The Challenge of Sequential Modelling

Stanford University, 26 Sept. 2016

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PLUG: **Deep Learning**, MIT Press book in pre-sale,
Chapters will remain online

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CIFAR | ICRA

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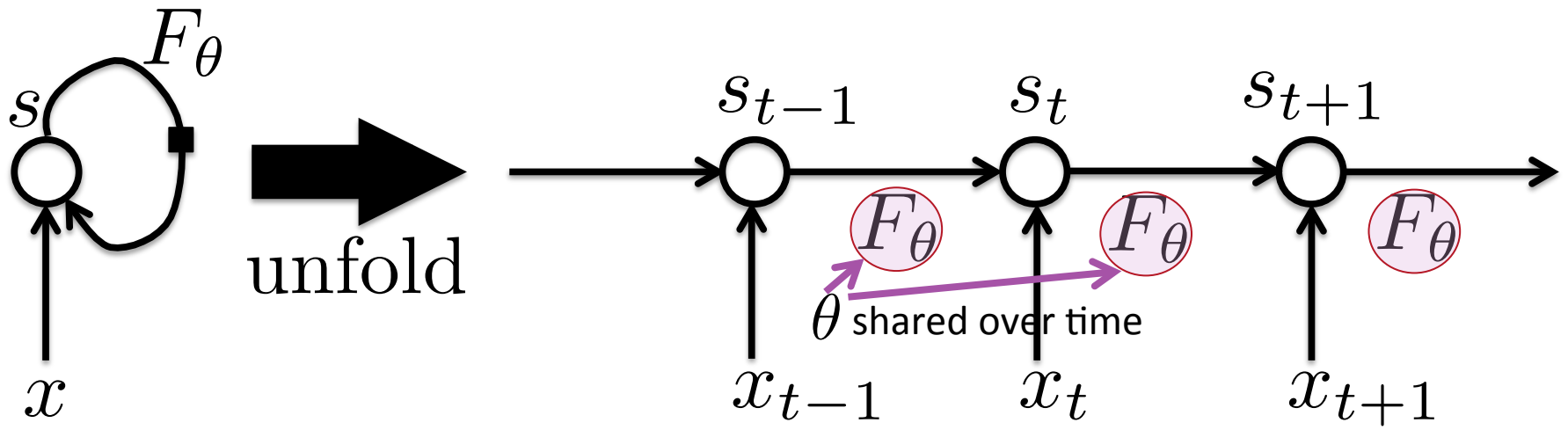
Recurrent Neural Networks

- Selectively summarize an input sequence in a fixed-size state vector via a recursive update

$$s_t = F_\theta(s_{t-1}, x_t)$$



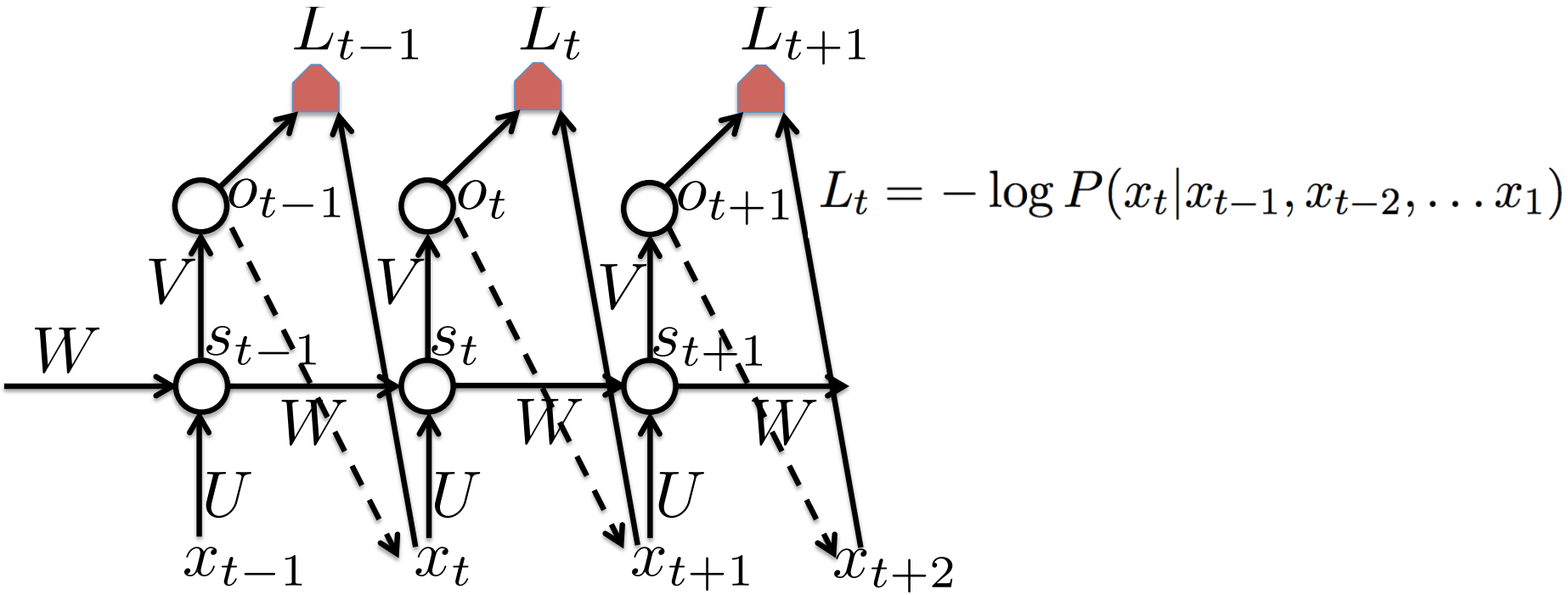
$$s_t = G_t(x_t, x_{t-1}, x_{t-2}, \ldots, x_2, x_1)$$

➔ **Generalizes naturally to new lengths not seen during training**

Generative RNNs

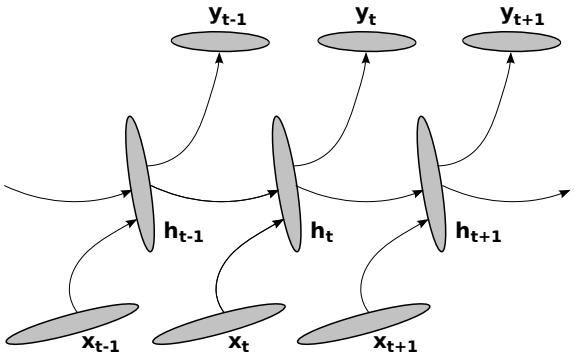
- An RNN can represent a fully-connected **directed generative model**: every variable predicted from all previous ones.

$$P(\mathbf{x}) = P(x_1, \ldots x_T) = \prod_{t=1}^{T} P(x_t | x_{t-1}, x_{t-2}, \ldots x_1)$$



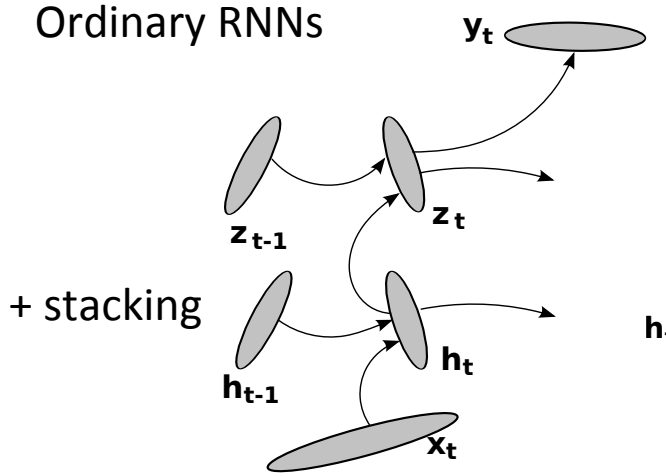
Increasing the Expressive Power of RNNs with more Depth

- ICLR 2014, *How to construct deep recurrent neural networks*

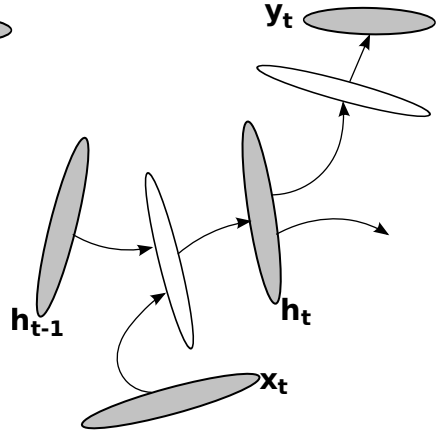


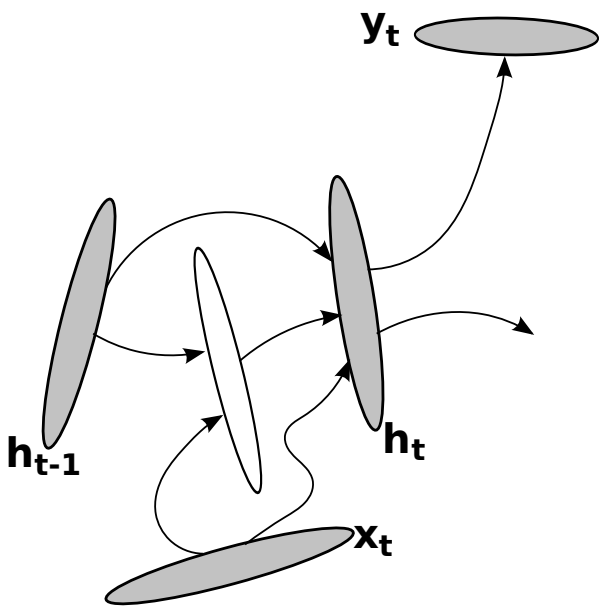
Ordinary RNNs

+ deep hid-to-out
+ deep hid-to-hid
+deep in-to-hid



+ stacking

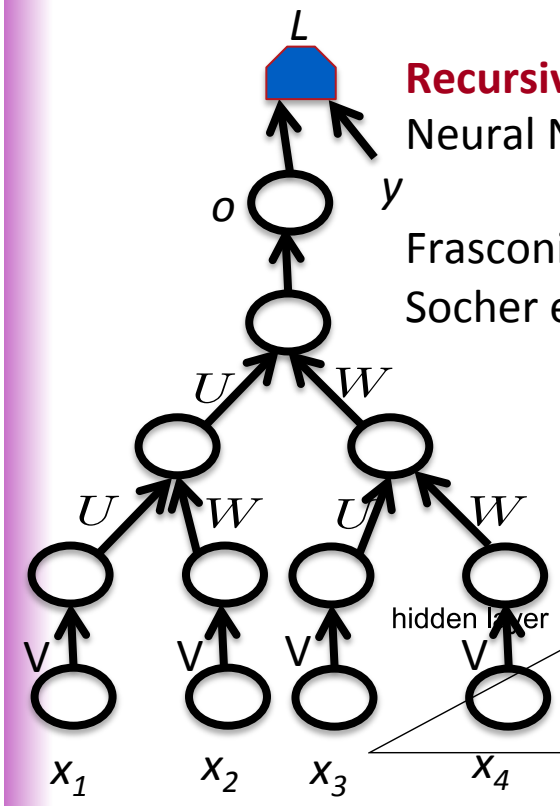




+ skip connections for creating shorter paths

Bidirectional RNNs, Recursive Nets, Multidimensional RNNs, etc.

- The unfolded architecture needs not be a straight chain

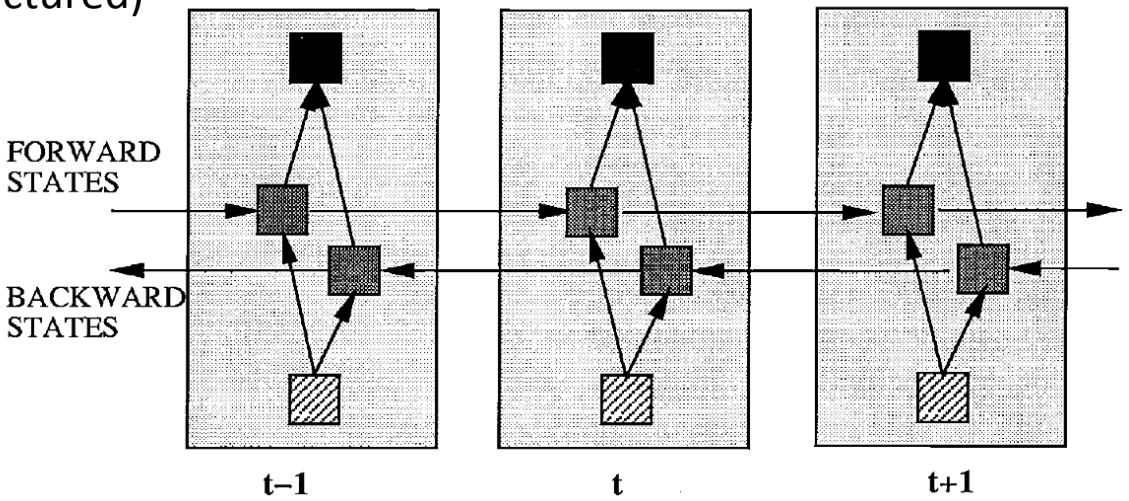


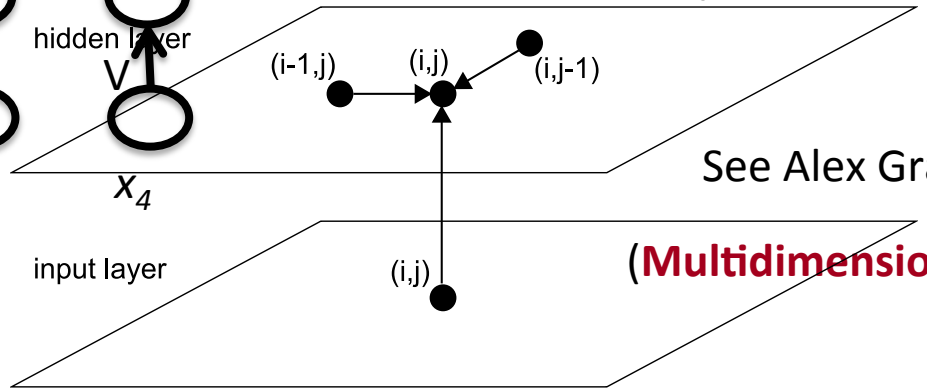
Recursive (tree-structured) Neural Nets:

Frasconi et al 97

Socher et al 2011

Bidirectional RNNs (Schuster and Paliwal, 1997)





See Alex Graves's work, e.g., 2012

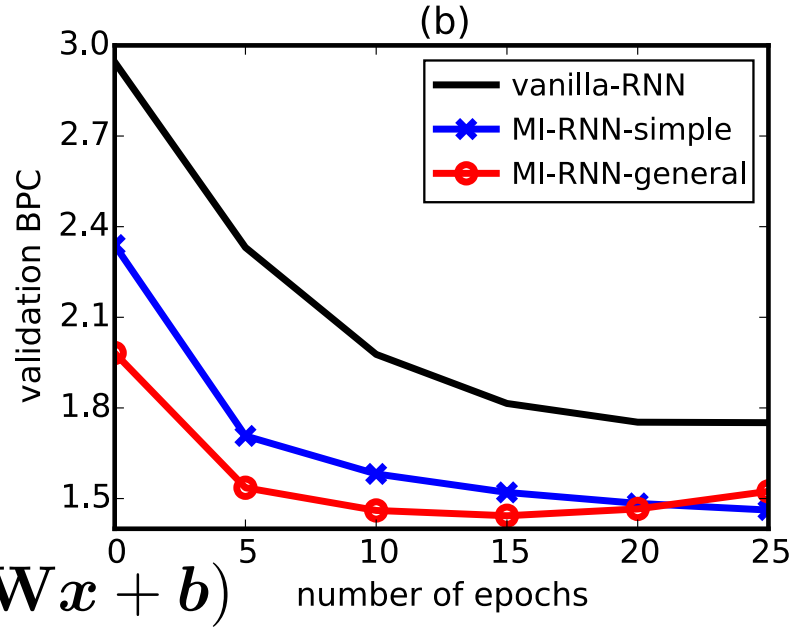
(**Multidimensional** RNNs, Graves et al 2007)

Multiplicative Interactions

(Wu et al, 2016, arXiv: 1606.06630, NIPS2016)

- Multiplicative Integration RNNs:
  - Replace $\phi(\mathbf{W}\boldsymbol{x} + \mathbf{U}\boldsymbol{z} + \mathbf{b})$
  - By $\phi(\mathbf{W}\boldsymbol{x} \odot \mathbf{U}\boldsymbol{z} + \mathbf{b})$
  - Or more general:

$$\phi(\boldsymbol{\alpha} \odot \mathbf{W}\boldsymbol{x} \odot \mathbf{U}\boldsymbol{z} + \boldsymbol{\beta}_1 \odot \mathbf{U}\boldsymbol{z} + \boldsymbol{\beta}_2 \odot \mathbf{W}\boldsymbol{x} + \boldsymbol{b})$$



Experiments on speech recognition, but also on language modeling and learning word embeddings.

WSJ Corpus	CER	WER
DRNN+CTCbeamsearch [17]	10.0	14.1
Encoder-Decoder [18]	6.4	9.3
LSTM+CTCbeamsearch [19]	9.2	8.7
Eesen [20]	-	**7.3**
LSTM+CTC+WFST (ours)	6.5	8.7
MI-LSTM+CTC+WFST (ours)	**6.0**	8.2

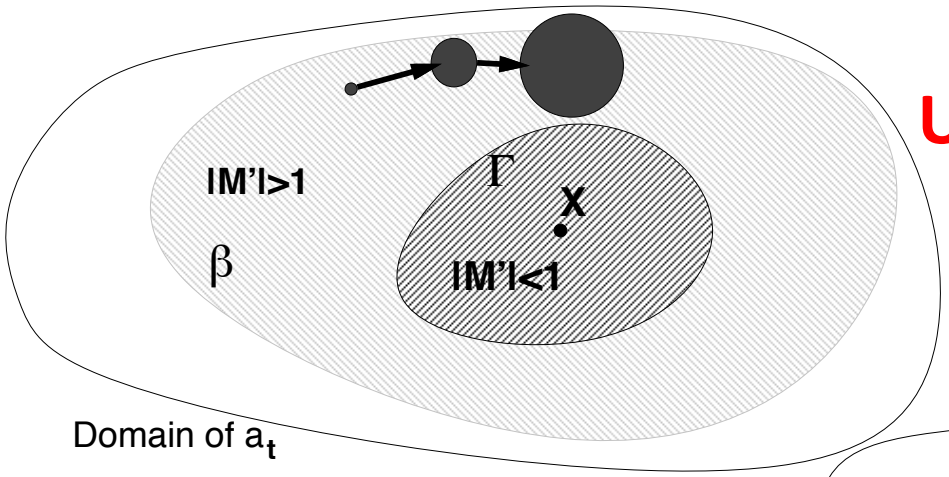
Learning Long-Term Dependencies with Gradient Descent is Difficult



Y. Bengio, P. Simard & P. Frasconi, IEEE Trans. Neural Nets, **1994**

Robustly storing 1 bit in the presence of bounded noise

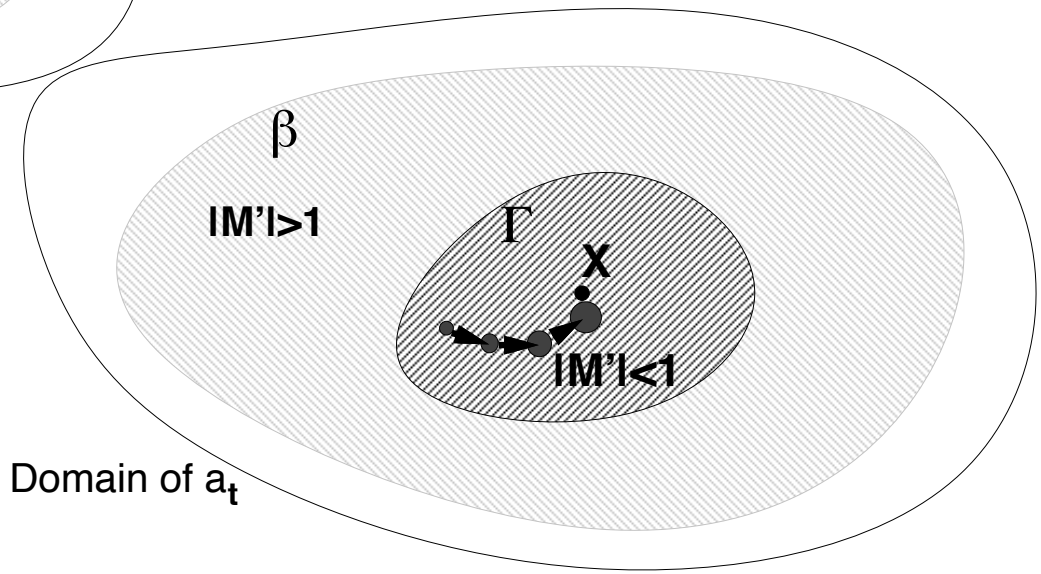
- With spectral radius > 1, noise can kick state out of attractor



UNSTABLE

- Not so with radius<1

CONTRACTIVE
→ STABLE



Storing Reliably → Vanishing gradients

- Reliably storing bits of information requires spectral radius<1
- The product of T matrices whose spectral radius is < 1 is a matrix whose spectral radius converges to 0 at exponential rate in T

$$L = L(s_T(s_{T-1}(\dots s_{t+1}(s_t, \dots))))$$

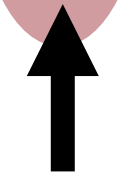
$$\frac{\partial L}{\partial s_t} = \frac{\partial L}{\partial s_T}\frac{\partial s_T}{\partial s_{T-1}}\cdots\frac{\partial s_{t+1}}{\partial s_t}$$

- If spectral radius of Jacobian is < 1 → propagated gradients vanish

Why it hurts gradient-based learning

- Long-term dependencies get a weight that is exponentially smaller (in T) compared to short-term dependencies

$$\frac{\partial C_t}{\partial W} = \sum_{\tau \leq t} \frac{\partial C_t}{\partial a_\tau} \frac{\partial a_\tau}{\partial W} = \sum_{\tau \leq t} \frac{\partial C_t}{\partial a_t} \frac{\partial a_t}{\partial a_\tau} \frac{\partial a_\tau}{\partial W}$$



Becomes exponentially smaller for longer time differences, when spectral radius < 1

To store information robustly the dynamics must be contractive

- The RNN gradient is a product of Jacobian matrices, each associated with a step in the forward computation. To store information robustly in a finite-dimensional state, the dynamics must be contractive [Bengio et al 1994].

$$L = L(s_T(s_{T-1}(\ldots s_{t+1}(s_t, \ldots))))$$

$$\frac{\partial L}{\partial s_t} = \frac{\partial L}{\partial s_T}\frac{\partial s_T}{\partial s_{T-1}}\cdots\frac{\partial s_{t+1}}{\partial s_t}$$

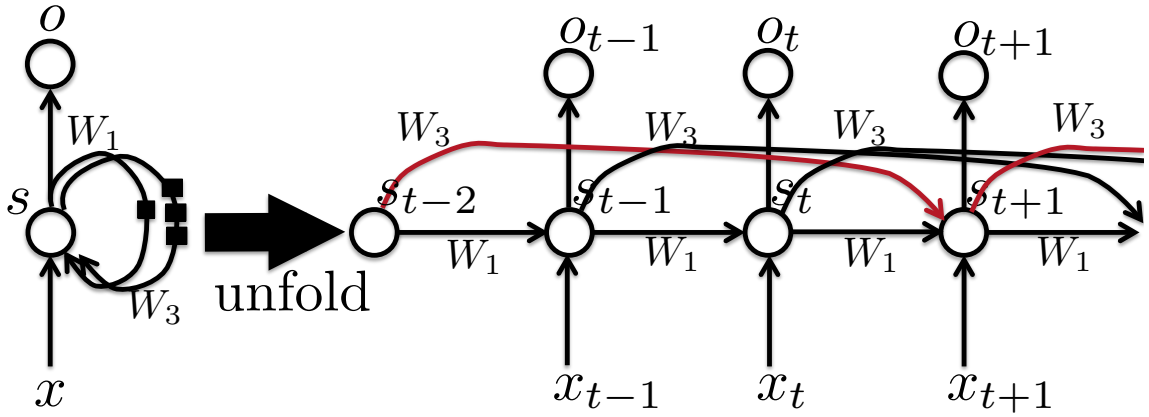
Storing bits robustly requires e-values<1

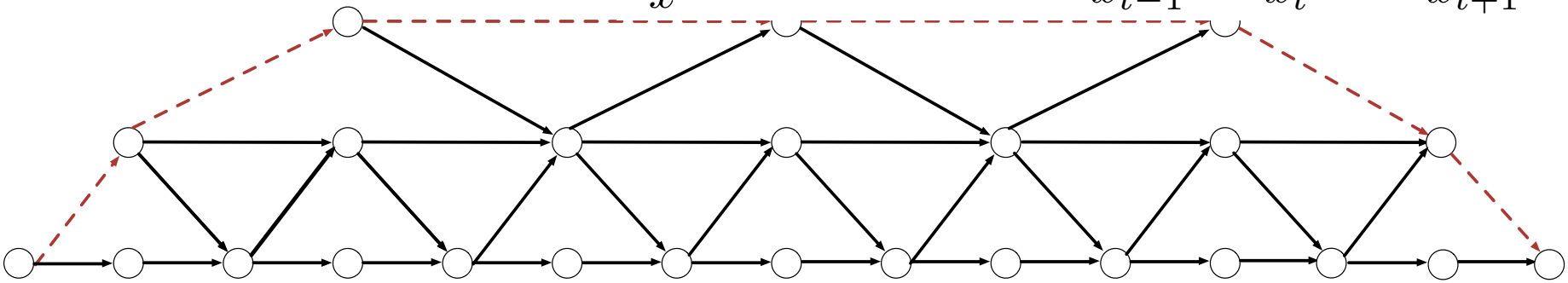
- Problems:
  - e-values of Jacobians > 1 → *gradients explode*
  - **or e-values < 1 → *gradients shrink & vanish***
  - or random → variance grows exponentially

→ **Gradient clipping** *(Pascanu, Mikolov & Bengio ICML2013)*

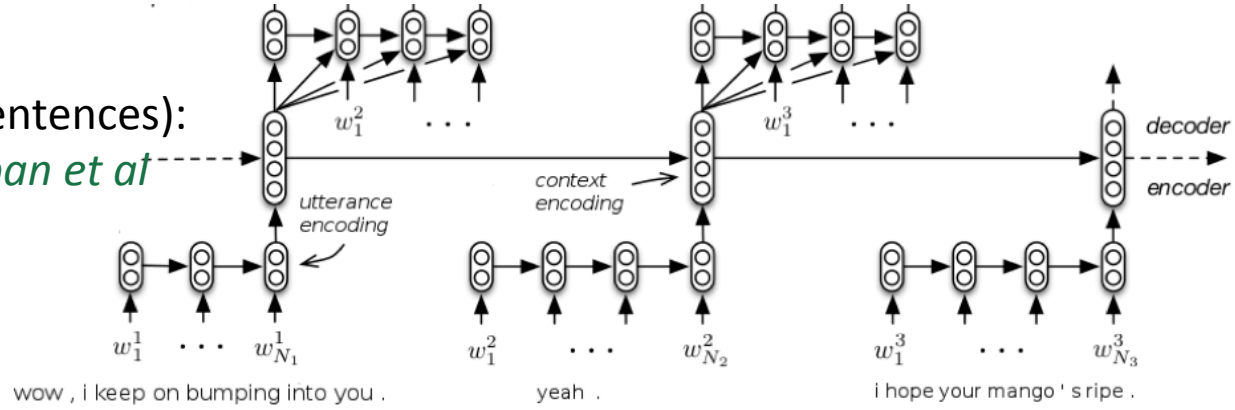
Delays & Hierarchies to Reach Farther

- Delays and multiple time scales, *Elhihi & Bengio NIPS 1995, Koutnik et al ICML 2014*
- *How to do this right?*
- *How to automatically and adaptively do it?*





Hierarchical RNNs (words / sentences):
Sordoni et al CIKM 2015, Serban et al AAAI 2016

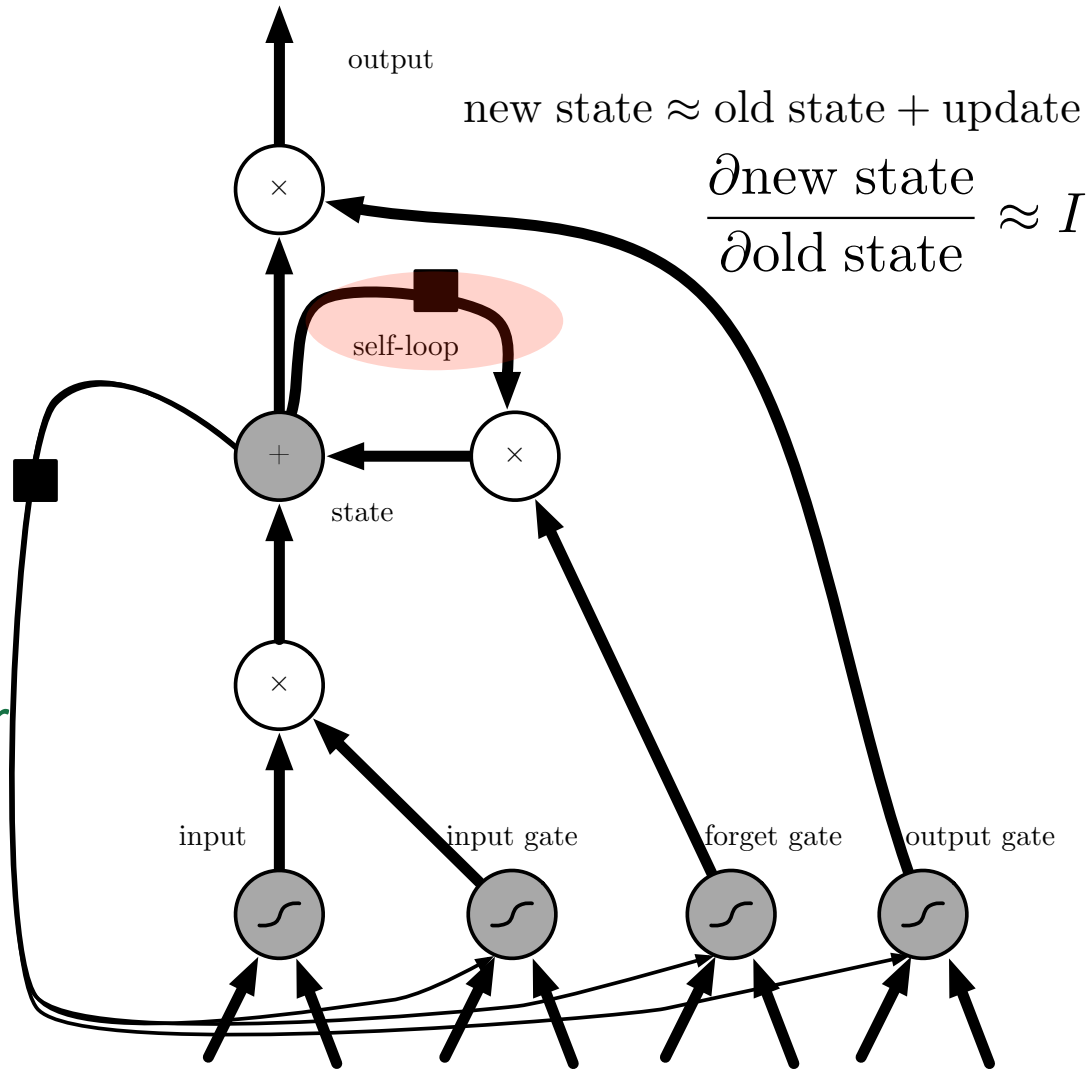


Fighting the vanishing gradient: LSTM & GRU

(Hochreiter 1991); first version of the LSTM, called Neural Long-Term Storage with self-loop

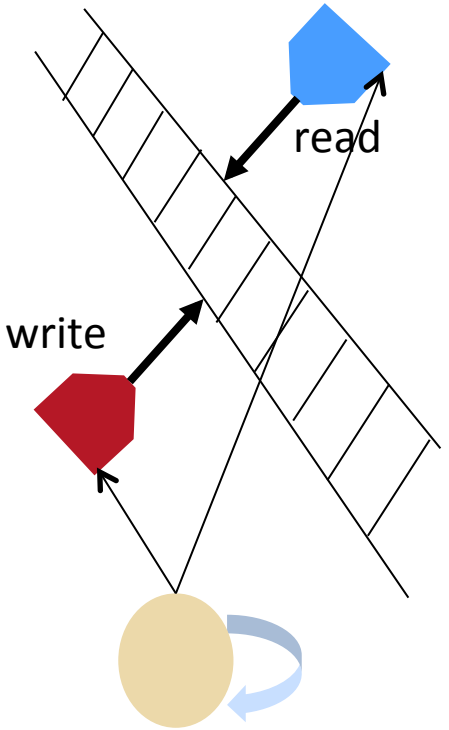
- Create a path where gradients can flow for longer with a self-loop
- Corresponds to an eigenvalue of Jacobian slightly less than 1
- LSTM is now **heavily used** *(Hochreiter & Schmidhuber 1997)*
- GRU light-weight version *(Cho et al 2014)*

LSTM: (Hochreiter & Schmidhuber 1997)



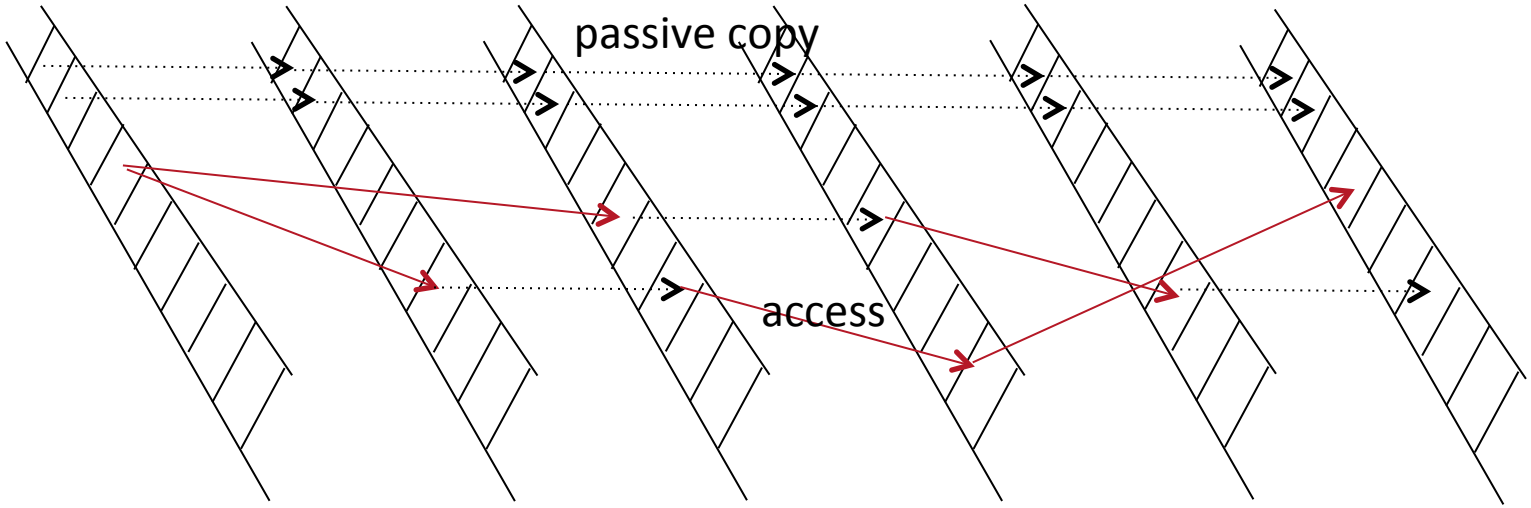
Fast Forward 20 years: Attention Mechanisms for Memory Access

- Neural Turing Machines *(Graves et al 2014)*
- and Memory Networks *(Weston et al 2014)*
- Use a content-based attention mechanism *(Bahdanau et al 2014)* to control the read and write access into a memory
- The attention mechanism outputs a softmax over memory locations

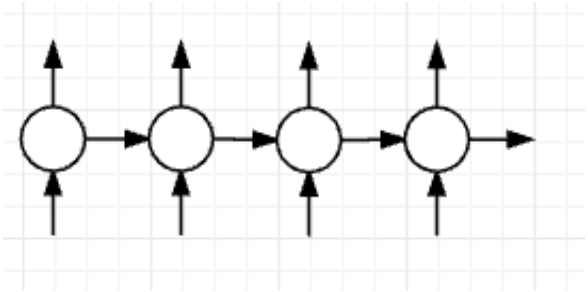


Large Memory Networks: Sparse Access Memory for Long-Term Dependencies

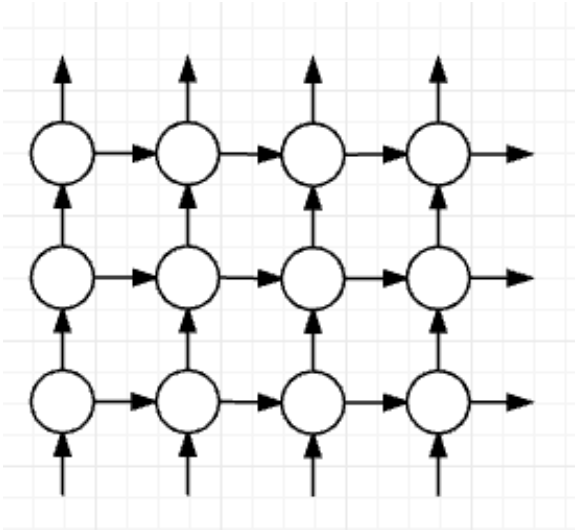
- Memory = part of the state
- Memory-based networks are special RNNs
- A mental state stored in an external memory can stay for arbitrarily long durations, until it is overwritten (partially or not)
- Forgetting = vanishing gradient.
- Memory = **higher-dimensional state**, avoiding or reducing the need for forgetting/vanishing



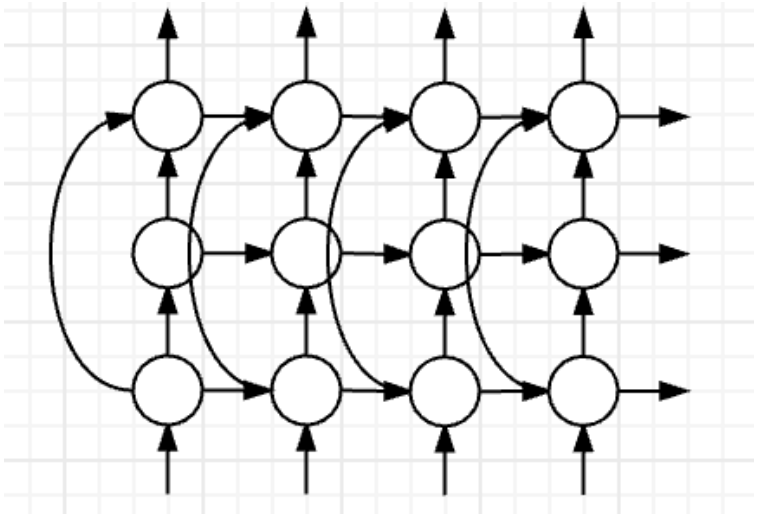
What RNN architectures do you use in your research?



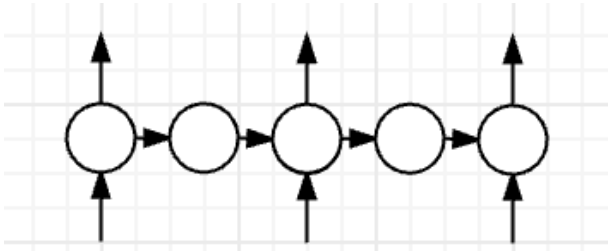
Simple RNN



Stacked RNN



Skip connections
Graves et al. 2013



"Deep" RNN
Pascanu et al. 2013

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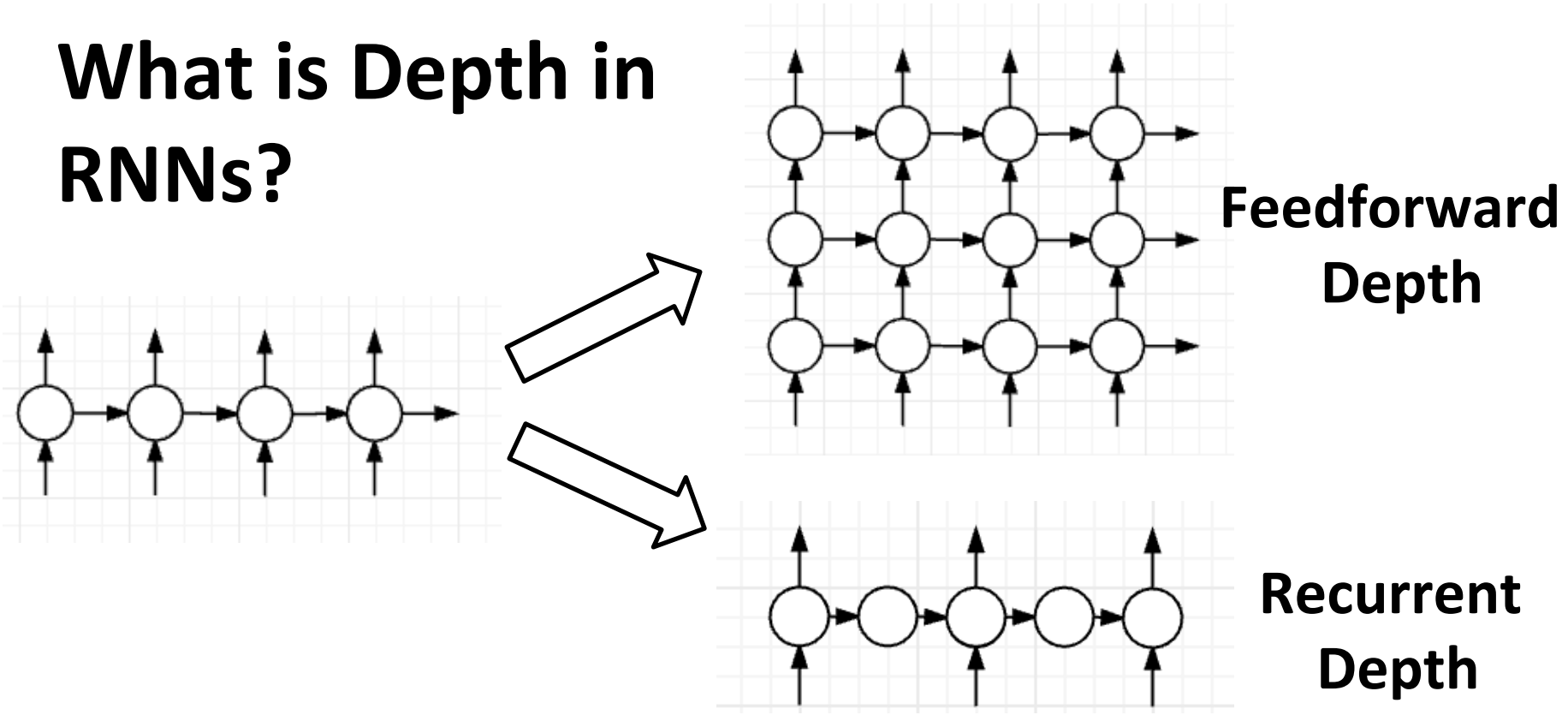
Depth in RNNs

What is Depth in RNNs?



(Architectural Complexity Measures of Recurrent Neural Networks Zhang et al 2016, arXiv:1602.08210)

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Depth in RNNs

- Recurrent depth :

the length of the *longest* path over time, divide by time

$$d_r = \lim_{n \to +\infty} \frac{\mathfrak{D}_i(n)}{n} = \max_{\vartheta \in C(\mathcal{G}_c)} \frac{l(\vartheta)}{\sigma_s(\vartheta)}$$

- Feedforward depth :

consider the input-output nonlinearities while eliminating the effect of recurrent depth

$$d_f = \sup_{i,n \in \mathbb{Z}} \mathfrak{D}_i^*(n) - n \cdot d_r$$

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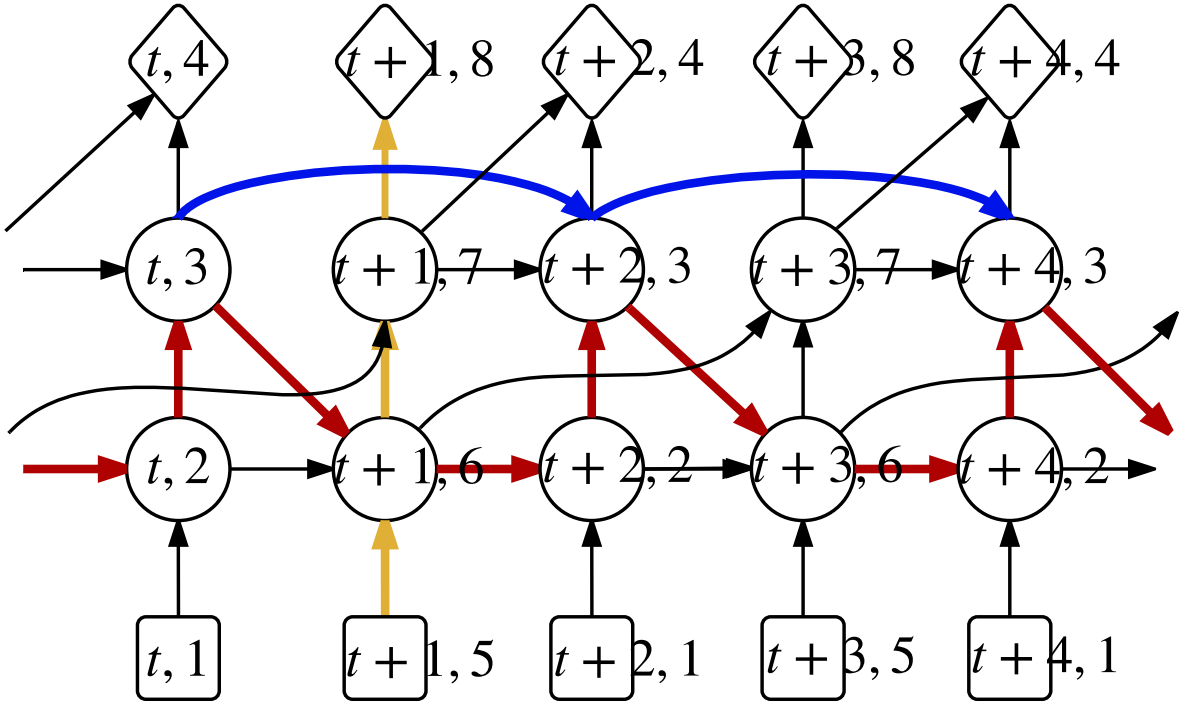
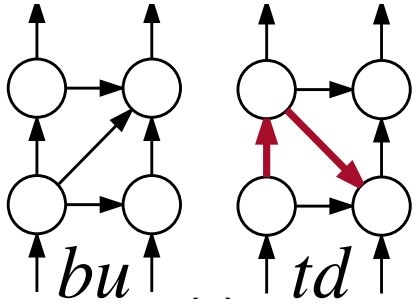
Designing the RNN Architecture

(Architectural Complexity Measures of Recurrent Neural Networks Zhang et al 2016, arXiv:1602.08210)

- **Recurrent depth**: max path length divided by sequence length
- **Feedforward depth**: max length from input to nearest output
- **Skip coefficient**: shortest path length divided sequence length



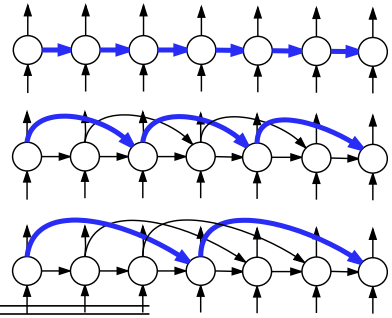
It makes a difference



- Impact of change in recurrent depth

DATASET	MODELS\ARCHS	*sh*	*st*	*bu*	*td*
PennTreebank	*tanh* RNN	1.54	1.59	1.54	**1.49**
text8	*tanh* RNN-SMALL	1.80	1.82	1.80	**1.77**
	tanh RNN-LARGE	1.69	1.67	1.64	**1.59**
	LSTM-SMALL	1.65	1.66	1.65	**1.63**
	LSTM-LARGE	1.52	1.53	1.52	**1.49**

- Impact of change in skip coefficient



RNN(*tanh*)	s = 1	s = 5	s = 9	s = 13	s = 21
MNIST	34.9	46.9	74.9	85.4	**87.8**
	s = 1	s = 3	s = 5	s = 7	s = 9
*p*MNIST	49.8	79.1	84.3	**88.9**	88.0

LSTM	s = 1	s = 3	s = 5	s = 7	s = 9
MNIST	56.2	**87.2**	86.4	86.4	84.8
	s = 1	s = 3	s = 4	s = 5	s = 6
*p*MNIST	28.5	25.0	60.8	62.2	**65.9**

Model	MNIST	*p*MNIST
*i*RNN[25]	97.0	≈82.0
*u*RNN[24]	95.1	91.4
LSTM[24]	**98.2**	88.0
RNN(*tanh*)[25]	≈35.0	≈35.0
*s*tanh(s = 21, 11)	98.1	**94.0**

Architecture, s		(1), 1	(2), 1	(3), $\frac{k}{2}$	(4), k
MNIST	k = 17	39.5	39.4	54.2	**77.8**
	k = 21	39.5	39.9	69.6	**71.8**
*p*MNIST	k = 5	55.5	66.6	74.7	**81.2**
	k = 9	55.5	71.1	78.6	**86.9**

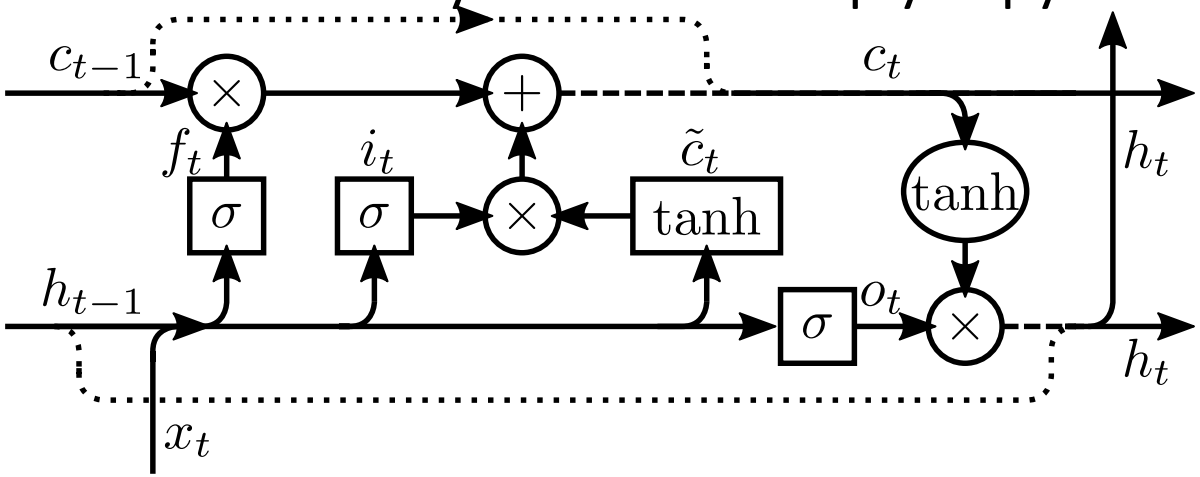
Table 2: Results for MNIST/*p*MNIST. **Top-left**: test accuracies with different s for *tanh* RNN. **Top-right**: test accuracies with different s for LSTM. **Bottom**: compared to previous results. **Bottom-right**: test accuracies for architectures (1), (2), (3) and (4) for *tanh* RNN.

Near-Orthogonality to Help Information Propagation

- Initialization to orthogonal recurrent W *(Saxe et al 2013, ICLR2014)*
- Unitary matrices: all e-values of matrix are 1 *(Arjowski, Amar & Bengio ICML 2016)*

$$\mathbf{W} = \mathbf{D}_3\mathbf{R}_2\mathcal{F}^{-1}\mathbf{D}_2\mathbf{\Pi}\mathbf{R}_1\mathcal{F}\mathbf{D}_1$$

- Zoneout: randomly choose to simply copy the state unchanged

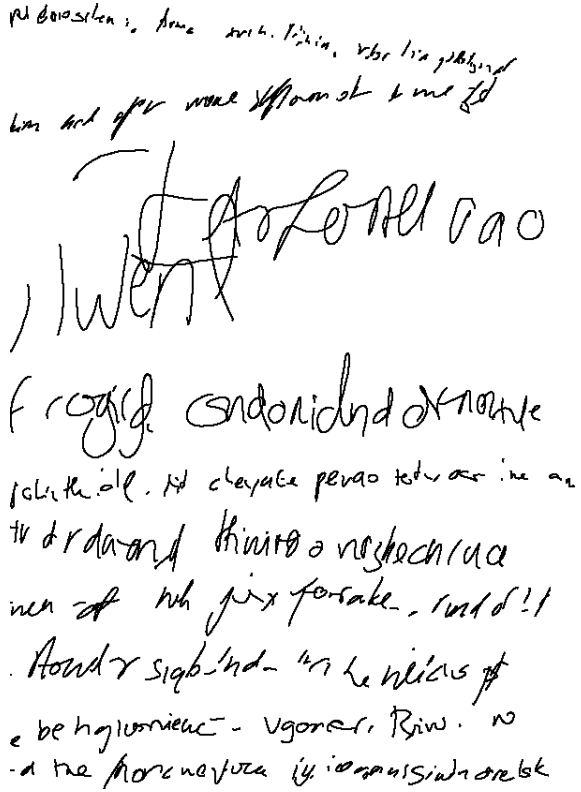


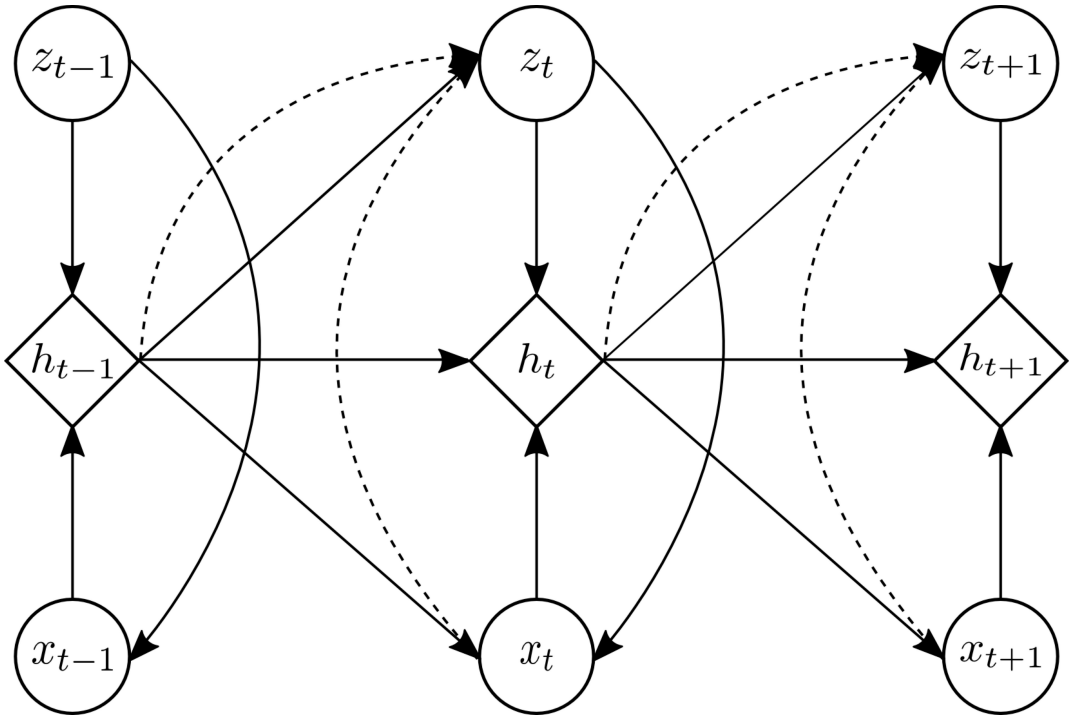
(Krueger et al 2016, submitted)

Variational Generative RNNs

Injecting higher-level variations / latent variables in RNNs

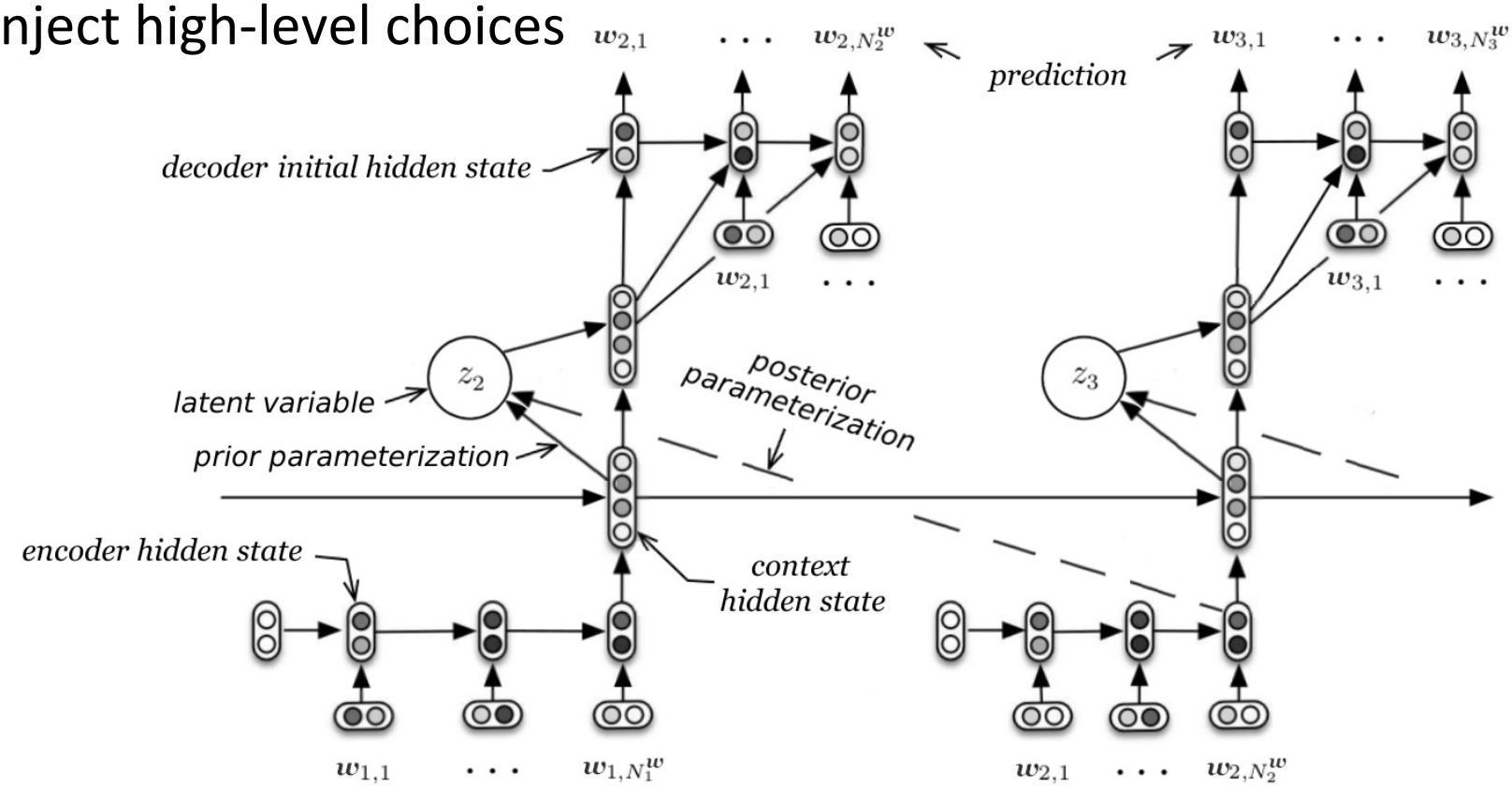
- *(Chung et al, NIPS'2015)*
- Regular RNNs have noise injected only in input space
- VRNNs also allow noise (latent variable) injected in top hidden layer; more « high-level » variability





Variational Hierarchical RNNs for Dialogue Generation (Serban et al 2016)

- Lower level = words of an utterance (turn of speech)
- Upper level = state of the dialogue
- Inject high-level choices

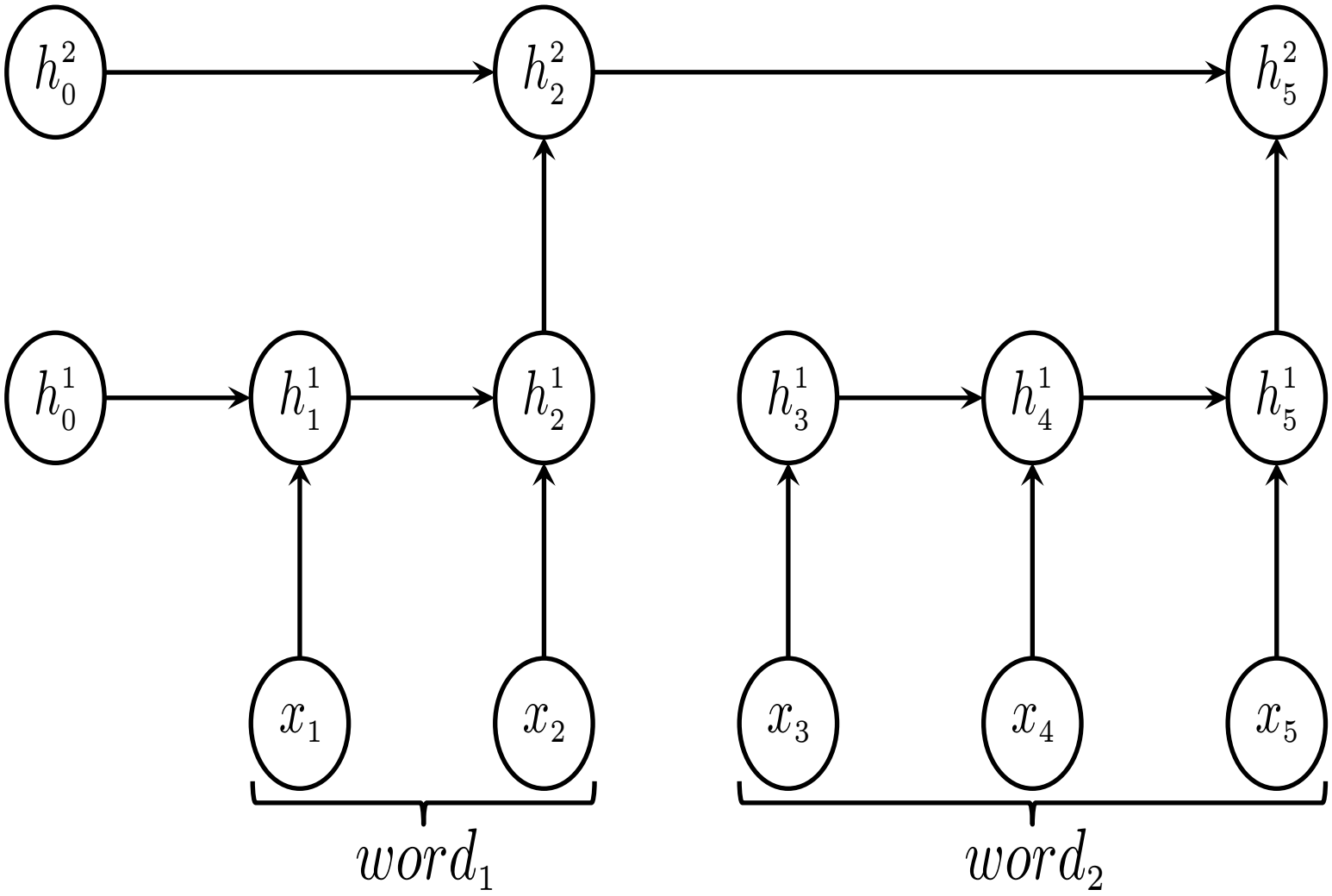


Hierarchical Multiscale Recurrent Neural Networks (Chung, Ahn & Bengio arXiv:1609.01704)

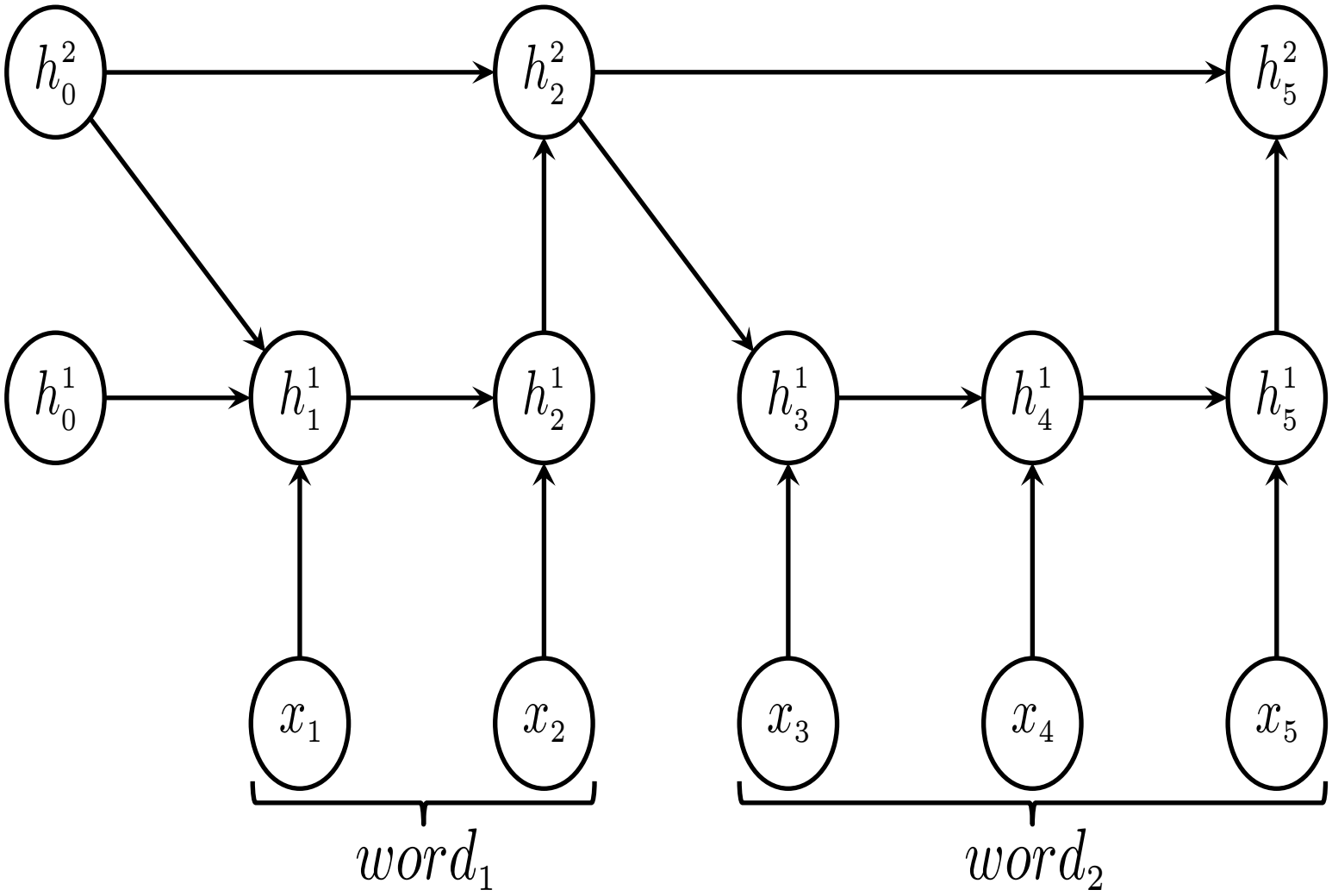
- How to learn to update higher-level states at the right points in time, corresponding to slower time scales, but not necessarily on a fixed (clockwork) schedule, rather an adaptive, dynamic one?

- If each level updates once every 10 steps of lower level then 3rd level updates once every 1000 raw steps!
  - Advantage for propagating gradients for long-term dependencies
  - Computational advantage (but need HARD decisions)

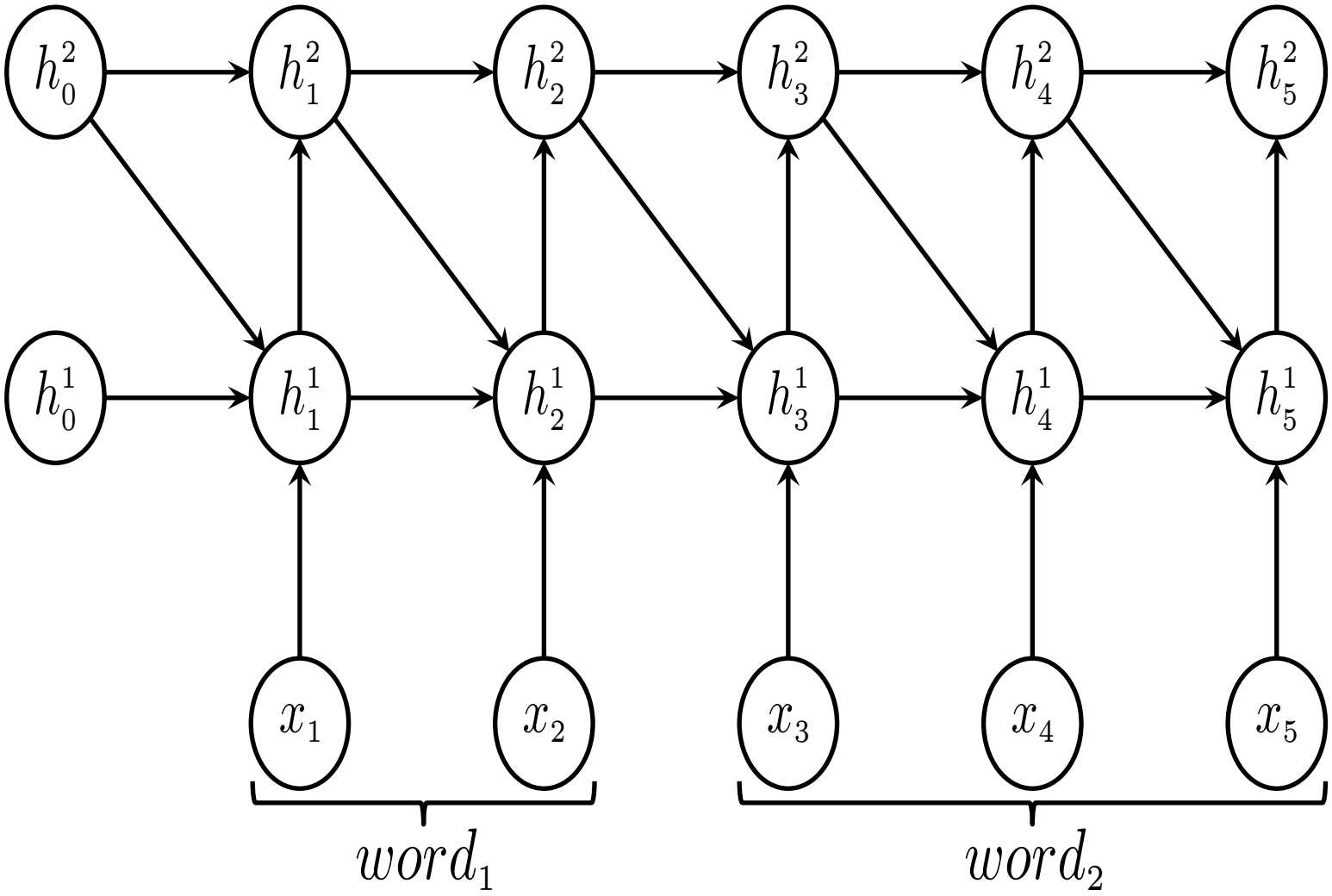
Motivation



Motivation



Motivation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x_1
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x_3
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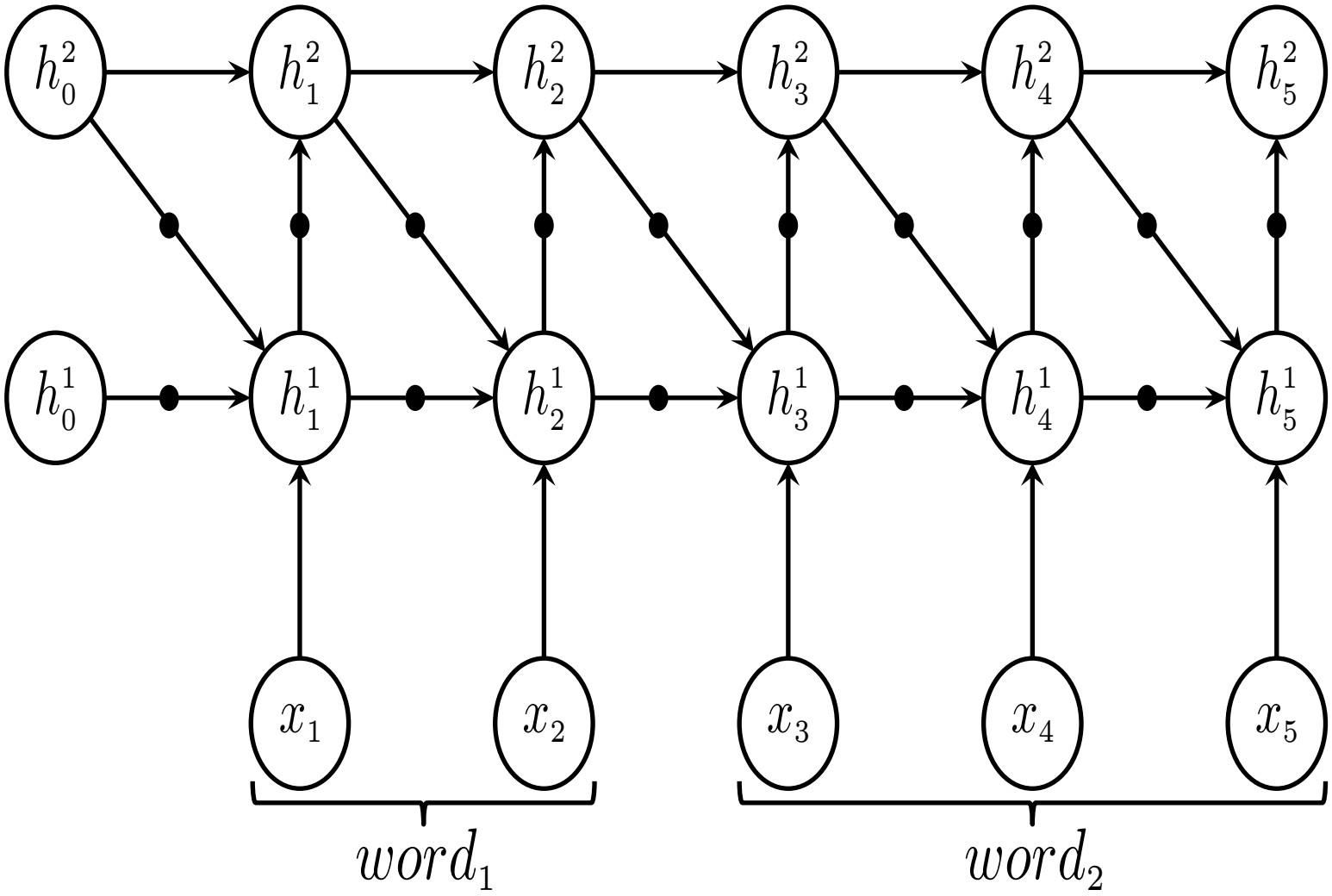
$word_1$
$word_2$

Motivation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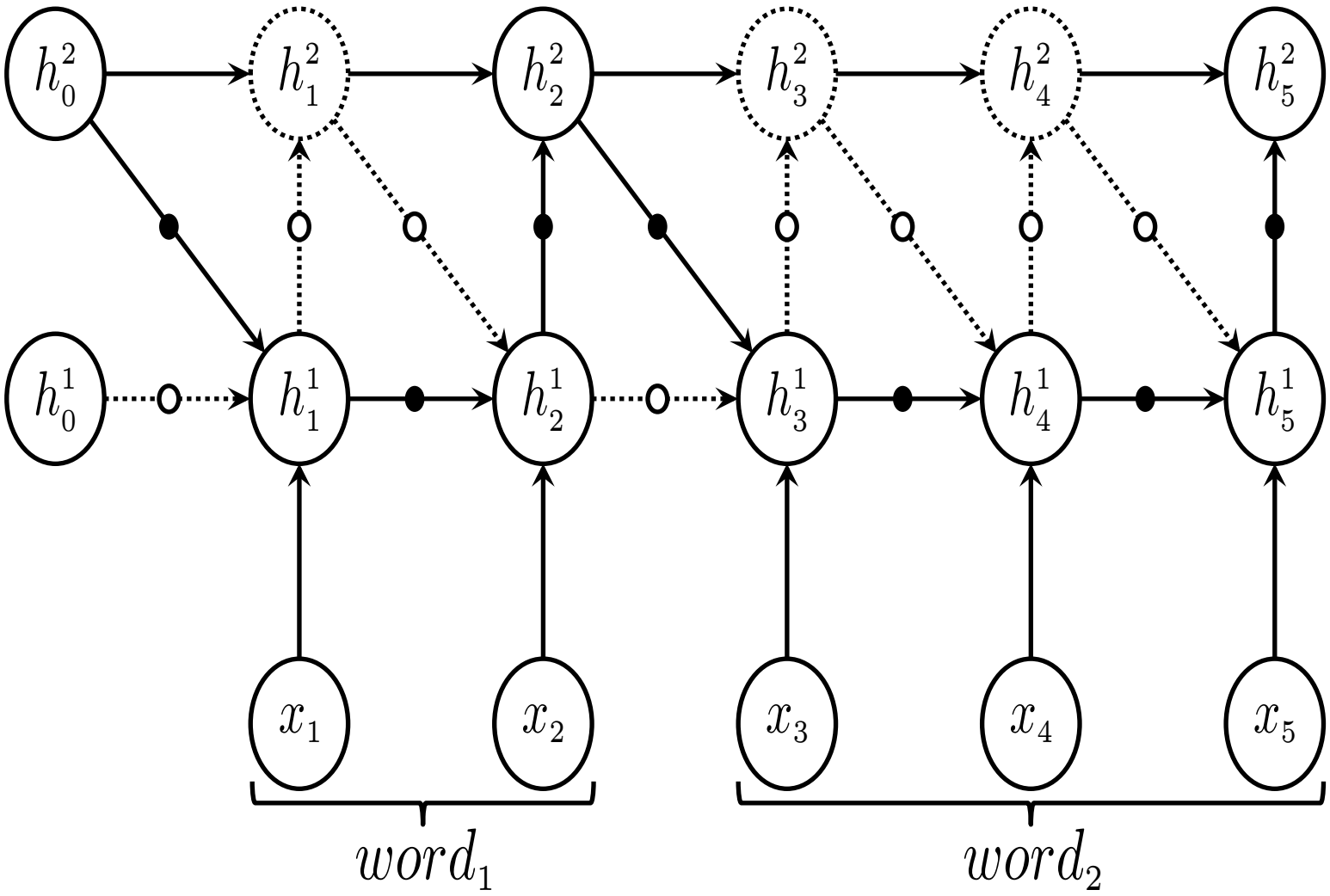
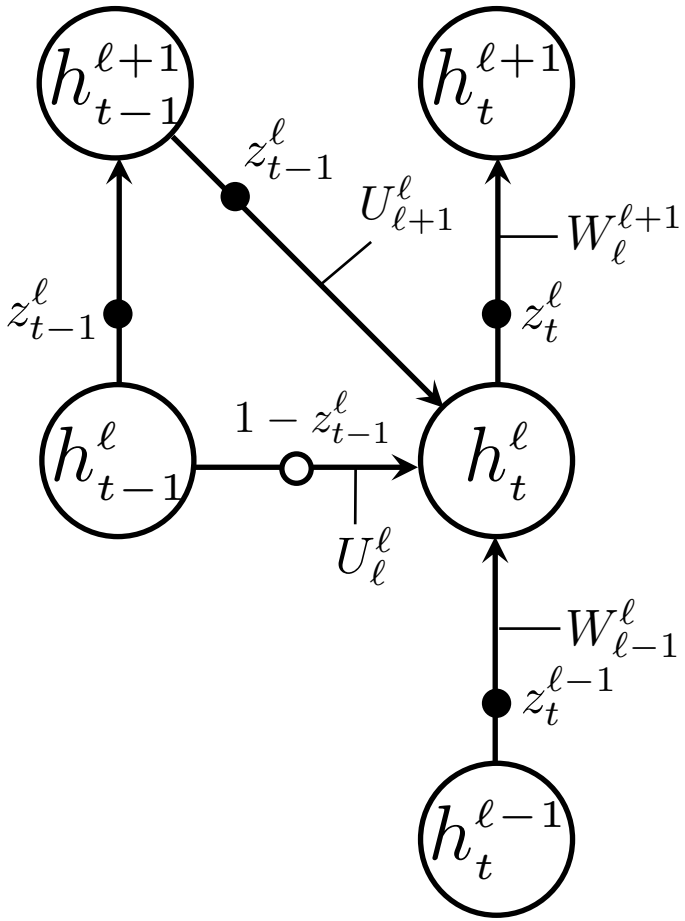
h_3^1
h_4^1
h_5^1
x_1
x_2
x_3
x_4
x_5
$word_1$
$word_2$

Motivation


h_0^2
h_1^2
h_2^2
h_3^2
h_4^2
h_5^2
h_0^1
h_1^1
h_2^1
h_3^1
h_4^1
h_5^1
x_1
x_2
x_3
x_4
x_5
$word_1$
$word_2$

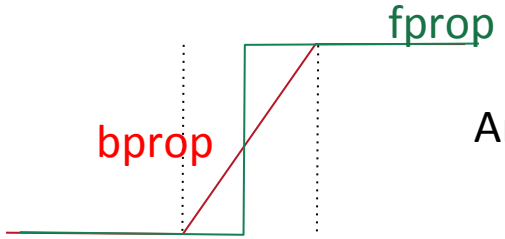
Hierarchical Multiscale Recurrent Neural Networks: Gating Mechanism

- Decide to copy the current level state vs updating it from lower-level state
- If updating it, restart the lower level RNN, conditioned on upper-level state
- Need to take a binary decision in order to actually get a pure copy and avoid leakage of gradients
- Need that binary decision to be stochastic to have a chance to learn it



How to backprop through stochastic binary decisions?

- REINFORCE: correlate the action with the reward, very high variance estimator
- **Straight-through estimator** (Hinton 2012, Bengio, Leonard & Courville arXiv 2013, Courbariaux et al NIPS 2015)
  - Heuristic but worked very well in several settings



Anneal the slope during training

- MuProp (Gu et al ICLR 2016, arXiv 1511.05176) combines a soft decision with a hard one, unbiased baseline of REINFORCE uses the gradient of the loss wrt soft decision
- VIMCO (Mnih & Rezende ICLR 2016, arXiv:1602.06725)

Hierarchical Multiscale Recurrent Neural Networks: Results (Chung, Ahn & Bengio arXiv:1609.01704)

- Automatically segmenting so as to better predict the next character

- SF-LSTM is not comparable (changes parameters on the fly)

Text8	
Model	**BPC**
td-LSTM (Zhang *et al.*, 2016)	1.63
HF-MRNN (Mikolov *et al.*, 2012)	1.54
MI-RNN (Wu *et al.*, 2016)	1.52
Skipping-RNN (Pachitariu and Sahani, 2013)	1.48
MI-LSTM (Wu *et al.*, 2016)	1.44
Batch-normalized LSTM (Cooijmans *et al.*, 2016)	1.36
HM-LSTM	**1.30**

Hutter Prize Wikipedia	
Model	**BPC**
SF-LSTM (Rocki, 2016b)*	1.37
Stacked LSTM (Graves, 2013)	1.67
MRNN (Sutskever *et al.*, 2011)	1.60
GF-LSTM (Chung *et al.*, 2015a)	1.58
Grid-LSTM (Kalchbrenner *et al.*, 2015)	1.47
MI-LSTM (Wu *et al.*, 2016)	1.44
Recurrent Highway Networks (Zilly *et al.*, 2016)	1.42
Recurrent Memory Array Structures (Rocki, 2016a)	1.40
Layer-normalized LSTM (Ba *et al.*, 2016)†	1.46
HM-LSTM	**1.37**

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Analysis

Penn Treebank Line 1

Penn Treebank Line 2

Penn Treebank Line 3

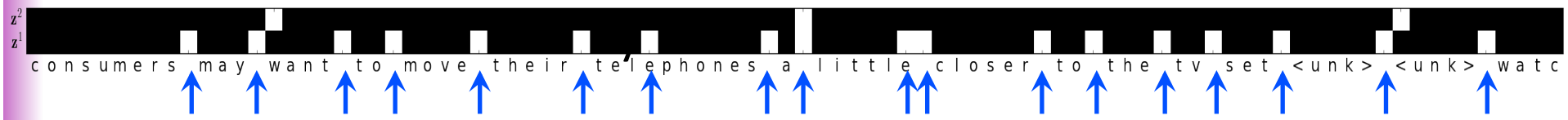
Wikipedia Line 1

Wikipedia Line 2

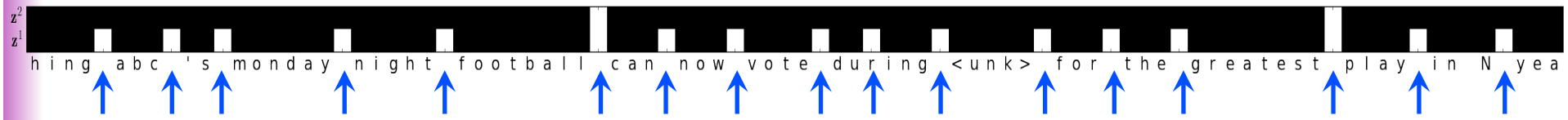
Wikipedia Line 3

Analysis

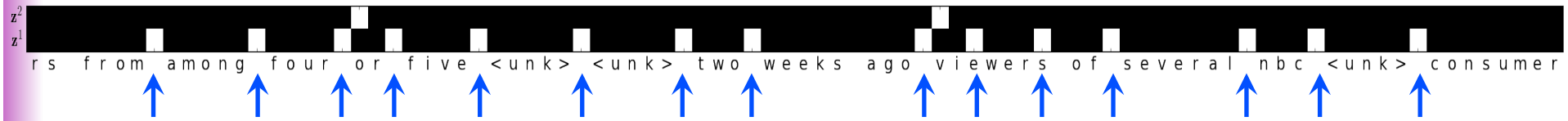
Penn Treebank Line 1



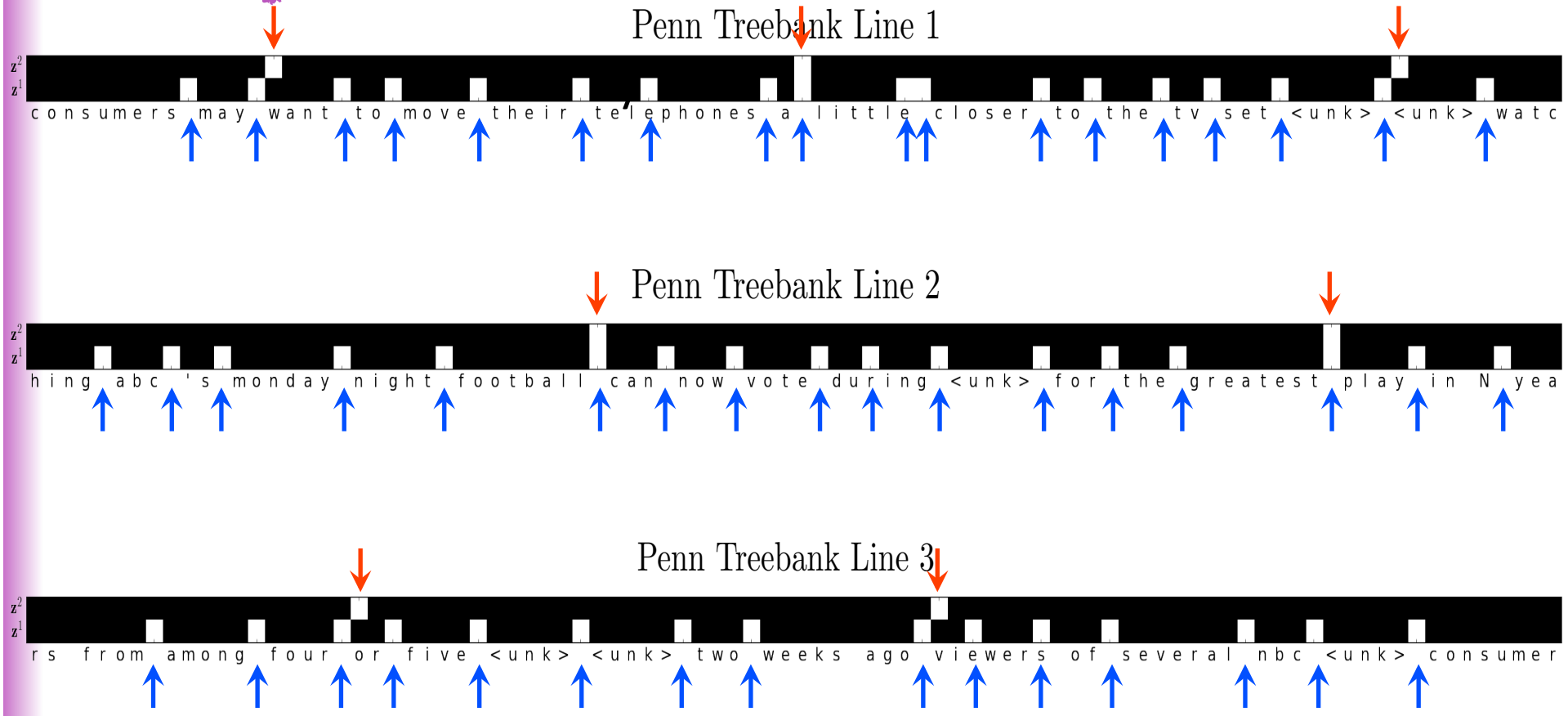
Penn Treebank Line 2



Penn Treebank Line 3

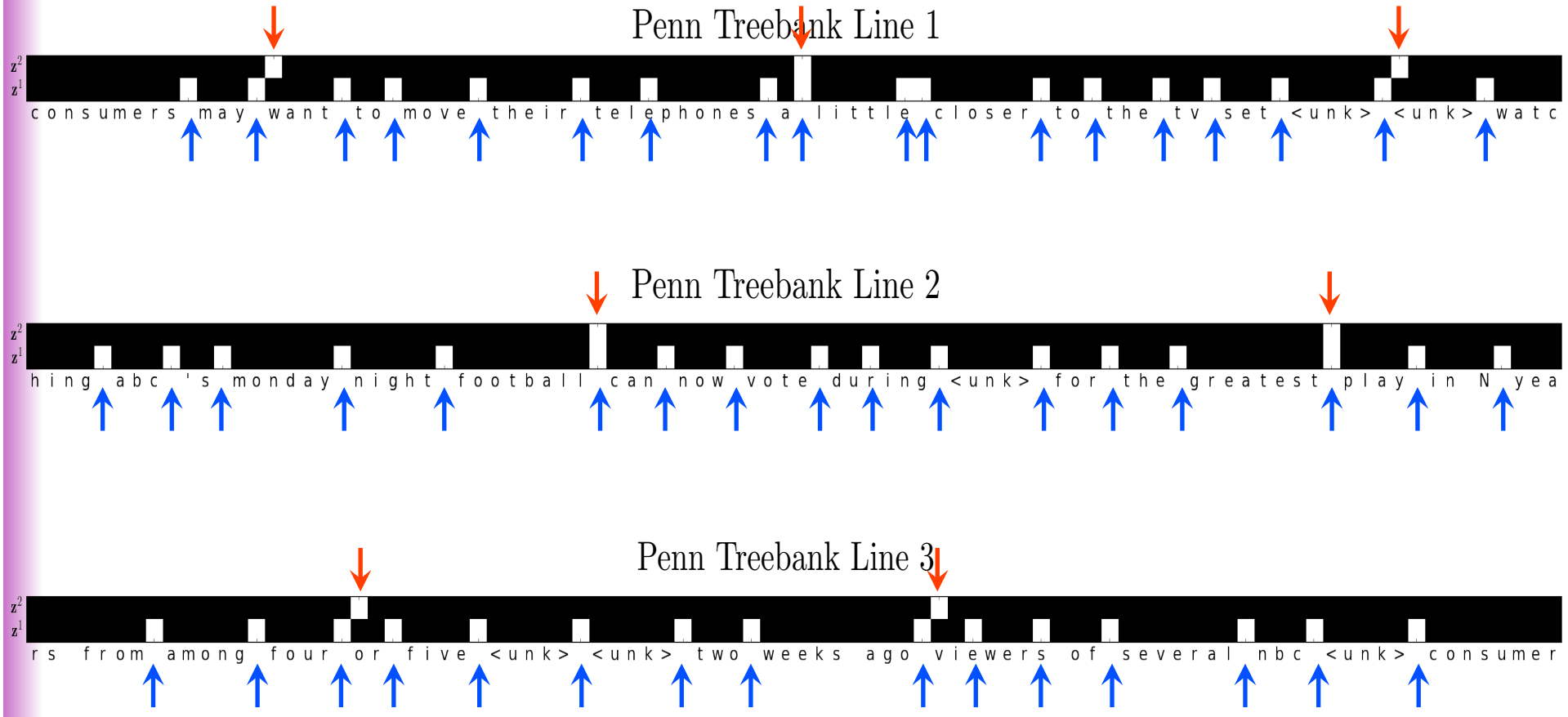


Analysis


Penn Treebank Line 1
consumers may want to move their telephones a little closer to the tv set <unk> <unk> watc
Penn Treebank Line 2
hing abc 's monday night football can now vote during <unk> for the greatest play in N yea
Penn Treebank Line 3
rs from among four or five <unk> <unk> two weeks ago viewers of several nbc <unk> consumer

Analysis

- Detected Boundaries by the HM-LSTM



$= 270 \times \frac{1}{5}$ $\approx 54 \times \frac{1}{8}$

1st layer: 270 updates, 2nd layer: 54 updates, 3rd layer: 7 updates

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