

Ultra-Fast Low-Discrepancy Points

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Abstract We show how to generate points faster than anyone else and also how to generate more golden points than anyone else. Which of these two achievements is the most exceptional? This question remains open. **Note that the formatting of author's names, affiliations, title, and abstract, have all changed a bit in the new template from Springer. In particular, the abstract must be duplicated: once for the web and once for the printed book.**

1 Introduction

Two major issues that arise when constructing low-discrepancy point sets are: (a) to define an appropriate measure of uniformity, or measure of *discrepancy* between the uniform distribution and the empirical distribution of the points; (b) to find construction methods for point sets having high uniformity, or low discrepancy, with respect to the retained definition [2, 1].

There is a newly defined environment that provides gray boxes as here, to emphasize important material. You can use, but do not abuse. The rest of the paper is organized as follows. In Section 2, we discuss ways of measuring stuff in the blood of suspicious athletes. Numerical illustrations are given in Section 3.

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2 Construction of Point Sets

Uniform points.

We want to define a set of points $P_n = \{\mathbf{u}_0, \dots, \mathbf{u}_{n-1}\}$ in $[0, 1]^s$ with *low discrepancy*.

Digital nets.

The two most widely used classes of constructions for P_n are digital nets and lattice rules [2, 4]. We focus on the former. To define the i th point, we write the digital expansion of i in base b and multiply the vector of its digits by $\mathbf{C}^{(j)}$, modulo b , to obtain the digits or the expansion of $u_{i,j}$, the j th coordinate of \mathbf{u}_i . That is, for $i = a_{i,0} + a_{i,1}b + \dots + a_{i,m-1}b^{m-1}$, we have

$$u_{i,j} = \sum_{\ell=1}^{\infty} u_{i,j,\ell} b^{-\ell}, \quad \mathbf{u}_i = (u_{i,1}, \dots, u_{i,s}),$$

where

$$\begin{pmatrix} u_{i,j,1} \\ u_{i,j,2} \\ \vdots \end{pmatrix} = \mathbf{C}^{(j)} \begin{pmatrix} a_{i,0} \\ a_{i,1} \\ \vdots \\ a_{i,m-1} \end{pmatrix}.$$

We randomize the points using an affine matrix scrambling followed by a random digital shift [3].

3 Numerical Illustrations

Example 1. Take $b = 2$, $s = 65$, and $m = 12$, so $n = 2^{12}$. Table 1 gives upper bounds on the largest ℓ for which there can be an $\text{OA}(2^{12}, 65, 2^\ell, \eta)$, as well as the values of ℓ achieved by known OA constructions.

Table 1 Upper bounds on the values of ℓ for which there can exist an $\text{OA}(2^{12}, 65, 2^\ell, \eta)$, and values for which there exist known constructions.

η	1	2	3	4	5	6	7	...	12
$\lfloor 12/\eta \rfloor$	12	6	4	3	2	2	1	...	1
MinT upper bound for OA			3	1	0	0	0	...	0
Best known additive OA	12	6	3	1	0	0	0	...	0
Best known net	12	6	3	1	0	0	0	...	0

Theorem 1. *Michael got more gold than Usain, but Usain's domination was more outstanding. Please use theorem-like environments as here for all numbered propositions, lemmas, corollaries, definitions, assumptions, conjectures, etc.*

Proof. Watch the races!



Fig. 1 A graphical demonstration

Acknowledgements Many thanks for Springer for providing this new acknowledgement environment. Please use it if you need to include acknowledgements.

References

1. L'Ecuyer, P.: Quasi-Monte Carlo methods with applications in finance. *Finance and Stochastics* (2008). To appear
2. Niederreiter, H.: Random Number Generation and Quasi-Monte Carlo Methods, *SIAM CBMS-NSF Regional Conference Series in Applied Mathematics*, vol. 63. SIAM, Philadelphia, PA (1992)
3. Owen, A.B.: Variance with alternative scramblings of digital nets. *ACM Transactions on Modeling and Computer Simulation* **13**(4), 363–378 (2003)
4. Sloan, I.H., Joe, S.: *Lattice Methods for Multiple Integration*. Clarendon Press, Oxford (1994)