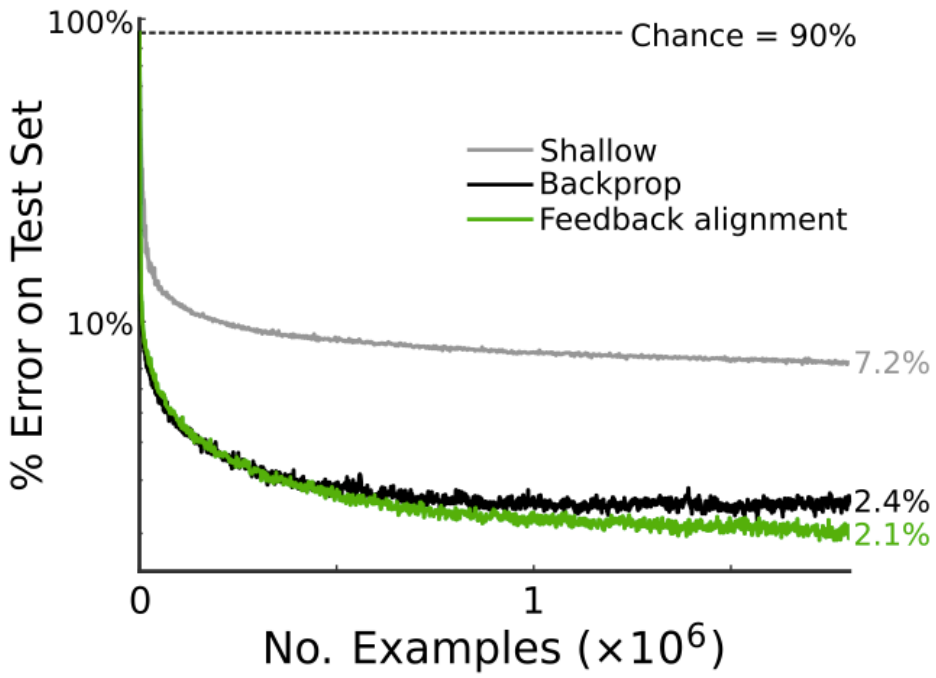
and becomes amplified by $W'_{ji}\sigma'(a_j)$ to yield temporal change in h_j

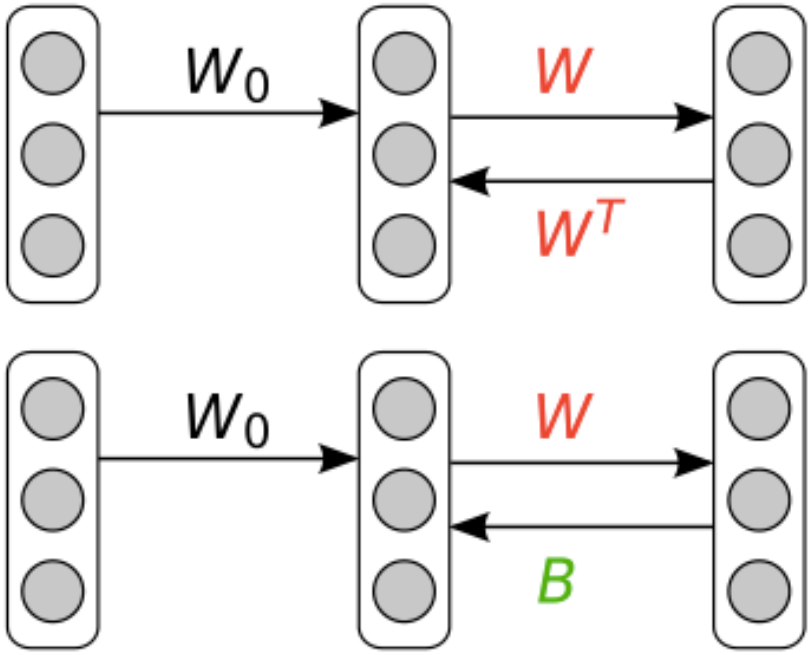
- STDP update = delta-rule (pre *x* d/dt post)



Feedback Alignment

- *(Lillicrap et al arxiv 2014, "Random feedback weights support learning in deep networks")*
- Feedback weights B (instead of W^T in backprop) are fixed
- As good as back-prop on not too deep nets: angle(W,B) decreases during training (but does not reach 0)





Generalizing gradients to non-differentiable credit assignment: Target Propagation

- Gradient = in which direction each unit should make infinitesimal change towards reducing loss function

not infinitesimal

- Target = **nearby value** which would yield a smaller loss

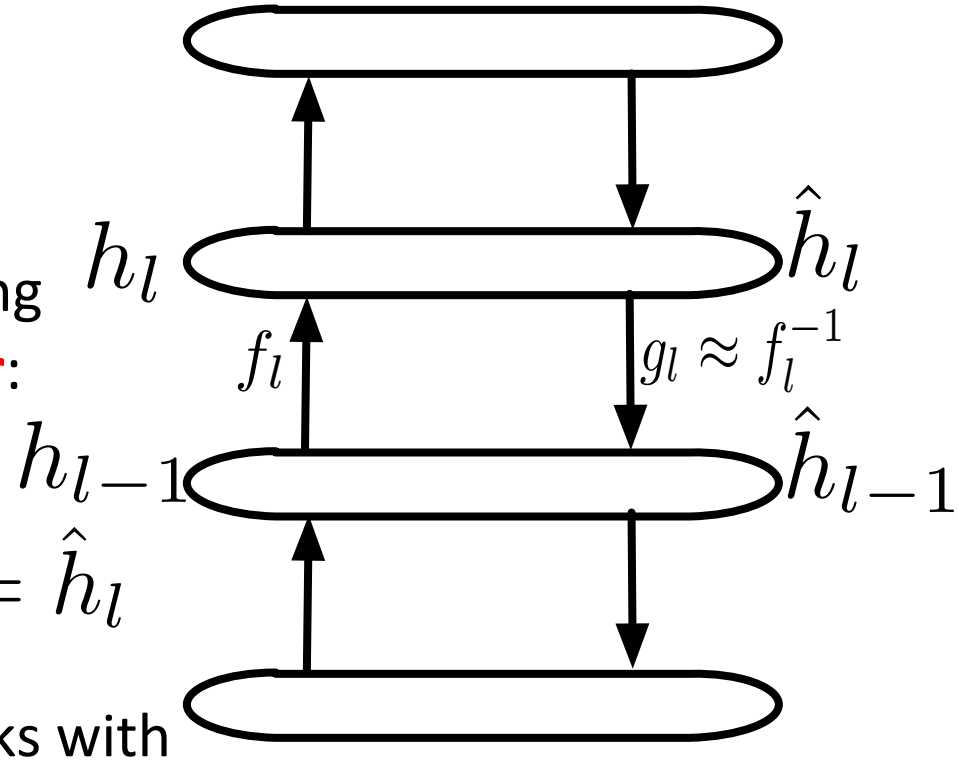
- When target - value is small → equivalent to backprop via Lagrange multipliers (LeCun thesis, 1986, 1987)

- Principle of target propagation can work for discrete values

Target Prop (Bengio 2014, arXiv 1407.7906)

- Instead of propagating the effect of an infinitesimal change, propagate a target that would be
  - Near the original value
  - Yielding to a lower loss
- Can be obtained by maintaining each layer as an **auto-encoder**:

To deal with highly non-linear or even discrete functions



good target $\hat{h}_{l-1}$ s.t.

$$f_l(\hat{h}_{l-1}) = f_l(g_l(\hat{h}_l)) = \hat{h}_l$$

- Preliminary experiments: works with correction for imperfect inverse:

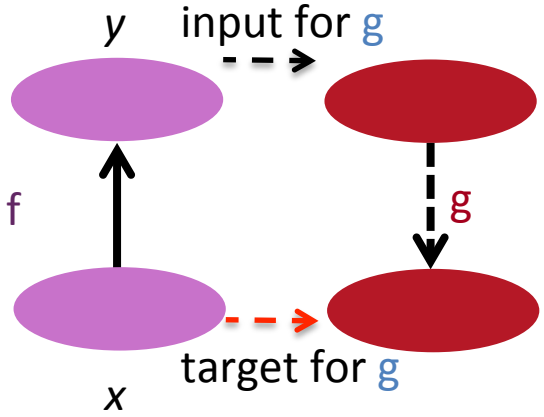
$$\hat{h}_{l-1} = h_{l-1} + g_l(\hat{h}_l) - g_l(h_l)$$

How to train an auto-encoder without backprop

- To learn a shallow auto-encoder **without backprop**: could be potentially applied to discrete units, biologically more plausible
- If you observe the output $y=f(x)$ for some x, that gives you a training example

  (input = y, target = x)

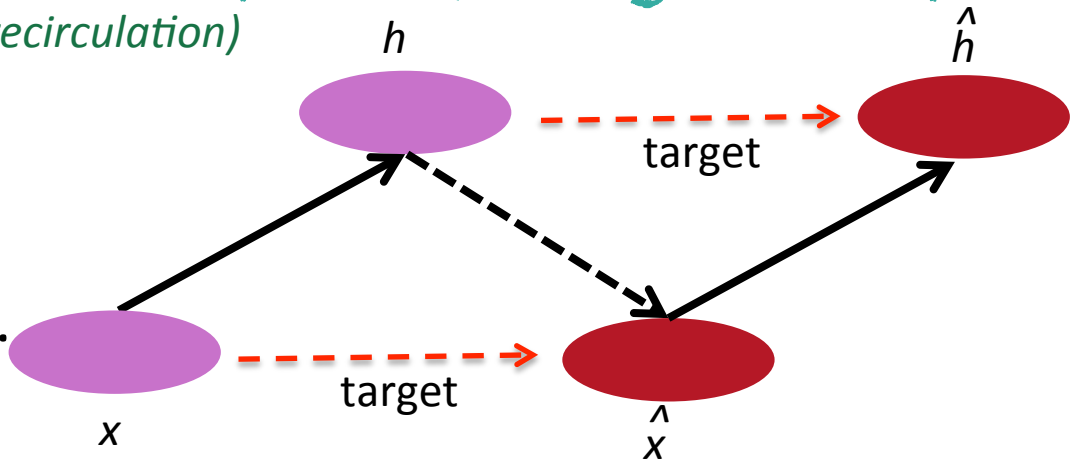
  for training a function g that tries to invert f
- Here we want both the encoder f and the decoder g to invert each other

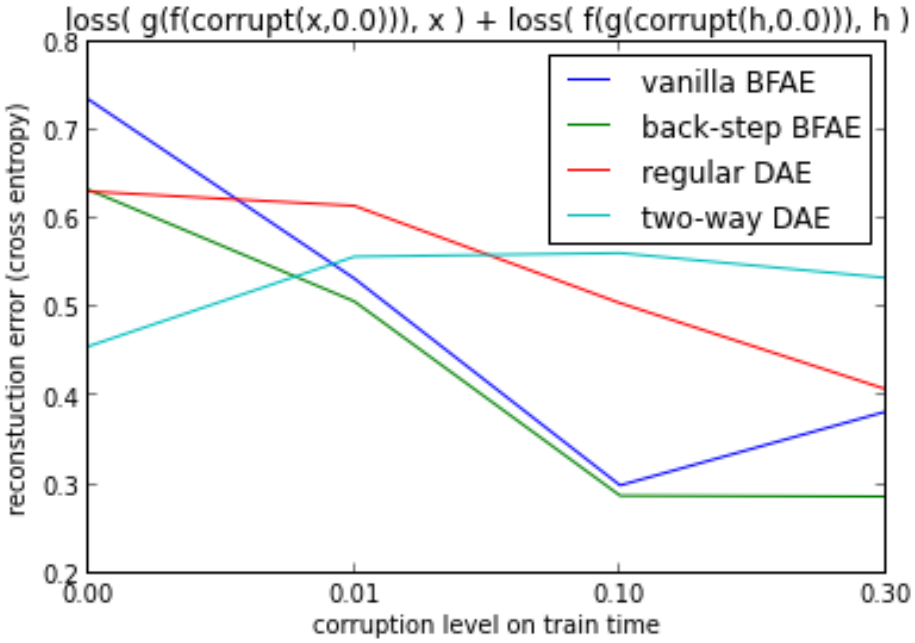


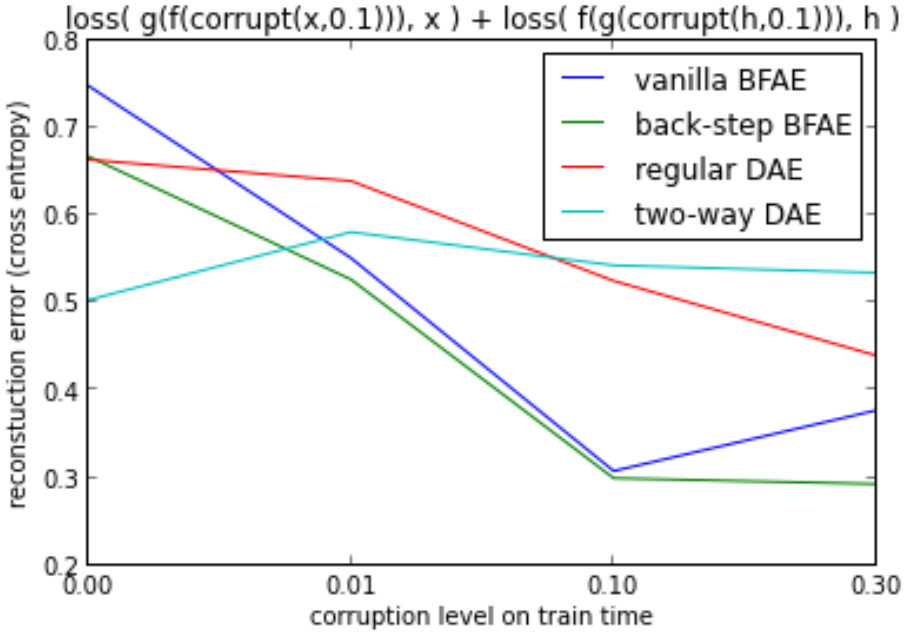
Recirculation & Backprop-free AEs
(Hinton & McClelland 1988, Lee & Bengio 2014)

Also: *(O'Reilly 1996, generalized recirculation)*

- Experiments with back-prop free auto-encoder training similar to Recirculation algorithm.
- Minimizes 2-way reconstruction losses



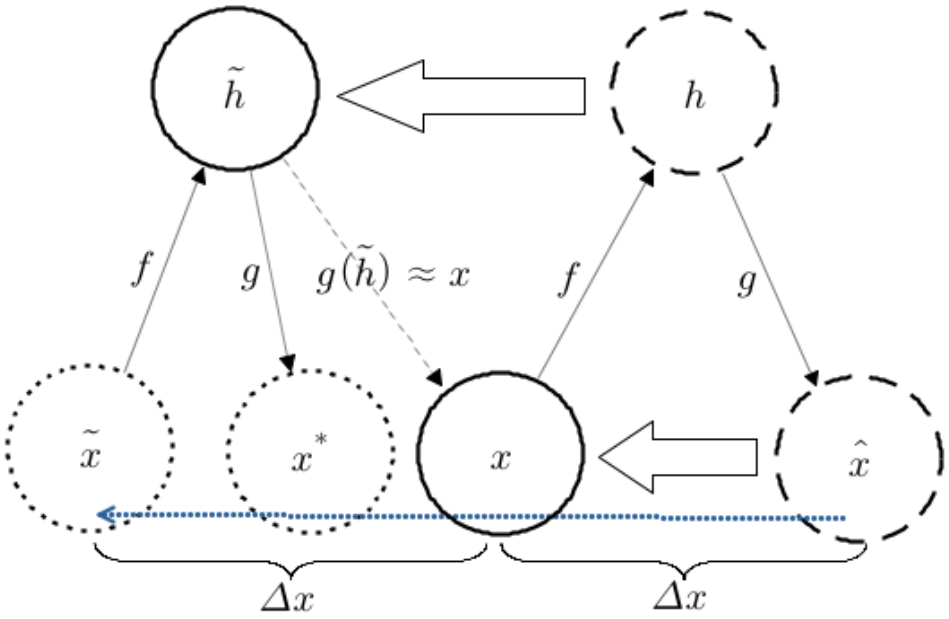




Back-Step BFAE

(Lee & Bengio, NIPS 2014 Deep Learning workshop)

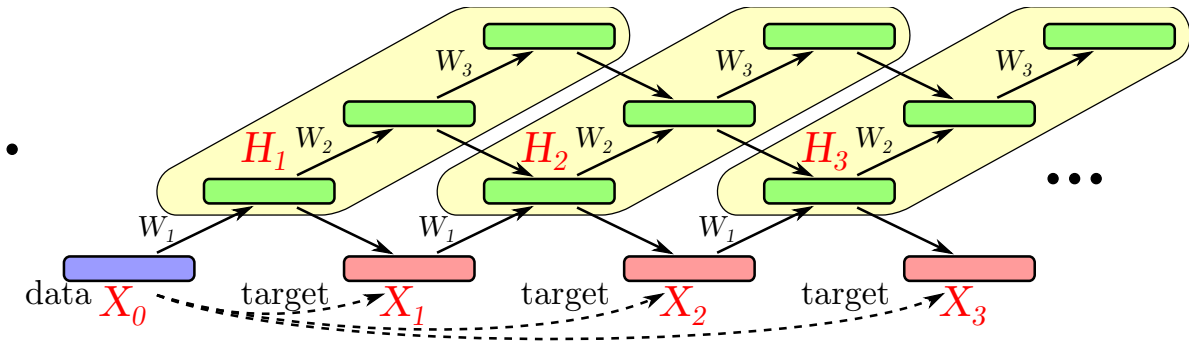
- By symmetry, we minimize both encode/decode and decode/encode reconstruction errors, unlike in ordinary auto-encoders
- To make the learning of encoder more relevant, we consider the decode/encode step where the input of the encoder is approximately *x*, using a local approximation:



Denoising Auto-Encoders (Minimizing Reconstruction Error) Learn to Model the Input Distribution

- *(Alain & Bengio ICLR 2013)*: reconstruction-input=dlogp(x)/dx
- (Alain & Bengio ICLR 2013; Bengio et al, arxiv 2013)

  (Bengio et al NIPS'2013; Bengio et al ICML'2014)
  - Encode-Decode iterations without noise = local MAP
  - Encode-Decode iterations with noise = MCMC samples from estimated generative model
  - Clamped Encode-Decode iterations fill-in missing values
  - GSNs generalize this to arbitrary recurrent net with injected noise



Denoising Auto-Encoders Learn a Small Move Towards Higher Probability

(Alain & Bengio ICLR 2013)

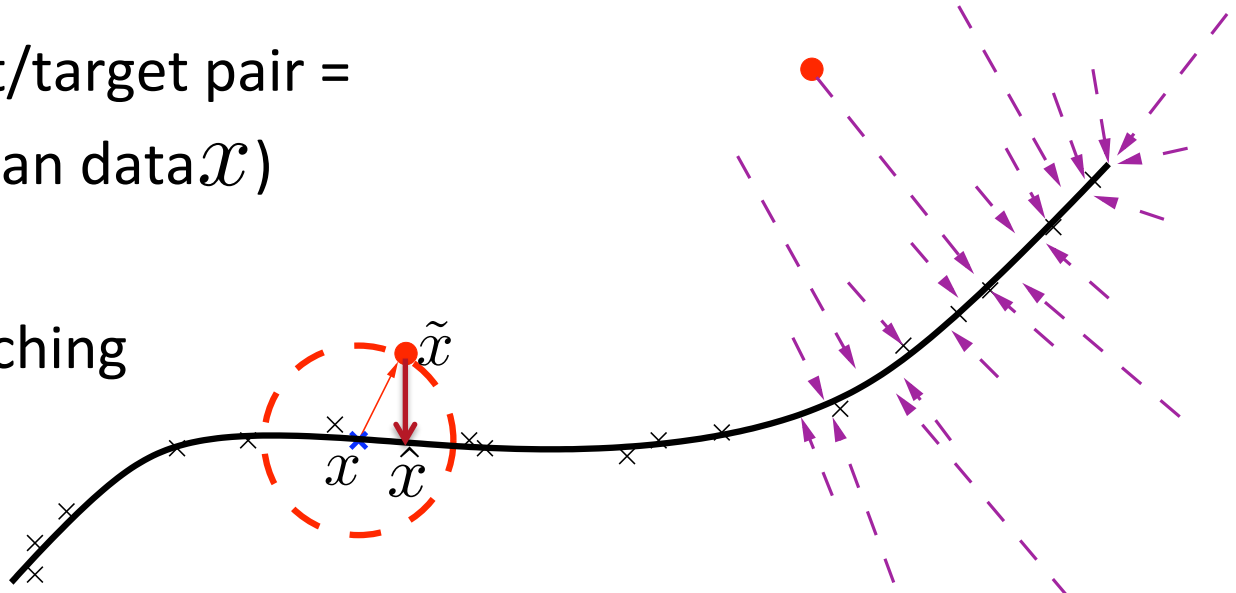
- Reconstruction $\hat{x}$ points in direction of higher probability

$$\hat{x} - x \propto \frac{\partial \log P(x)}{\partial x}$$

gradient

- Trained with input/target pair = (corrupted $\tilde{x}$ → clean data x)

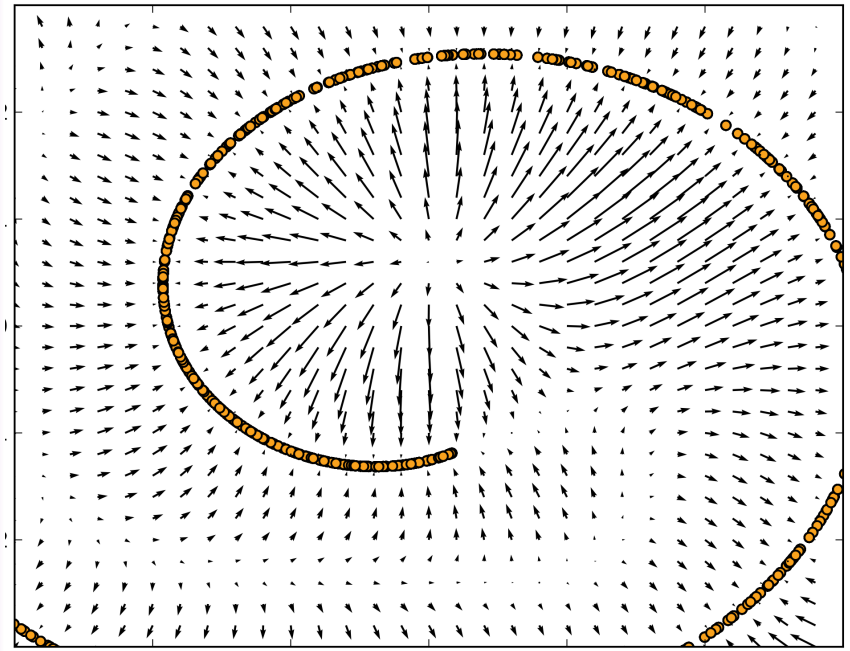
- DAE → Score matching (Vincent 2011)

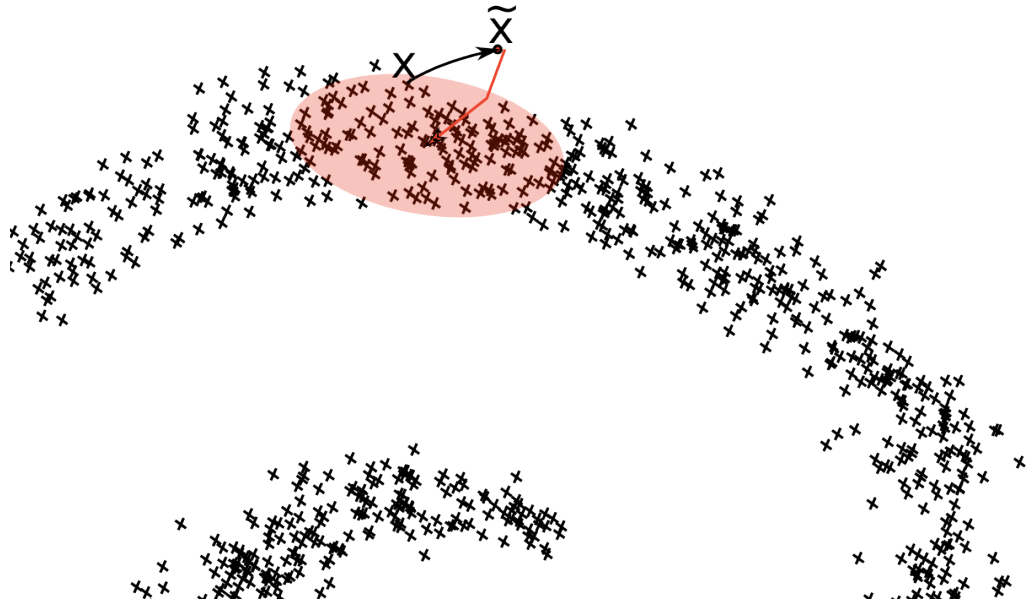


Reconstruction is towards more probable configuration according to AE

Regularized Auto-Encoders Learn a Vector Field or a Markov Chain Transition Distribution

- (Bengio, Vincent & Courville, TPAMI 2013) review paper
- (Alain & Bengio ICLR 2013; Bengio et al, arxiv 2013)
- (Bengio et al NIPS'2013; Bengio et al ICML'2014)

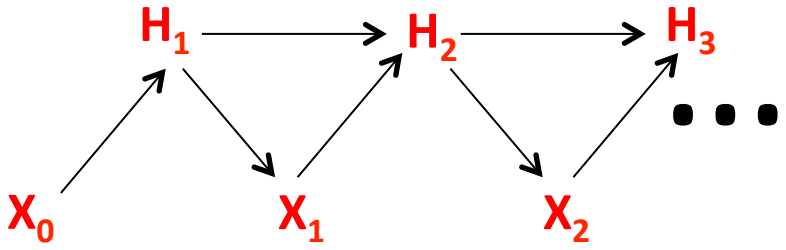




Generative Stochastic Networks

- Generalizes the denoising auto-encoder training scheme
  - Introduce latent variables in the Markov chain (over X,H)
  - Instead of a fixed corruption process, have a deterministic function with parameters θ_1 and a noise source Z as input

$$H_{t+1} = f_{\theta_1}(X_t, Z_t, H_t)$$

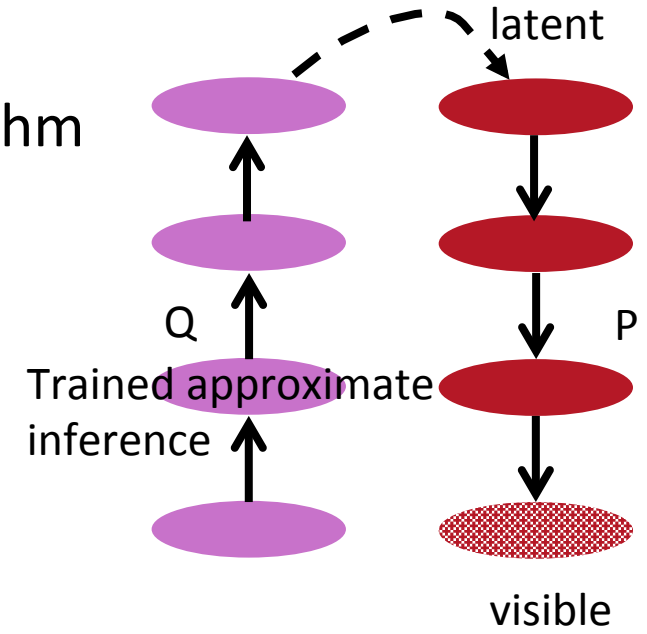


$$H_{t+1} \sim P_{\theta_1}(H|H_t, X_t)$$
$$X_{t+1} \sim P_{\theta_2}(X|H_{t+1})$$

  - DAE special case of GSN, both generate a Markov chain whose stationary distribution is a consistent estimator of the data generating distribution *(Bengio et al, NIPS'2013; ICML'2014)*

Ancestral Sampling with Learned Approximate Inference

- Helmholtz machine & Wake-Sleep algorithm
  - (Dayan, Hinton, Neal, Zemel 1995)
- Variational Auto-Encoders
  - (Kingma & Welling 2013, ICLR 2014)
  - (Gregor et al ICML 2014)
  - (Rezende et al ICML 2014)
  - (Mnih & Gregor ICML 2014)
- Reweighted Wake-Sleep (Bornschein & Bengio 2014)
- Target Propagation (Bengio 2014)
- Deep Directed Generative Auto-Encoders (Ozair & Bengio 2014)
- NICE (Dinh et al 2014)

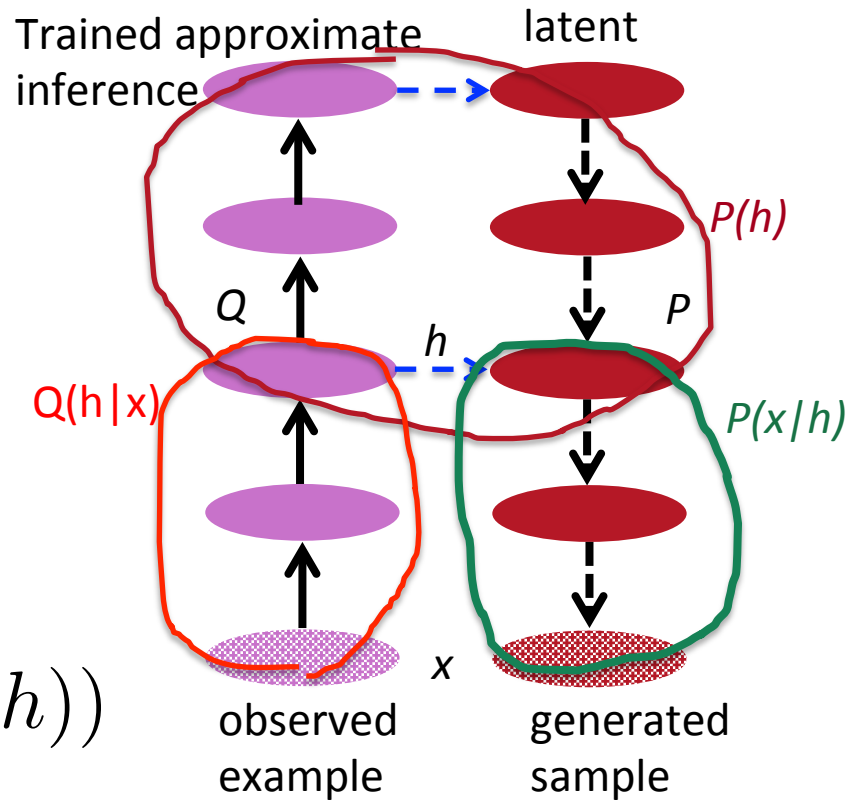


Variational Auto-Encoder Training Objective

- Two distributions:
  - data $Q(x) \rightarrow h \sim Q(h|x)$: $Q(x,h)$
  - model $P(h) \times P(x|h)$: $P(x,h)$
- Variational bound is equivalent to the following natural objective:

$$\min KL(Q(x,h)||P(x,h))$$

Reconstruction of *h* is towards Configuration of *h* more probable according to higher levels



Can consistently be applied at every layer *h* of a deep net

All the pieces are tractable

$$KL(Q(x,h)||P(x,h))$$

Decomposes into

- - *H(Q)*
- Reconstruction error:

$$E_{Q(h,x)}[-\log P(x|h)]$$

- Bottom-up / Top-down match:

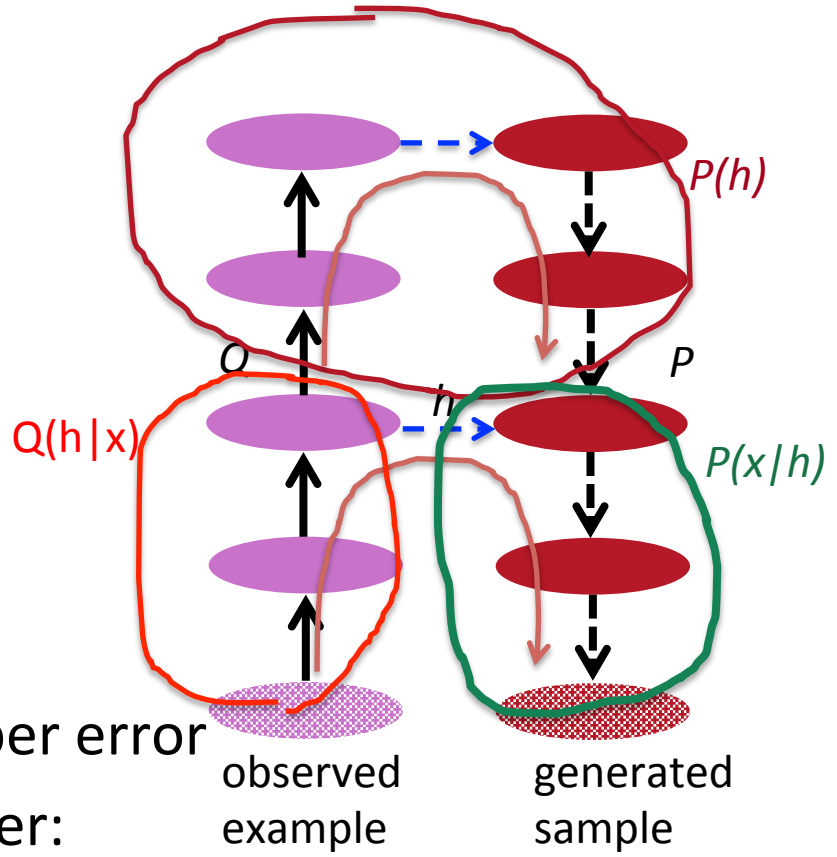
$$E_{Q(h,x)}[-\log P(h)]$$

→ upper model likelihood

→ lower encoder tries to reduce upper error

tractable if top *P(h)* is an auto-encoder:

$$\hat{h} - h \propto \frac{\partial \log P(h)}{\partial h}$$

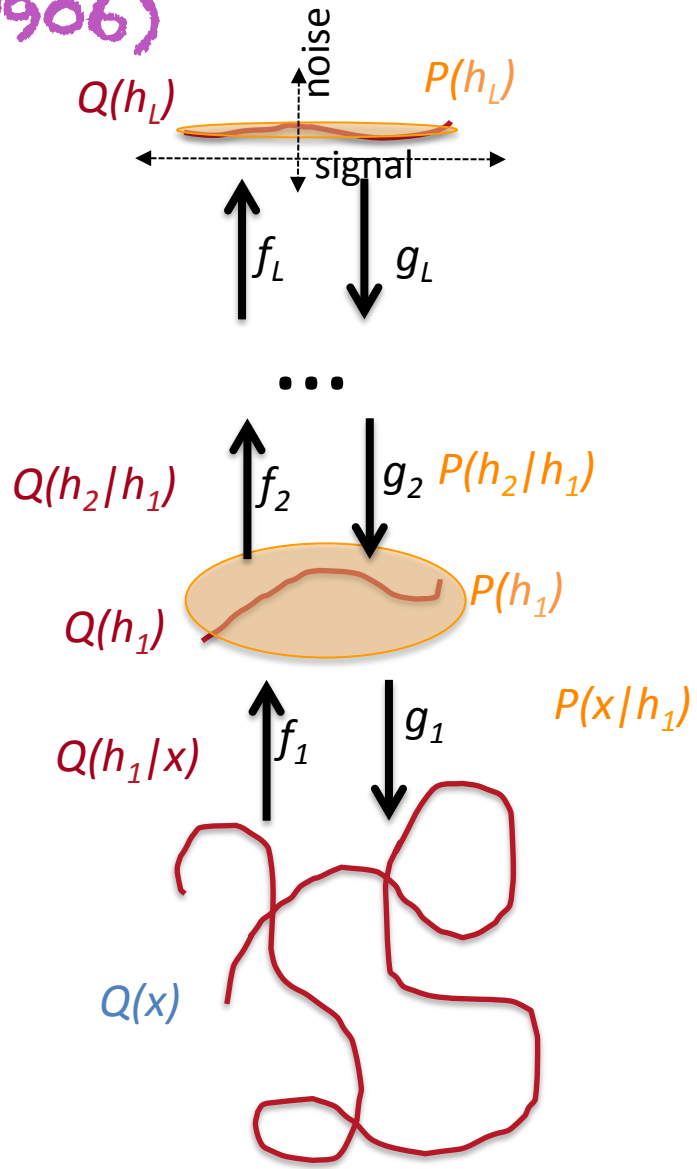


Extracting Structure By Gradual Disentangling and Manifold Unfolding (Bengio 2014, arXiv 1407.7906)

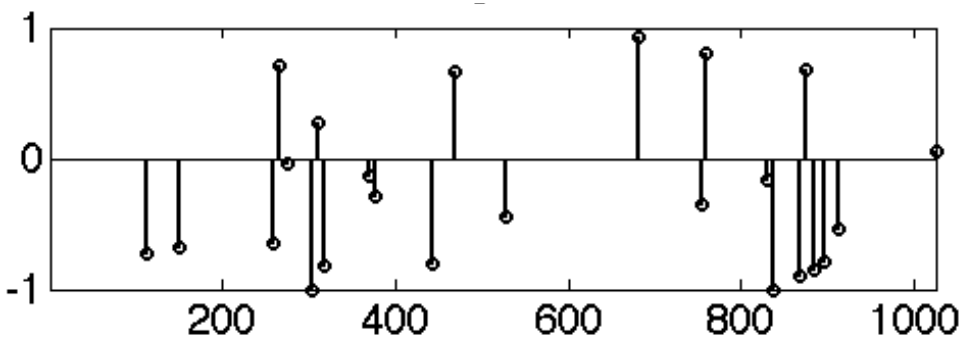
Each level transforms the data into a representation in which it is easier to model, unfolding it more, contracting the noise dimensions and mapping the signal dimensions to a factorized (uniform-like) distribution.

$$\min KL(Q(x,h)||P(x,h))$$

for each intermediate level h



Disentangling with Deep Nets



- With sparse auto-encoders, the pressure to make *P(h)* simple comes from the sparsity penalty

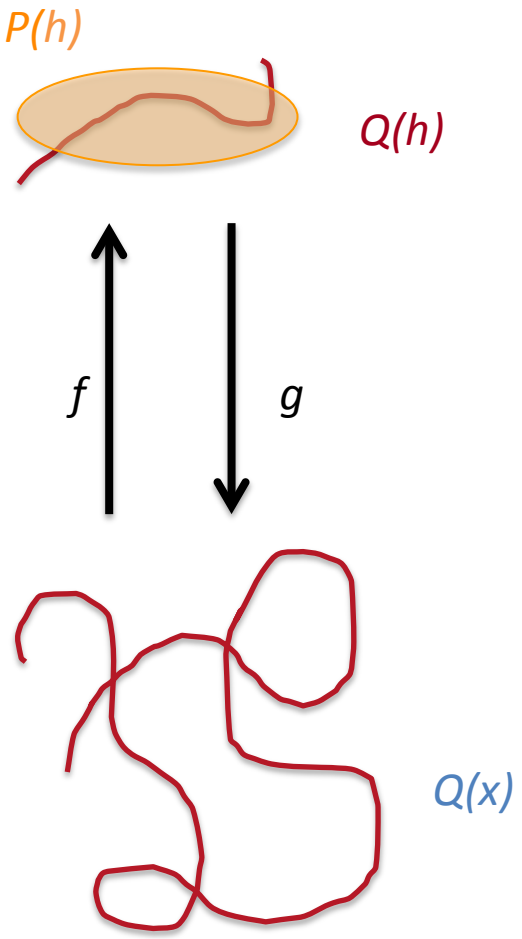
- However, we would also like lower-level encoders to help the higher-level encoders achieve a better *P(h)*

- Minimizing $KL(Q(h_l,x) \,||\, P(h_l,x))$ at each level h_l achieves that!

- However, with INSUFFICIENT DEPTH, the sparsity (or other simplicity preference) of *P(h)* cannot be achieved without yielding a mismatch between *P(h)* and *Q(h)*, hence a poor *P(x)*

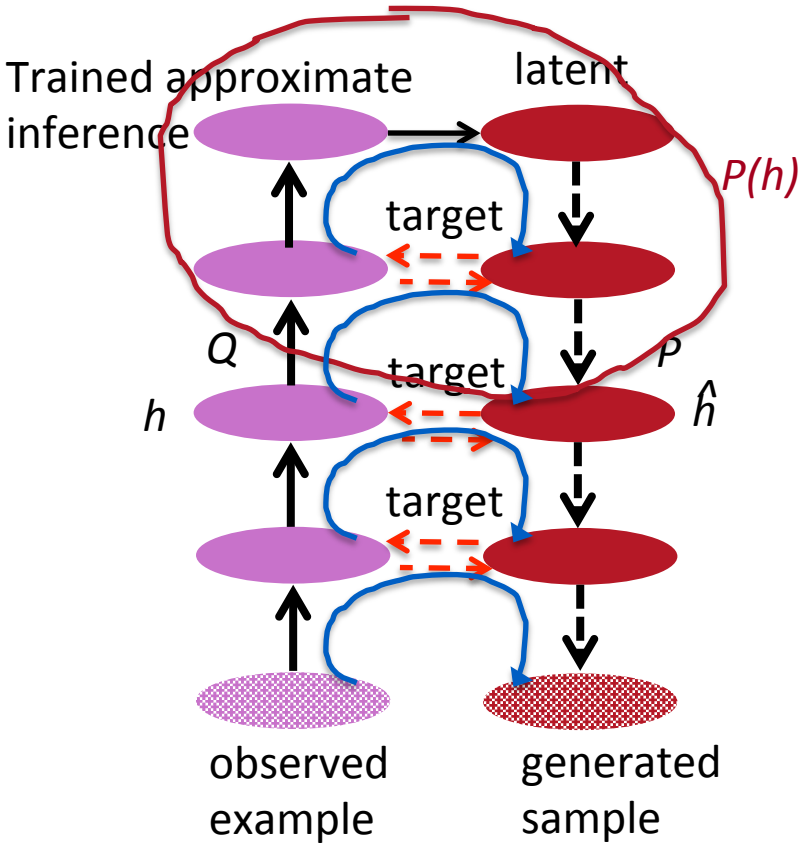
The Importance of Contraction

- Denoising → contractive *g*
- Max. determinant of f' → contractive *g*
- Contraction → removes unnecessary directions
- The *log P(h = f(x)+noise)* force on the encoder *f* makes *f* contractive, making it insensitive to directions of non-variation in *x*-space
- Making *g* contractive helps to manage the mismatch between *P(h)* and *Q(h)*
- Adding noise at the top-level in *Q(h|x)* shows to the decoder which directions of *h* need to be contracted out, making it contractive too



Purely Local Training Signals

- Each layer tries to be a good denoising auto-encoder while transforming the lower-level data into a form *h* easier to model by higher levels: higher *P(h)*
- This basically makes the long-path reconstructions (going all the way up) a target $\hat{h}$ for the original *h*, and vice-versa, while the long-path auto-encoder is trained with *h* as data

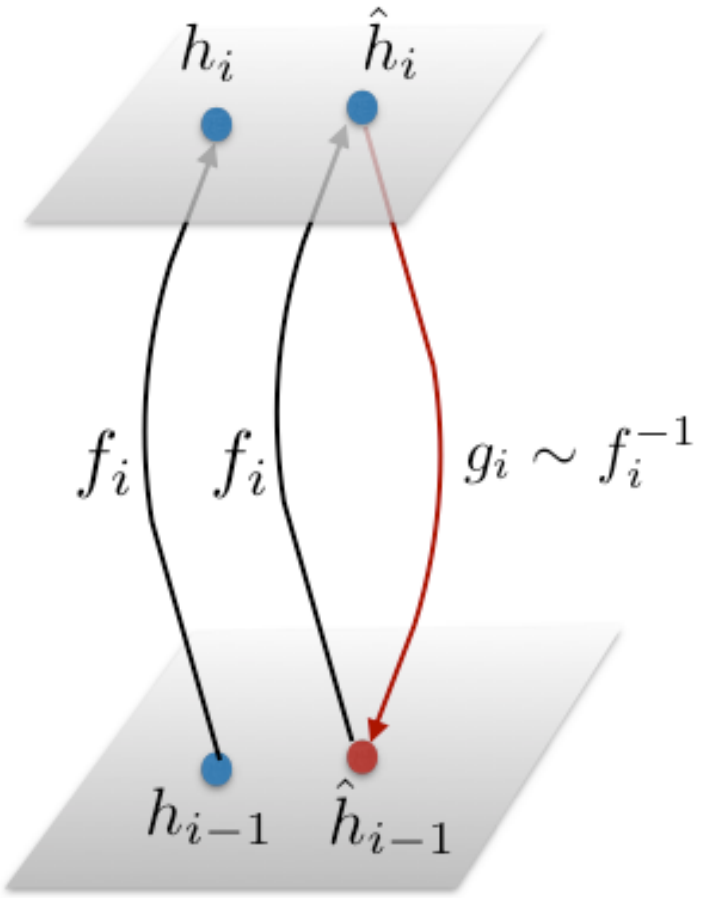


Target-Prop as an alternative to Back-Prop

- Weights at each layer are trained to match targets associated with the layer output
- How to propagate targets?
- If g_i is the inverse of f_i, then it guarantees an improvement in the cost.

$$cost(h_i) > cost(\hat{h}_i)$$

$$cost(f_i(h_{i-1})) > cost(f_i(\hat{h}_{i-1}) \approx \hat{h}_i)$$

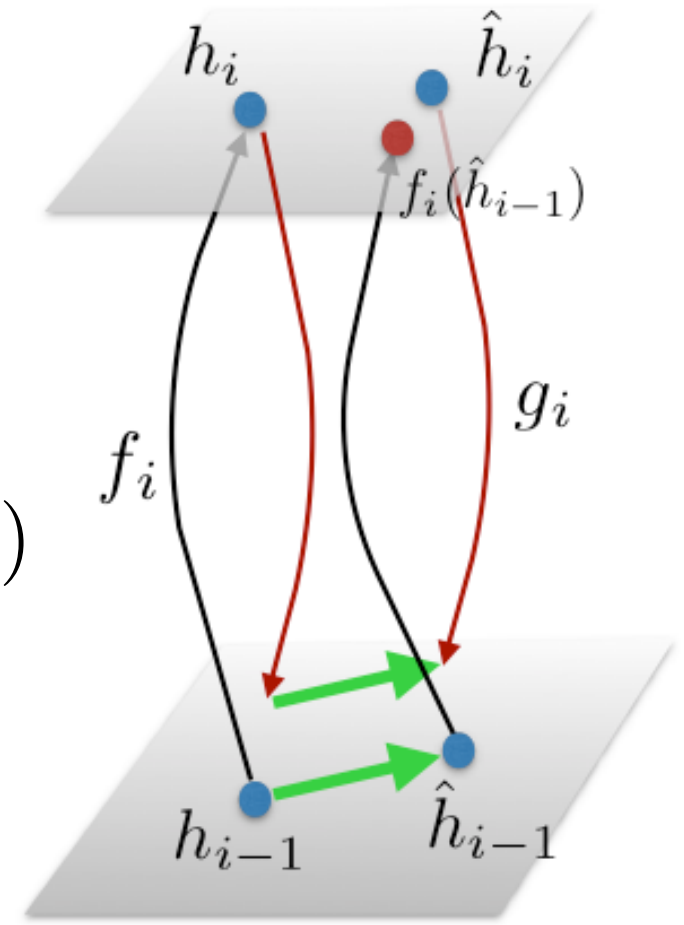


Difference Target-Prop for Inexact Inverse

- Make a correction that guarantees to first order that the projection estimated target is closer to the correct target than the original value

$$\hat{h}_{i-1} = h_{i-1} - g_i(h_i) + g_i(\hat{h}_i)$$

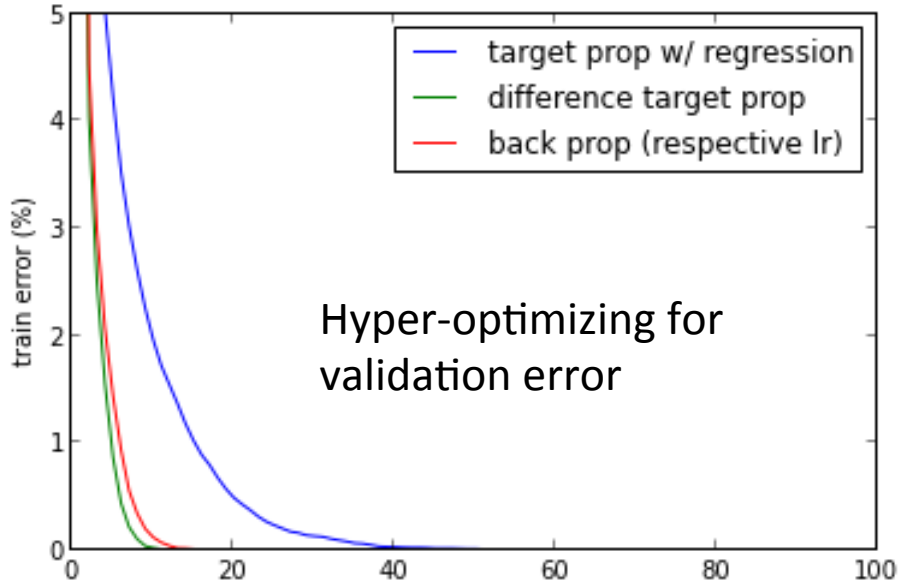
- Special case: feedback alignement, if $g_i(h) = B\,h$

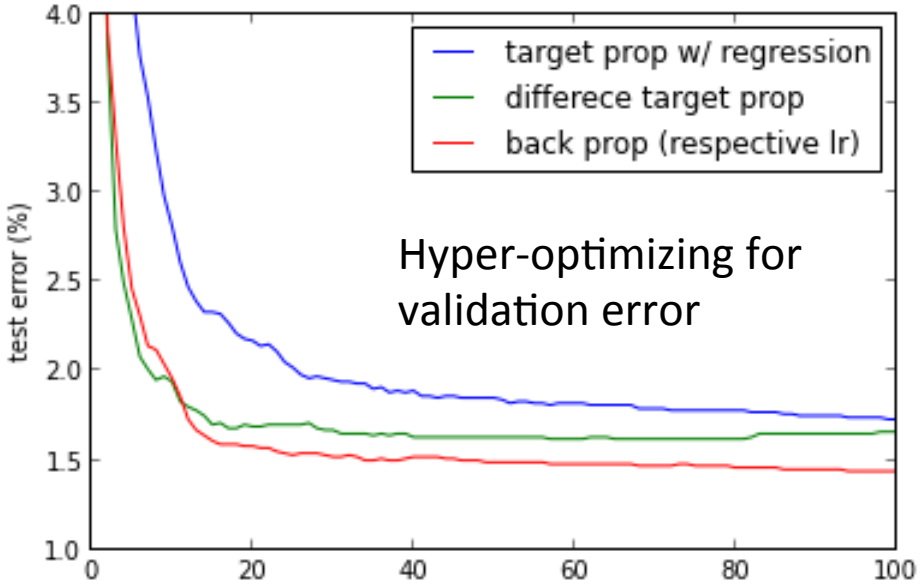


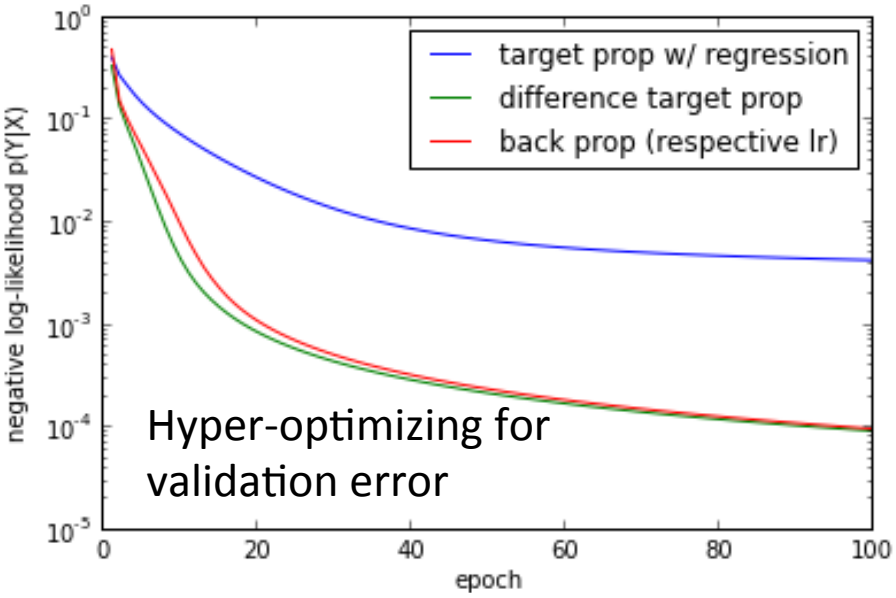
$$\left\|\hat{h}_i - f_i(\hat{h}_{i-1})\right\|^2 < \left\|\hat{h}_i - h_i\right\|^2$$

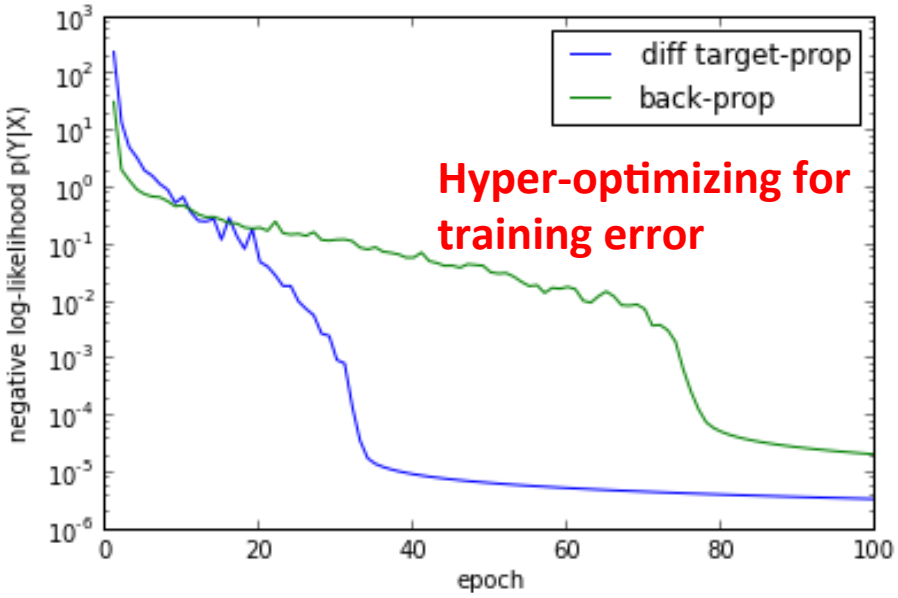
$$\text{if } \; 1 \; > \; max\ eigen\ value \left[(I - f_i'(h_{i-1})g_i'(h_i))^T\,(I - f_i'(h_{i-1})g_i'(h_i))\right.$$

Obligatory MNIST Results



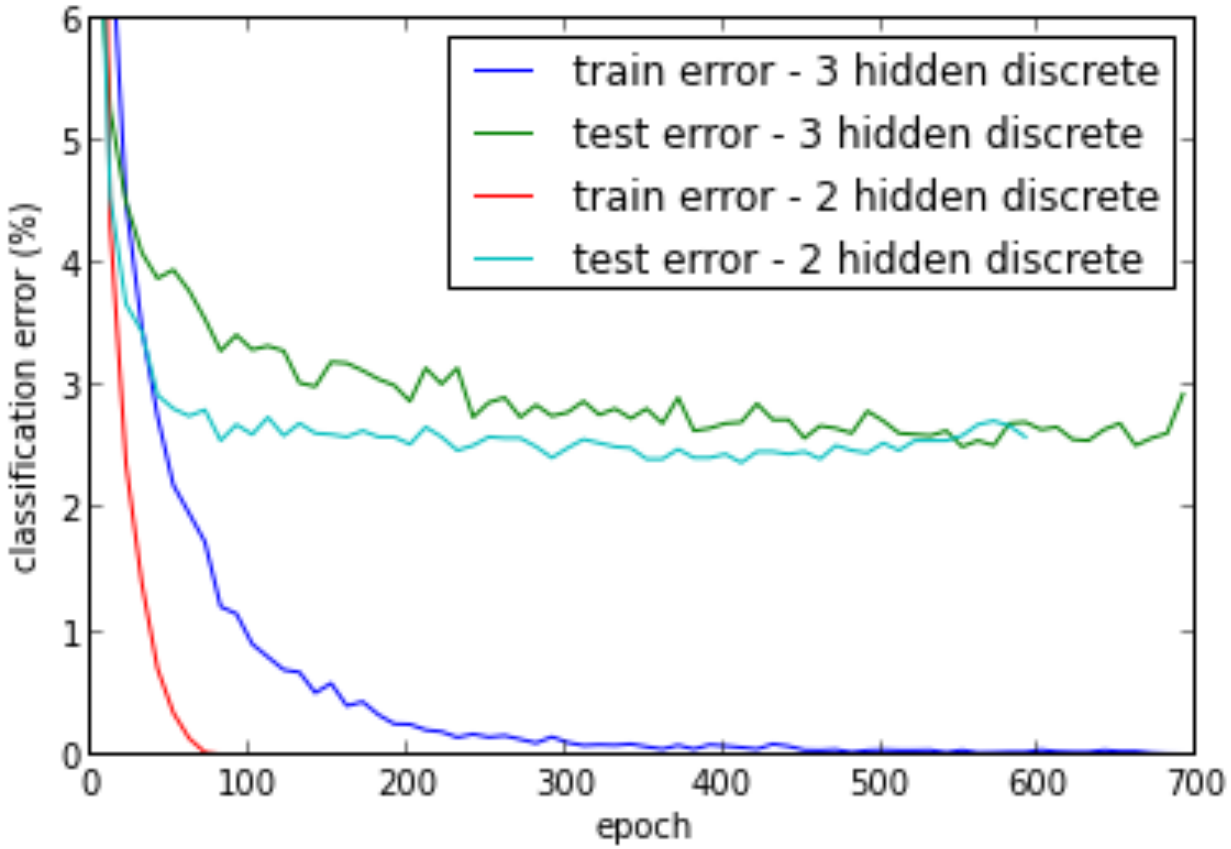






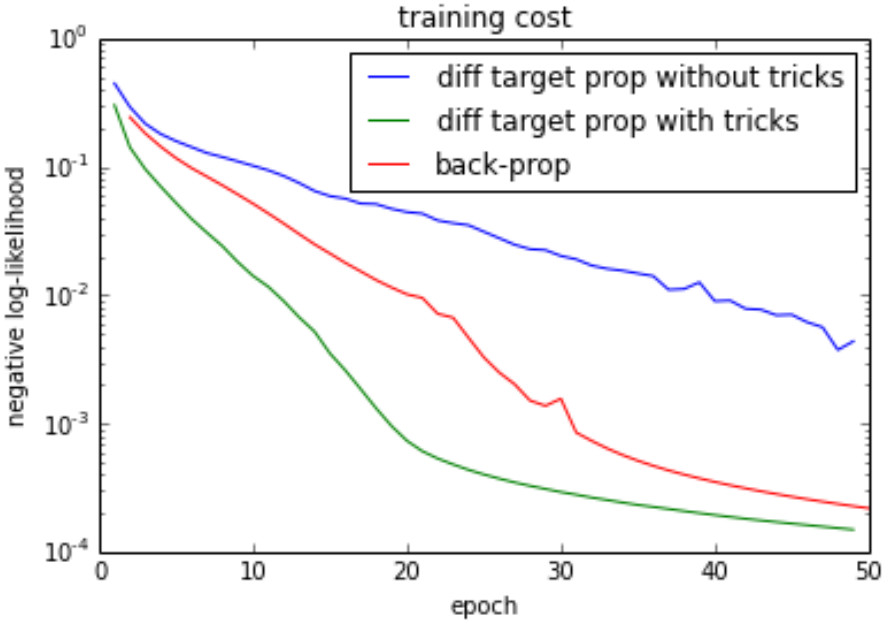
Targetprop can work for discrete activations

- Work in progress

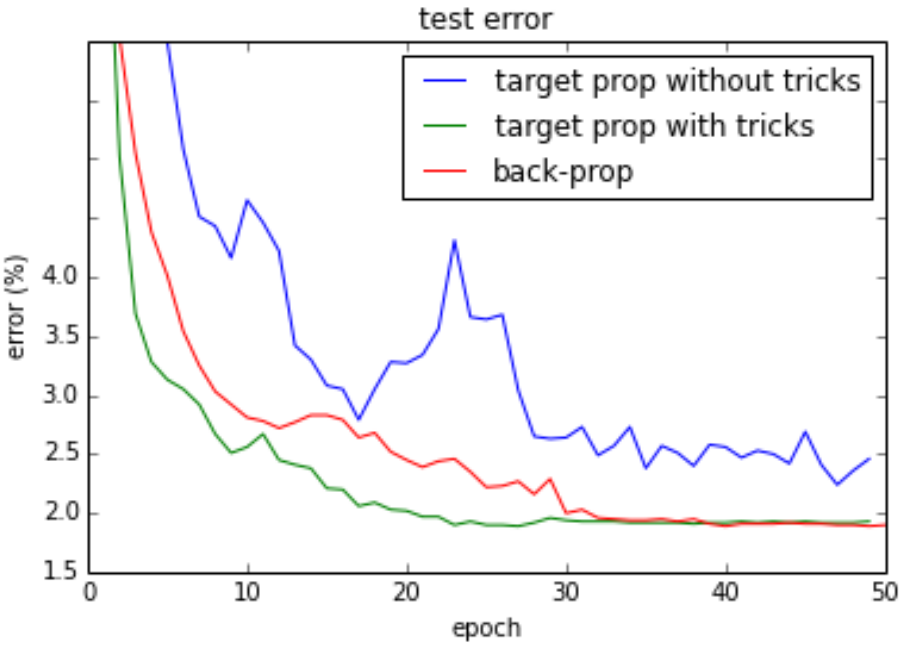


Target-Prop on Deep Nets

- MNIST 784-240-240-240-240-240-240-10 tanh net

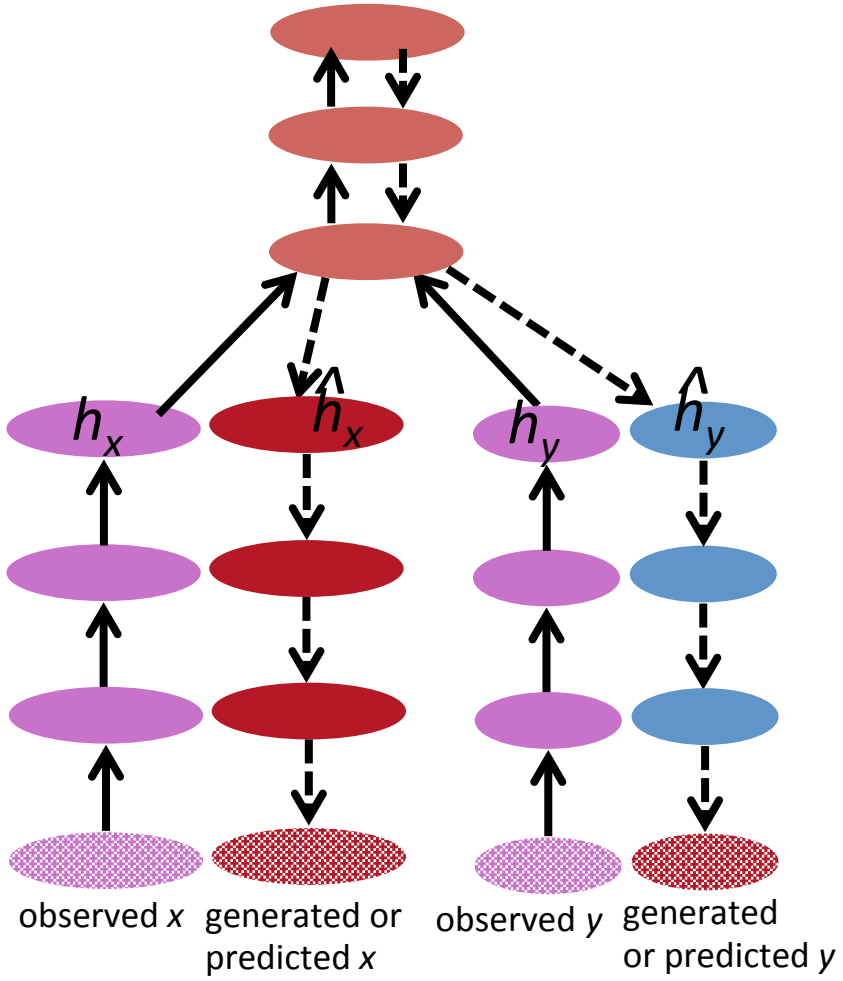


Tricks: noise injection, combine feedforward and feedback values, train lower layers first



From Supervised Targets to Reconstructions from Other Modalities: Multi-Modal / Structured Output

- *y* is complex and needs its own *P(y)* (modeled by it's own stack of auto-encoders) and non-trivial *P(y|x)*. Model joint of $h_x(x)$ and $h_y(y)$ with another stack on top.
- Inference (MAP or MCMC) is done with the top stack, then projected back in the *x* or *y* space.
- Top level is a DAE or GSN

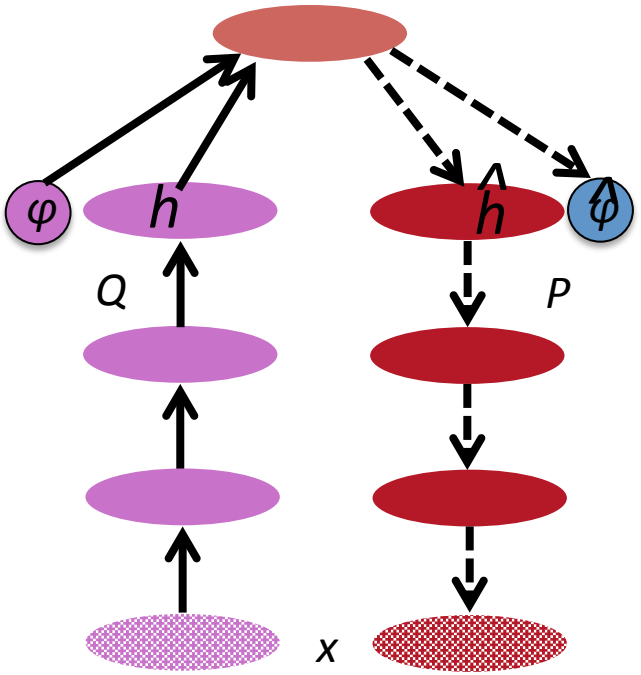


(Conditional) Sampling & MAP

- Two things we want from our models:
  - Probabilistic inference:
    - Sample some variables given others (or none)
  - MAP inference:
    - Choose likely values for some variables given others
- Both can be done here:
  - Unconditional sampling by ancestral sampling from P
  - Conditional sampling by GSN-like MCMC, clamping the given variables and resampling others:
    - Iteratively encode/decode with noise injected (top level stack)
  - Local ascent for approximate MAP:
    - Iteratively encode/decode with no noise injected (top level stack)

Ambiguous Multi-Modal Posteriors on Latent Variables

- The simple stack model aims at a factorized posterior $Q(h|x)$
- However, plausible latent variables must have a **multi-modal posterior**
- Latent variables can be thought of as unobserved labels φ
- Stick an AE/GSN on top with the latent variables φ and h as input



Multiple Iterations of Target-Prop

Set *h* to opt. reconstruction (et al & LeCun 200x) or prediction error (Perpinan & Wang AISTATS 2014)

- Want time-invariant computation/update without f/bprop phases
- If target = new value of *h*, target-prop update

$$\hat{h}^l = h^l + g(\hat{h}^{l+1}) - g(h^{l+1})$$

can be interpreted as

$$\text{new } h^l - \text{old } h^l = g(\text{new } h^{l+1}) - g(\text{old } h^{l+1})$$

$$\Delta h = \Delta r$$

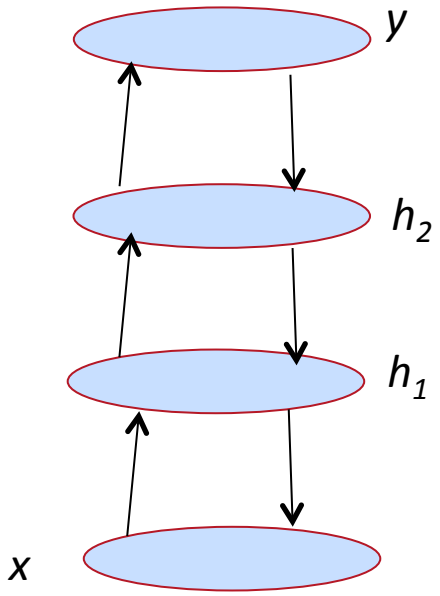
- Where *r* = output of reconstruction (feedback path) units

$$r = g(\text{upper level } h)$$

- If we do several iterations, *h* continues to move towards better matching the top-level target but may become unrealistic for realization by feedforward path
- Iterations stop when target is reached as well as possible

Reconstructing Both x and y

- Feedback path = target for feedforward path (better predict y)
- Feedforward path = target for feedback path (better predict x)
- Good 2-way auto-encoders give that for free
- Iterative targetprop is run both ways



Multiple Iterations of Target-Prop → MAP or EM training

- One option is to force *h* to remain close to its feedforward value
- More appealing: look for *h* that is compatible with both *x* and *y*
- Achievable by moving *h* towards better reconstruction of both *x* and *y*
- If we add a sparsity prior, this corresponds to

$$\text{argmax}_h \log P(h|x,y)$$

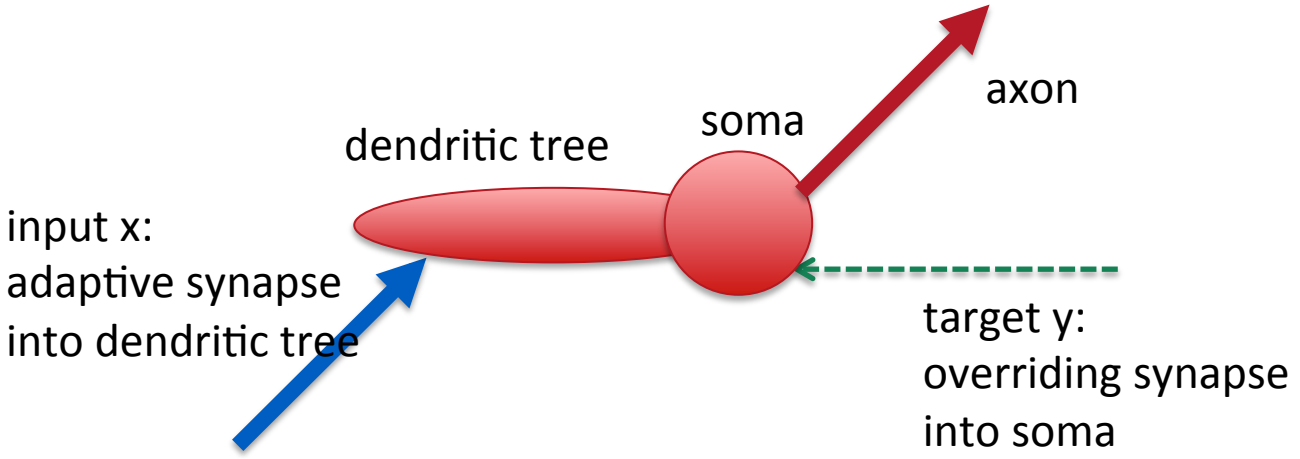
- And if we inject noise, this corresponds a GSN MCMC towards

$$h \sim P(h|x,y)$$

- And we can then do a layer-wise update (delta-rule = STDP) which corresponds to an EM update of *P(x,y,h)*

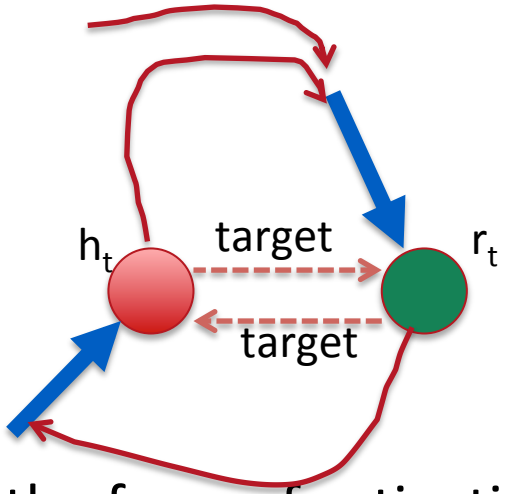
Delta-Rule Updates in Cortex

- *(Urbanczik & Senn, Neuron Report 2014, "Learning by the dendritic prediction of somatic spiking")*
- Dendritic synapses learn by matching the target imposed by soma-level synapses, as in Delta-Rule
- Spiking rate at soma depends on both, dominated by soma-synapses



How Brains Might Learn Without Backprop

- Two kinds of nodes: feedforward & feedback
- Feedback nodes = targets for feedforward ones and vice-versa
- Initially clamp x, fprop, then clamp y, then settle, delta-rule update along the way (new x,y can arrive at any time)



- Does not depend on the form of activation function, tied symmetric weights, differentiability of anything, using rates vs spikes, etc.

Extends Hinton's proposal made at the first DL workshop NIPS'2007 based on encoding gradients as temporal derivatives

Conclusion: Beautiful Coincidences

- Same basic mechanism (auto-encoders) basically does it all
- Local training signals provided only by using activity at the neurons and synapses (like in the Boltzmann machine, but no need to achieve mixing in inner loop of training)
- Not requiring the differentiability of neuronal activation and noise helps to achieve to needed contraction (could be used to train spiking neurons?)
- Same mechanism for supervised, unsupervised, and multi-modal learning, and structured-output sampling or MAP
- Handling sequences: relation to Jaeger's Conceptors…

PLUG: Deep Learning, MIT Press book in preparation, draft chapters online for feedback

LISA team: Merci! Questions?