

Exponential inequalities in probability theory

Proposition 1 (*Hoeffding, 1963*). Let X_1, \dots, X_n be independent random variables with $a_i \leq X_i \leq b_i$, and let

$$s = \sum_{i=1}^n (b_i - a_i)^2,$$

then

$$\text{Prob} \left[\left| \sum_{i=1}^n (X_i - \mathbb{E}[X_i]) \right| > t \right] \leq 2e^{-\frac{2t^2}{s}}$$

for $t > 0$. ▲

Proposition 2. Let $X_i \sim \text{Bernoulli}[p_i]$ for $1 \leq i \leq n$ be independent, then

$$\text{Prob} \left[\left| \sum_{i=1}^n (X_i - p_i) \right| > t \right] \leq 2e^{-\frac{2t^2}{n}}$$

for $t > 0$. ▲

Proposition 3 (*Chernoff, 1952*). Let $X \sim \text{Binomial}[n, p]$, then

$$\text{Prob}[|X - np| > t] \leq 2e^{-\frac{2t^2}{n}}$$

for $t > 0$. ▲

Proposition 4. Let $X \sim \text{Normal}[0, 1]$, then

$$\text{Prob}[X > t] \leq e^{-\frac{t^2}{2}}$$

for $t > 0$.



References

- [Dev99] Luc Devroye. Probabilistic analysis of algorithms and data structures: Lecture notes. School of Computer Science, McGill University, Fall 1999.