

# Call Center Routing Policy Using Call Waiting and Agent Idle Times

## Online Appendix

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In this appendix, we present an additional numerical example, the N-model. We also give the complete routing solutions for all the numerical examples, including the N-model. Finally, we give an empirical illustration on the convergence of the optimization algorithms presented in this paper.

### Appendix A: Experiments with the N-model

For the N-model, there are 2 call types and 2 agent groups. Group 1 (the specialists) can only serve call type 1 and group 2 (the generalists) can serve both call types. The skill sets are  $\mathcal{S}_1 = \{1\}$  and  $\mathcal{S}_2 = \{1, 2\}$ . We consider three different sets of parameters:

1. Arrival rates  $\lambda_1 = \lambda_2 = 100$ , SL targets  $t_1 = t_2 = 80\%$ ,  $y_1 = 11$ , and  $y_2 = 12$  agents.
2. Arrival rates  $\lambda_1 = \lambda_2 = 100$ , SL targets  $t_1 = t_2 = 85\%$ ,  $y_1 = 5$ , and  $y_2 = 18$  agents.
3. Arrival rates  $\lambda_1 = 110, \lambda_2 = 10$ , SL targets  $t_1 = 80\%, t_2 = 90\%$ ,  $y_1 = 10$ , and  $y_2 = 4$  agents.

In cases 1 and 3, the numbers of specialists and generalists are almost proportional to the arrival rates of call types 1 and 2, respectively, which is the intuitive staffing. In case 2, we have an unbalanced staffing, with too many generalists and too few specialists.

Tables 1 and 2 show the results and the routing solutions under the objective function  $F_S^1$ . The notations used to present the solutions are described in the section B. In the solutions, the delays of less than 1 second have negligible effects and they can be replaced with 0. Policy P does not work well for the balanced staffing cases 1 and 3, because too many generalists end up serving calls of type 1 and this results in a large imbalance between  $\Delta S_1$  and  $\Delta S_2$ . For the policy PD, in case 1, the solution gives a delay between group 2 and call type 1 of 63.4 seconds, well above the AWT of 20 seconds! The explanation is that due to the convex penalty function, it is preferable to degrade the SL of call type 1 to avoid a very bad SL for call type 2. The policies PT and PDT provide the best results for the balanced staffing cases 1 and 3. For case 1, the PDT solution gives priority to the generalists for calls of type 1, but with a delay of 13.3 seconds and there must be more than three idle generalists for them to serve a call of type 1 (that is, at least three generalists are reserved for call of type 2). The generalists giving priority to call type 1 seems to be in contradiction with the delay and idle agent threshold. But for the N-model, the priority list  $\mathcal{L}_2$  of the generalists is never used when  $m_{1,2} \geq 1$ . Indeed,  $\mathcal{L}_2$  would be used when a generalist completes a call and becomes idle. But when the number of idle generalists gets above  $m_{1,2}$ , either the waiting queue of call type 2 is empty or all calls of type 2 are currently

**Table 1** Results for the N-model examples under the objective function  $F_S^1$ .

Routing policy	Case 1					Case 2					Case 3				
	$\Delta S_1$	$\Delta S_2$	$A_1$	$A_2$	$f^*$	$\Delta S_1$	$\Delta S_2$	$A_1$	$A_2$	$f^*$	$\Delta S_1$	$\Delta S_2$	$A_1$	$A_2$	$f^*$
G	9.4	-21.6	1.9	9.7	468	-1.9	-8.4	3.0	4.4	74	-2.8	-38.9	4.9	15.2	1521
P	7.7	-16.1	2.5	8.1	260	-3.7	-2.8	4.4	2.9	21	-3.3	-25.6	5.1	10.2	665
PD	-6.6	-9.6	4.7	6.7	135	-3.7	-2.7	4.4	2.9	21	-3.9	-20.2	6.0	8.6	421
PT	-2.9	-7.2	5.1	6.2	60	-3.7	-2.8	4.4	2.9	21	-12.2	-1.2	7.6	3.2	149
PDT	-3.6	-6.8	5.4	6.0	<b>59</b>	-3.7	-2.7	4.4	2.9	21	-11.8	-2.5	7.5	6.8	<b>146</b>
WR	-3.4	-7.2	5.8	6.1	63	-1.2	-1.7	5.1	4.4	<b>4</b>	-10.4	-7.3	9.3	5.0	161

**Table 2** Solutions for the N-model examples under the objective function  $F_S^1$ .

Case	Routing solutions
1	<p><b>P:</b> <math>\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{2\}, \{1\})</math>.</p> <p><b>PD:</b> <math>\mathcal{G}_1 = (\{1, 2\}), \mathcal{L}_2 = (\{2\}, \{1\}), d_{1,2} = 63, d_{2,2} = 0.02</math>.</p> <p><b>PT:</b> <math>\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{1\}, \{2\}), m_{1,2} = 3</math>.</p> <p><b>PDT:</b> <math>\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{1\}, \{2\}), d_{1,1} = 0.008, d_{1,2} = 13, m_{1,2} = 3</math>.</p> <p><b>WR:</b> <math>q_{1,1} = 0.0004, q_{1,2} = -0.5, q_{2,2} = 0.0003, a_{1,1} = -12, a_{1,2} = -31, a_{2,2} = 34, b_{1,1} = 10000, b_{1,2} = 21, b_{2,2} = 1500</math>.</p>
2	<p><b>P:</b> <math>\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{2\}, \{1\})</math>.</p> <p><b>PD:</b> <math>\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{2\}, \{1\}), d_{1,2} = 0.1, d_{2,2} = 0.002</math>.</p> <p><b>PT:</b> <math>\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{2\}, \{1\}),</math> No threshold.</p> <p><b>PDT:</b> <math>\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{2\}, \{1\}), d_{1,2} = 0.2,</math> No threshold.</p> <p><b>WR:</b> <math>q_{1,1} = 13, q_{1,2} = 0.38, q_{2,2} = 0.51, a_{1,1} = -52, a_{1,2} = -65, a_{2,2} = -42, b_{1,1} = 54, b_{1,2} = 140, b_{2,2} = 142</math>.</p>
3	<p><b>P:</b> <math>\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{2\}, \{1\})</math>.</p> <p><b>PD:</b> <math>\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{2\}, \{1\}), d_{1,1} = 0.2, d_{1,2} = 19.9</math>.</p> <p><b>PT:</b> <math>\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{1, 2\}), m_{1,2} = 1</math>.</p> <p><b>PDT:</b> <math>\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{1\}, \{2\}), d_{1,1} = 0.001, d_{1,2} = 0.01, d_{2,2} = 14, m_{1,2} = 1</math>.</p> <p><b>WR:</b> <math>q_{1,1} = 2.9, q_{1,2} = -0.42, q_{2,2} = 0.2, a_{1,1} = -104, a_{1,2} = -114, a_{2,2} = -3.3, b_{1,1} = 444, b_{1,2} = 32, b_{2,2} = 289</math>.</p>

**Table 3** Results for the N-model examples under the objective function  $F_S^2$ .

Routing policy	Case 1					Case 2					Case 3				
	$\Delta S_1$	$\Delta S_2$	$A_1$	$A_2$	$f^*$	$\Delta S_1$	$\Delta S_2$	$A_1$	$A_2$	$f^*$	$\Delta S_1$	$\Delta S_2$	$A_1$	$A_2$	$f^*$
G	9.4	-21.6	1.9	9.7	1403	-1.9	-8.4	3.0	4.4	214	-2.8	-38.9	4.9	15.2	4547
P	7.7	-16.1	2.5	8.1	779	-3.7	-2.8	4.4	2.9	37	-3.3	-25.6	5.1	10.2	1974
PD	-12.8	-4.5	7.5	5.6	223	-4.9	-1.5	5.6	2.7	31	-26.7	-6.5	9.2	4.8	841
PT	-6.2	-5.7	6.0	5.8	135	-3.7	-2.8	4.4	2.9	37	-12.2	-1.2	7.6	3.2	152
PDT	-6.9	-5.4	6.2	5.8	<b>134</b>	-4.8	-1.6	5.5	2.7	31	-12.1	-1.3	7.6	3.6	<b>151</b>
WR	-6.7	-5.6	6.6	5.8	139	-4.2	-0.8	5.4	2.6	<b>19</b>	-13.4	-3.6	9.7	4.0	218

delayed ( $d_{2,2} > 0$ ), so the only available choices for an idle generalist are to serve a call of type 1 if one is waiting, or to stay idle. Although the WR policy is not the best performer for cases 1 and 3, its performance is close to that of policies PDT and PT. Note that most of the coefficients  $a_{k,g}$  are negative in the given solutions. There can be several good local optima for the weight parameters in WR, so good solutions can often be obtained with very different combinations of parameter values. For example, the following solution for case 1 is quite different from that given in Table 2 and also has a cost of 63:  $q_{1,1} = 15.0, q_{1,2} = -1.6, q_{2,2} = 13.3, a_{1,1} = 177.4, a_{1,2} = -158.6, a_{2,2} = 52.4, b_{1,1} = 73.8, b_{1,2} = 75.5, b_{2,2} = 63.0$ .

For the unbalanced staffing case 2, WR is the best performer. The policy P performs well in this case, because there are very few specialists and a large portion of the type 1 calls must be served by the generalists. Adding a delay or an idle agent threshold just degrades performance in this case. The optimal values of these parameters for policies PD, PT, and PDT are therefore all zero, and the solutions are the same as for policy P.

In Table 3, the objective function is  $F_S^2$ , which gives a higher penalty to the SL violation on call type 2. As expected, and as for the V-model, the SL of call type 2 increases or remains unchanged and the SL of call type 1 decreases or remains unchanged. Similar to what we saw in Table 1, the PDT policy gives the best results for cases 1 and 3, and WR is the best performer for case 2.

Table 4 shows the results for the objective function  $F_{SA}$ . They are similar to those obtained with  $F_S^1$  and  $F_S^2$ . In fact, the policies P, PD and PT have almost the same solutions as with  $F_S^1$ . The abandonments would have to be penalized much more (for example, by increasing  $b_1$  and  $b_2$ ) to see a significant difference with  $F_S^1$ . Again, the policies PT and PDT give the best results for the balanced staffing cases 1 and 3, and the WR policy gives the best result for the unbalanced case 2.

Table 5 presents the results with the objective function  $F_{SO}$ , where the unfairness in agent occupancies is penalized. Because the generalists have more skills than the specialists, they tend to be more busy if the unfairness is not penalized. After adding this penalty, we obtain solutions with slightly smaller SL than for  $F_S^1$ . The PDT policy is the best performer in cases 1 and 3. The WR policy is the best in case 2, and is very close to the best in case 1, but not so good in case 3. It also provides very good occupancy fairness in all cases,

**Table 4** Results for the N-model examples under the objective function  $F_{SA}$ .

Routing policy	Case 1					Case 2					Case 3				
	$\Delta S_1$	$\Delta S_2$	$A_1$	$A_2$	$f^*$	$\Delta S_1$	$\Delta S_2$	$A_1$	$A_2$	$f^*$	$\Delta S_1$	$\Delta S_2$	$A_1$	$A_2$	$f^*$
G	9.4	-21.6	1.9	9.7	565	-1.9	-8.4	3.0	4.4	103	-2.8	-38.9	4.9	15.2	1776
P	7.7	-16.1	2.5	8.1	332	-3.7	-2.8	4.4	2.9	49	-3.3	-25.6	5.1	10.2	795
PD	-6.6	-9.6	4.7	6.7	202	-3.7	-2.8	4.4	2.9	49	-3.9	-20.1	6.0	8.6	531
PT	-2.9	-7.2	5.1	6.2	<b>124</b>	-3.7	-2.8	4.4	2.9	49	-12.2	-1.2	7.6	3.2	<b>217</b>
PDT	-3.0	-7.1	5.2	6.1	<b>124</b>	-3.7	-2.8	4.4	2.9	49	-12.2	-1.2	7.6	3.2	<b>217</b>
WR	-2.9	-7.4	5.6	6.2	134	-1.8	-3.6	3.8	3.5	<b>43</b>	-10.6	-7.7	8.2	5.1	264

**Table 5** Results for the N-model examples under the objective function  $F_{SO}$ .

Routing policy	Case 1					Case 2					Case 3				
	$\Delta S_1$	$\Delta S_2$	$O_1$	$O_2$	$f^*$	$\Delta S_1$	$\Delta S_2$	$O_1$	$O_2$	$f^*$	$\Delta S_1$	$\Delta S_2$	$O_1$	$O_2$	$f^*$
G	9.4	-21.6	74	89	1494	-1.9	-8.4	80	85	194	-2.8	-38.9	80	83	1559
P	7.7	-16.1	79	86	505	-3.5	-3.7	80	85	134	-3.0	-35.9	80	83	1334
PD	-7.6	-8.8	82	82	137	-3.5	-3.7	80	85	134	-3.0	-35.8	80	83	1328
PT	-6.2	-5.7	82	82	71	-3.5	-3.7	80	85	134	-11.4	-6.4	81	76	280
PDT	-3.9	-6.8	81	82	<b>63</b>	-6.5	-2.5	80	83	95	-12.0	-6.9	78	77	<b>194</b>
WR	-4.8	-6.5	81	82	66	-3.8	-1.7	84	84	<b>18</b>	-15.3	-13.9	73	71	447

## Appendix B: Solutions for the numerical examples

In this section, we report the parameters of the best solution found for each routing policy for all the tables shown in the numerical section. We show the parameters of the policies P, PD, PT, PDT and WR. The policy G has no parameters.

The delays for the policies PD and PDT are given in seconds. For the WR policy, the call waiting times  $w_k$  and agent idle times  $v_g$  are measured in hours. To save space, we do not report the trivial cases where the type-to-group and group-to-type priority lists have a single group or a single call type. We also do not report the delay when  $d_{k,g} = 0$ , nor the idle agent threshold when  $m_{k,g} = 0$ . We mention ‘‘No delay’’ if all  $d_{k,g} = 0$  and ‘‘No threshold’’ if all  $m_{k,g} = 0$ . The parameters  $q_{k,g}$ ,  $a_{k,g}$  and  $b_{k,g}$  for the WR policy are not reported when their values are 0. Refer to the descriptions of the routing policies for the definitions of the parameters.

### B.1. Solutions for the V-model

V-model, objective function  $F_S^1$ , case 1:

**P:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ .

**PD:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ ,  $d_{2,1} = 5.889$ .

**PT:**  $\mathcal{L}_1 = (\{1\}, \{2\})$ , No threshold.

**PDT:**  $\mathcal{L}_1 = (\{1\}, \{2\})$ ,  $d_{1,1} = 5.76$ , No threshold.

**WR:**  $q_{1,1} = 10.071$ ,  $q_{2,1} = 10.233$ ,  $a_{1,1} = -113.495$ ,  $a_{2,1} = -128.399$ ,  $b_{1,1} = 219.662$ ,  $b_{2,1} = 62.436$ .

V-model, objective function  $F_S^1$ , case 2:

**P:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ .

**PD:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ ,  $d_{1,1} = 19.995$ .

**PT:**  $\mathcal{L}_1 = (\{1\}, \{2\})$ ,  $m_{1,1} = 1$ .

**PDT:**  $\mathcal{L}_1 = (\{1, 2\})$ ,  $d_{2,1} = 10.449$ ,  $m_{1,1} = 1$ .

**WR:**  $q_{1,1} = -0.088$ ,  $q_{2,1} = 12.660$ ,  $a_{1,1} = -80.544$ ,  $a_{2,1} = 125.006$ ,  $b_{1,1} = 189.448$ ,  $b_{2,1} = 104.838$ .

V-model, objective function  $F_S^1$ , case 3:

**P:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ .

**PD:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ ,  $d_{1,1} = 19.999$ .

**PT:**  $\mathcal{L}_1 = (\{1\}, \{2\})$ ,  $m_{1,1} = 1$ .

**PDT:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ ,  $d_{1,1} = 19.961$ , No threshold.

**WR:**  $q_{1,1} = -0.0129$ ,  $q_{2,1} = 1.0$ ,  $a_{1,1} = -493.409$ ,  $a_{2,1} = 9.977$ ,  $b_{1,1} = 1713.836$ ,  $b_{2,1} = 10.0$ .

V-model, objective function  $F_S^2$ , case 1:

**P:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ .

**PD:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ ,  $d_{2,1} = 4.027$ .

**PT:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ , No threshold.

**PDT:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ ,  $d_{2,1} = 4.027$ , No threshold.

**WR:**  $q_{1,1} = 0.123$ ,  $q_{2,1} = 0.374$ ,  $a_{1,1} = -3.332$ ,  $a_{2,1} = -14.872$ ,  $b_{1,1} = 12.262$ ,  $b_{2,1} = 35.962$ .

V-model, objective function  $F_S^2$ , case 2:

**P:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ .

**PD:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ ,  $d_{1,1} = 19.999$ .

**PT:**  $\mathcal{L}_1 = (\{1\}, \{2\})$ ,  $m_{1,1} = 1$ .

**PDT:**  $\mathcal{L}_1 = (\{1\}, \{2\})$ ,  $d_{2,1} = 4.028$ ,  $m_{1,1} = 1$ .

**WR:**  $q_{1,1} = -0.132$ ,  $q_{2,1} = 7.802$ ,  $a_{1,1} = -73.728$ ,  $a_{2,1} = 68.006$ ,  $b_{1,1} = 168.349$ ,  $b_{2,1} = 93.498$ .

V-model, objective function  $F_S^2$ , case 3:

**P:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ .

**PD:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ ,  $d_{1,1} = 19.996$ .

**PT:**  $\mathcal{L}_1 = (\{1\}, \{2\})$ ,  $m_{1,1} = 1$ .

**PDT:**  $\mathcal{L}_1 = (\{1\}, \{2\})$ ,  $d_{2,1} = 19.996$ ,  $m_{1,1} = 1$ .

**WR:**  $q_{1,1} = 0.245$ ,  $q_{2,1} = 1.508$ ,  $a_{1,1} = -144.980$ ,  $a_{2,1} = 10.0$ ,  $b_{1,1} = 424.014$ ,  $b_{2,1} = 10.0$ .

V-model, objective function  $F_{SA}$ , case 1:

**P:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ .

**PD:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ ,  $d_{2,1} = 0.870$ .

**PT:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ , No threshold.

**PDT:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ ,  $d_{2,1} = 0.870$ , No threshold.

**WR:**  $q_{1,1} = 13.452$ ,  $q_{2,1} = 14.113$ ,  $a_{1,1} = -13.328$ ,  $a_{2,1} = -76.445$ ,  $b_{1,1} = 35.849$ ,  $b_{2,1} = 95.283$ .

V-model, objective function  $F_{SA}$ , case 2:

**P:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ .

**PD:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ ,  $d_{1,1} = 0.002$ ,  $d_{2,1} = 0.002$ .

**PT:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ , No threshold.

**PDT:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ ,  $d_{1,1} = 0.067$ , No threshold.

**WR:**  $q_{1,1} = -0.717$ ,  $q_{2,1} = 14.429$ ,  $a_{1,1} = -3.899$ ,  $a_{2,1} = 220.591$ ,  $b_{1,1} = 162.404$ ,  $b_{2,1} = 10.886$ .

V-model, objective function  $F_{SA}$ , case 3:

**P:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ .

**PD:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ ,  $d_{1,1} = 0.156$ .

**PT:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ , No threshold.

**PDT:**  $\mathcal{L}_1 = (\{2\}, \{1\})$ ,  $d_{1,1} = 0.159$ , No threshold.

**WR:**  $q_{1,1} = -0.377$ ,  $q_{2,1} = 20.255$ ,  $a_{1,1} = -6.442$ ,  $a_{2,1} = 51.652$ ,  $b_{1,1} = 161.680$ ,  $b_{2,1} = 52.142$ .

## B.2. Solutions for the N-model

N-model, objective function  $F_S^1$ , case 1:

**P:**  $\mathcal{G}_1 = (\{1\}, \{2\})$ ,  $\mathcal{L}_2 = (\{2\}, \{1\})$ .

**PD:**  $\mathcal{G}_1 = (\{1, 2\})$ ,  $\mathcal{L}_2 = (\{2\}, \{1\})$ ,  $d_{1,2} = 63.372$ ,  $d_{2,2} = 0.016$ .

**PT:**  $\mathcal{G}_1 = (\{1\}, \{2\})$ ,  $\mathcal{L}_2 = (\{1\}, \{2\})$ ,  $m_{1,2} = 3$ .

**PDT:**  $\mathcal{G}_1 = (\{1\}, \{2\})$ ,  $\mathcal{L}_2 = (\{1\}, \{2\})$ ,  $d_{1,1} = 0.008$ ,  $d_{1,2} = 13.325$ ,  $m_{1,2} = 3$ .

**WR:**  $q_{1,1} = 4.182E-4$ ,  $q_{1,2} = -0.500$ ,  $q_{2,2} = 2.555E-4$ ,  $a_{1,1} = -12.042$ ,  $a_{1,2} = -31.186$ ,  $a_{2,2} = 34.129$ ,  $b_{1,1} = 10010.0$ ,  $b_{1,2} = 21.083$ ,  $b_{2,2} = 1456.523$ .

N-model, objective function  $F_S^1$ , case 2:

**P:**  $\mathcal{G}_1 = (\{1\}, \{2\})$ ,  $\mathcal{L}_2 = (\{2\}, \{1\})$ .

**PD:**  $\mathcal{G}_1 = (\{1\}, \{2\})$ ,  $\mathcal{L}_2 = (\{2\}, \{1\})$ ,  $d_{1,2} = 0.129$ ,  $d_{2,2} = 0.002$ .

**PT:**  $\mathcal{G}_1 = (\{1\}, \{2\})$ ,  $\mathcal{L}_2 = (\{2\}, \{1\})$ , No threshold.

**PDT:**  $\mathcal{G}_1 = (\{1\}, \{2\})$ ,  $\mathcal{L}_2 = (\{2\}, \{1\})$ ,  $d_{1,2} = 0.225$ , No threshold.

**WR:**  $q_{1,1} = 13.210$ ,  $q_{1,2} = 0.378$ ,  $q_{2,2} = 0.511$ ,  $a_{1,1} = -51.814$ ,  $a_{1,2} = -65.235$ ,  $a_{2,2} = -42.404$ ,  $b_{1,1} = 54.222$ ,  $b_{1,2} = 139.623$ ,  $b_{2,2} = 142.054$ .

N-model, objective function  $F_S^1$ , case 3:

**P:**  $\mathcal{G}_1 = (\{1\}, \{2\})$ ,  $\mathcal{L}_2 = (\{2\}, \{1\})$ .

**PD:**  $\mathcal{G}_1 = (\{1\}, \{2\})$ ,  $\mathcal{L}_2 = (\{2\}, \{1\})$ ,  $d_{1,1} = 0.192$ ,  $d_{1,2} = 19.997$ .

**PT:**  $\mathcal{G}_1 = (\{1\}, \{2\})$ ,  $\mathcal{L}_2 = (\{1, 2\})$ ,  $m_{1,2} = 1$ .

**PDT:**  $\mathcal{G}_1 = (\{1\}, \{2\})$ ,  $\mathcal{L}_2 = (\{1\}, \{2\})$ ,  $d_{1,1} = 0.001$ ,  $d_{1,2} = 0.009$ ,  $d_{2,2} = 14.339$ ,  $m_{1,2} = 1$ .

**WR:**  $q_{1,1} = 2.907, q_{1,2} = -0.415, q_{2,2} = 0.196, a_{1,1} = -104.330, a_{1,2} = -113.483, a_{2,2} = -3.317, b_{1,1} = 444.360, b_{1,2} = 31.786, b_{2,2} = 288.747.$

N-model, objective function  $F_S^2$ , case 1:

**P:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{2\}, \{1\}).$

**PD:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{2\}, \{1\}), d_{1,2} = 178.946.$

**PT:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{1\}, \{2\}), m_{1,2} = 4.$

**PDT:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{L}_2 = (\{2\}, \{1\}), d_{1,2} = 14.405, m_{1,2} = 4.$

**WR:**  $q_{1,1} = 0.0790, q_{1,2} = -0.869, q_{2,2} = -0.052, a_{1,1} = -16.433, a_{1,2} = -76.653, a_{2,2} = 45.443, b_{1,1} = 45228.987, b_{1,2} = 25.065, b_{2,2} = 72.981.$

N-model, objective function  $F_S^2$ , case 2:

**P:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{2\}, \{1\}).$

**PD:**  $\mathcal{G}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}), d_{1,1} = 0.148, d_{1,2} = 9.933, d_{2,2} = 0.024.$

**PT:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{2\}, \{1\}),$  No threshold.

**PDT:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{2\}, \{1\}), d_{1,2} = 9.003,$  No threshold.

**WR:**  $q_{1,1} = 12.496, q_{1,2} = 0.701, q_{2,2} = 9.687, a_{1,1} = 75.631, a_{1,2} = -60.259, a_{2,2} = 19.064, b_{1,1} = 38.021, b_{1,2} = 181.091, b_{2,2} = 244.212.$

N-model, objective function  $F_S^2$ , case 3:

**P:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{2\}, \{1\}).$

**PD:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{L}_2 = (\{2\}, \{1\}), d_{1,1} = 0.006, d_{1,2} = 81.676.$

**PT:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{1\}, \{2\}), m_{1,2} = 1.$

**PDT:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{1\}, \{2\}), d_{2,2} = 1.446, m_{1,2} = 1.$

**WR:**  $q_{1,1} = 0.046, q_{1,2} = -0.993, q_{2,2} = -0.064, a_{1,1} = -8.241, a_{1,2} = -128.853, a_{2,2} = 16.765, b_{1,1} = 671.092, b_{1,2} = 30.230, b_{2,2} = 9.617.$

N-model, objective function  $F_{SA}$ , case 1:

**P:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{2\}, \{1\}).$

**PD:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{2\}, \{1\}), d_{1,2} = 63.247.$

**PT:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{1\}, \{2\}), m_{1,2} = 3.$

**PDT:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{1\}, \{2\}), d_{1,2} = 3.129, m_{1,2} = 3.$

**WR:**  $q_{1,1} = 13.541, q_{1,2} = -2.798, q_{2,2} = 14.013, a_{1,1} = 19.565, a_{1,2} = -147.893, a_{2,2} = 30.151, b_{1,1} = 228.445, b_{1,2} = 131.427, b_{2,2} = 184.431.$

N-model, objective function  $F_{SA}$ , case 2:

**P:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{2\}, \{1\}).$

**PD:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{2\}, \{1\}), d_{1,2} = 0.026.$

**PT:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{2\}, \{1\}),$  No threshold.

**PDT:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{2\}, \{1\}), d_{1,2} = 0.026,$  No threshold.

**WR:**  $q_{1,1} = 15.807, q_{1,2} = 7.680, q_{2,2} = 7.817, a_{1,1} = 90.144, a_{1,2} = -53.197, a_{2,2} = -27.130, b_{1,1} = 91.691, b_{1,2} = 19.076, b_{2,2} = 111.955.$

N-model, objective function  $F_{SA}$ , case 3:

**P:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{2\}, \{1\}).$

**PD:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{L}_2 = (\{2\}, \{1\}), d_{1,1} = 0.329, d_{1,2} = 19.997.$

**PT:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{1\}, \{2\}), m_{1,2} = 1.$

**PDT:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{1, 2\}), d_{2,2} = 0.005, m_{1,2} = 1.$

**WR:**  $q_{1,1} = 20.748, q_{1,2} = -1.302, q_{2,2} = 5.944, a_{1,1} = -58.767, a_{1,2} = -83.761, a_{2,2} = 206.231, b_{1,1} = 157.820, b_{1,2} = 80.487, b_{2,2} = 75.504.$

N-model, objective function  $F_{SO}$ , case 1:

**P:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{2\}, \{1\}).$

**PD:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{2\}, \{1\}), d_{1,1} = 0.001, d_{1,2} = 73.609, d_{2,2} = 0.007.$

**PT:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{1, 2\}), m_{1,2} = 4.$

**PDT:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{L}_2 = (\{1, 2\}), d_{1,1} = 0.287, d_{1,2} = 19.541, d_{2,2} = 2.516, m_{1,2} = 3.$

**WR:**  $q_{1,1} = 15.355, q_{1,2} = -2.074, q_{2,2} = 11.739, a_{1,1} = -54.469, a_{1,2} = -135.926, a_{2,2} = -49.187, b_{1,1} = 113.710, b_{1,2} = 77.483, b_{2,2} = 157.335.$

N-model, objective function  $F_{SO}$ , case 2:

**P:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{L}_2 = (\{2\}, \{1\}).$

**PD:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{L}_2 = (\{2\}, \{1\}),$  No delay.

**PT:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{L}_2 = (\{2\}, \{1\}),$  No threshold.

**PDT:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{L}_2 = (\{1\}, \{2\}), d_{2,2} = 16.77, m_{1,2} = 1.$

**WR:**  $q_{1,1} = -0.00195, q_{1,2} = 0.0197, q_{2,2} = 0.523, a_{1,1} = 10.259, a_{1,2} = -2.798, a_{2,2} = -7.385, b_{1,1} = 26.280, b_{1,2} = 13.596, b_{2,2} = 12.289.$

N-model, objective function  $F_{SO},$  case 3:

**P:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{L}_2 = (\{2\}, \{1\}).$

**PD:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{L}_2 = (\{2\}, \{1\}), d_{2,2} = 0.055.$

**PT:**  $\mathcal{G}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{1\}, \{2\}), m_{1,2} = 1.$

**PDT:**  $\mathcal{G}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{1\}, \{2\}), d_{1,1} = 14.614, m_{1,2} = 1.$

**WR:**  $q_{1,1} = 3.611, q_{1,2} = -4.718, q_{2,2} = 16.343, a_{1,1} = -113.613, a_{1,2} = -52.009, a_{2,2} = 172.383, b_{1,1} = 58.341, b_{1,2} = 198.681, b_{2,2} = 64.768.$

### B.3. Solutions for the X-model

X-model, objective function  $F_S^1,$  case 1:

**P:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{G}_2 = (\{1, 2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{1\}, \{2\}).$

**PD:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{G}_2 = (\{2\}, \{1\}), \mathcal{L}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{1\}, \{2\}), d_{1,1} = 13.382, d_{1,2} = 2.052, d_{2,1} = 0.162.$

**PT:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{G}_2 = (\{1\}, \{2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{1\}, \{2\}),$  No threshold.

**PDT:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{G}_2 = (\{1, 2\}), \mathcal{L}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{1\}, \{2\}), d_{1,1} = 7.784, d_{1,2} = 9.250,$  No threshold.

**WR:**  $q_{1,1} = 4.973, q_{1,2} = 16.167, q_{2,1} = 4.062, q_{2,2} = 16.662, a_{1,1} = -208.712, a_{1,2} = -68.253, a_{2,1} = -121.777, a_{2,2} = -120.391, b_{1,1} = 124.909, b_{1,2} = 334.419, b_{2,1} = 137.450, b_{2,2} = 112.981.$

X-model, objective function  $F_S^1,$  case 2:

**P:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{G}_2 = (\{1\}, \{2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}).$

**PD:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{G}_2 = (\{1\}, \{2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}), d_{1,1} = 16.066, d_{1,2} = 19.995.$

**PT:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{G}_2 = (\{1\}, \{2\}), \mathcal{L}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{2\}, \{1\}), m_{1,1} = 1.$

**PDT:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{G}_2 = (\{1\}, \{2\}), \mathcal{L}_1 = (\{1, 2\}), \mathcal{L}_2 = (\{2\}, \{1\}),$  No delay,  $m_{1,1} = 1.$

**WR:**  $q_{1,1} = 0.340, q_{1,2} = 0.989, q_{2,1} = 15.992, q_{2,2} = 13.862, a_{1,1} = -59.019, a_{1,2} = -148.732, a_{2,1} = 115.361, a_{2,2} = 74.036, b_{1,1} = 266.009, b_{1,2} = 201.371, b_{2,1} = 10.656, b_{2,2} = 230.207.$

X-model, objective function  $F_S^1,$  case 3:

**P:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{G}_2 = (\{2\}, \{1\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}).$

**PD:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{G}_2 = (\{2\}, \{1\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}), d_{1,1} = 19.936, d_{1,2} = 0.963.$

**PT:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{G}_2 = (\{1\}, \{2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}),$  No threshold.

**PDT:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{G}_2 = (\{1\}, \{2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}), d_{1,1} = 15.070, d_{1,2} = 19.696, d_{2,2} = 0.013,$  No threshold.

**WR:**  $q_{1,1} = 0.0708, q_{1,2} = 0.470, q_{2,1} = 0.330, q_{2,2} = 1.130, a_{1,1} = -8.991, a_{1,2} = -77.267, a_{2,1} = -9.081, a_{2,2} = 10.450, b_{1,1} = 41.560, b_{1,2} = 72.456, b_{2,1} = 10.007, b_{2,2} = 7.185.$

X-model, objective function  $F_S^3,$  case 1:

**P:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{G}_2 = (\{1\}, \{2\}), \mathcal{L}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{1\}, \{2\}).$

**PD:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{G}_2 = (\{1\}, \{2\}), \mathcal{L}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{1\}, \{2\}), d_{1,1} = 4.572, d_{1,2} = 8.967, d_{2,2} = 0.005.$

**PT:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{G}_2 = (\{1, 2\}), \mathcal{L}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{1\}, \{2\}),$  No threshold.

**PDT:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{G}_2 = (\{2\}, \{1\}), \mathcal{L}_1 = (\{1\}, \{2\}), \mathcal{L}_2 = (\{1\}, \{2\}), d_{1,1} = 10.601, d_{1,2} = 1.033,$  No threshold.

**WR:**  $q_{1,1} = 4.800, q_{1,2} = 18.779, q_{2,1} = 5.835, q_{2,2} = 3.332, a_{1,1} = -177.540, a_{1,2} = 144.163, a_{2,1} = -136.057, a_{2,2} = 68.336, b_{1,1} = 61.501, b_{1,2} = 99.231, b_{2,1} = 152.218, b_{2,2} = 98.691.$

X-model, objective function  $F_S^3,$  case 2:

**P:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{G}_2 = (\{1\}, \{2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}).$

**PD:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{G}_2 = (\{1\}, \{2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}), d_{1,1} = 14.254, d_{1,2} = 19.336.$

**PT:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{G}_2 = (\{1\}, \{2\}), \mathcal{L}_1 = (\{1, 2\}), \mathcal{L}_2 = (\{2\}, \{1\}), m_{1,1} = 1.$

**PDT:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{G}_2 = (\{1\}, \{2\}), \mathcal{L}_1 = (\{1, 2\}), \mathcal{L}_2 = (\{2\}, \{1\}), d_{2,1} = 2.058, d_{2,2} = 2.179, m_{1,1} = 1.$

**WR:**  $q_{1,1} = 0.833, q_{1,2} = 0.662, q_{2,1} = 13.073, q_{2,2} = 13.388, a_{1,1} = -138.430, a_{1,2} = -26.091, a_{2,1} = 129.897, a_{2,2} = -25.954, b_{1,1} = 101.493, b_{1,2} = 151.413, b_{2,1} = 122.030, b_{2,2} = 106.900.$

X-model, objective function  $F_S^3,$  case 3:

**P:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{G}_2 = (\{1\}, \{2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}).$

**PD:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{G}_2 = (\{1\}, \{2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}), d_{1,1} = 2.13, d_{1,2} = 19.549.$

**PT:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{G}_2 = (\{2\}, \{1\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}),$  No threshold.

**PDT:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{G}_2 = (\{1\}, \{2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}), d_{1,1} = 2.792, d_{1,2} = 19.323,$  No threshold.

**WR:**  $q_{1,1} = 0.139, q_{1,2} = 1.624, q_{2,1} = 13.954, q_{2,2} = 14.190, a_{1,1} = -15.728, a_{1,2} = -147.292, a_{2,1} = -19.845, a_{2,2} = 76.144, b_{1,1} = 147.175, b_{1,2} = 77.924, b_{2,1} = 7.284, b_{2,2} = 22.980.$

X-model, objective function  $F_{SA}$ , case 1:

**P:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{G}_2 = (\{1, 2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{1\}, \{2\}).$

**PD:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{G}_2 = (\{1, 2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{1\}, \{2\}),$  No delay.

**PT:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{G}_2 = (\{1, 2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{1\}, \{2\}),$  No threshold.

**PDT:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{G}_2 = (\{1, 2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{1\}, \{2\}),$  No delay, No threshold.

**WR:**  $q_{1,1} = 10.266, q_{1,2} = 14.952, q_{2,1} = 10.026, q_{2,2} = 15.232, a_{1,1} = -132.520, a_{1,2} = -47.639, a_{2,1} = -91.985, a_{2,2} = -89.493, b_{1,1} = 276.899, b_{1,2} = 258.094, b_{2,1} = 65.021, b_{2,2} = 300.096.$

X-model, objective function  $F_{SA}$ , case 2:

**P:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{G}_2 = (\{1\}, \{2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}).$

**PD:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{G}_2 = (\{1\}, \{2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}),$   $d_{1,1} = 17.432, d_{1,2} = 0.002.$

**PT:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{G}_2 = (\{1\}, \{2\}), \mathcal{L}_1 = (\{1, 2\}), \mathcal{L}_2 = (\{2\}, \{1\}),$   $m_{1,1} = 1.$

**PDT:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{G}_2 = (\{1\}, \{2\}), \mathcal{L}_1 = (\{1, 2\}), \mathcal{L}_2 = (\{2\}, \{1\}),$   $d_{2,1} = 0.004, d_{2,2} = 0.524, m_{1,1} = 1$

**WR:**  $q_{1,1} = -0.0865, q_{1,2} = 0.265, q_{2,1} = 1.521, q_{2,2} = 1.519, a_{1,1} = -3.124, a_{1,2} = 7.779, a_{2,1} = 48.300, a_{2,2} = -7.319, b_{1,1} = 29.252, b_{1,2} = 29.337, b_{2,1} = 108.793, b_{2,2} = 109.947.$

X-model, objective function  $F_{SA}$ , case 3:

**P:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{G}_2 = (\{1\}, \{2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}).$

**PD:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{G}_2 = (\{1, 2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}),$   $d_{1,2} = 19.215.$

**PT:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{G}_2 = (\{1\}, \{2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}),$  No threshold.

**PDT:**  $\mathcal{G}_1 = (\{1\}, \{2\}), \mathcal{G}_2 = (\{1, 2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}),$   $d_{1,2} = 19.837,$  No threshold.

**WR:**  $q_{1,1} = 0.0133, q_{1,2} = -0.0563, q_{2,1} = 1.000, q_{2,2} = 0.631, a_{1,1} = -2.454, a_{1,2} = 10.459, a_{2,1} = 10.000, a_{2,2} = 9.228, b_{1,1} = 10.466, b_{1,2} = 10.652, b_{2,1} = 10.027, b_{2,2} = 11.439.$

X-model, objective function  $F_{SO}$ , case 1:

**P:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{G}_2 = (\{1, 2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{1\}, \{2\}).$

**PD:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{G}_2 = (\{1, 2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{1\}, \{2\}),$  No delay.

**PT:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{G}_2 = (\{1, 2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{1\}, \{2\}),$  No threshold.

**PDT:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{G}_2 = (\{1, 2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{1\}, \{2\}),$  No delay, No threshold.

**WR:**  $q_{1,1} = 7.422, q_{1,2} = 2.963, q_{2,1} = 2.704, q_{2,2} = 11.814, a_{1,1} = -204.019, a_{1,2} = 79.552, a_{2,1} = -118.661, a_{2,2} = 6.096, b_{1,1} = 118.530, b_{1,2} = -9.673, b_{2,1} = 143.561, b_{2,2} = 24.693.$

X-model, objective function  $F_{SO}$ , case 2:

**P:**  $\mathcal{G}_1 = (\{2\}, \{1\}), \mathcal{G}_2 = (\{1\}, \{2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}).$

**PD:**  $\mathcal{G}_1 = (\{2\}, \{1\}), \mathcal{G}_2 = (\{1\}, \{2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}),$   $d_{1,1} = 19.938, d_{1,2} = 16.464.$

**PT:**  $\mathcal{G}_1 = (\{2\}, \{1\}), \mathcal{G}_2 = (\{1, 2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{1, 2\}),$   $m_{1,2} = 1.$

**PDT:**  $\mathcal{G}_1 = (\{2\}, \{1\}), \mathcal{G}_2 = (\{1, 2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}),$   $d_{1,1} = 3.932, d_{1,2} = 4.203, d_{2,1} = 5.101, m_{1,2} = 1.$

**WR:**  $q_{1,1} = 288.551, q_{1,2} = 314.543, q_{2,1} = 663.340, q_{2,2} = 720.027, a_{1,1} = -8726.969, a_{1,2} = -24265.252, a_{2,1} = 12336.010, a_{2,2} = 12310.970, b_{1,1} = 21984.524, b_{1,2} = 16637.667, b_{2,1} = 34882.602, b_{2,2} = 50173.418.$

X-model, objective function  $F_{SO}$ , case 3:

**P:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{G}_2 = (\{1, 2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}).$

**PD:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{G}_2 = (\{1, 2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}),$  No delay.

**PT:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{G}_2 = (\{1, 2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}),$  No threshold.

**PDT:**  $\mathcal{G}_1 = (\{1, 2\}), \mathcal{G}_2 = (\{1, 2\}), \mathcal{L}_1 = (\{2\}, \{1\}), \mathcal{L}_2 = (\{2\}, \{1\}),$  No delay, No threshold.

**WR:**  $q_{1,1} = 0.0285, q_{1,2} = -0.0235, q_{2,1} = 1.000, q_{2,2} = 0.167, a_{1,1} = -5.007, a_{1,2} = 10.317, a_{2,1} = -13.779, a_{2,2} = 609.984, b_{1,1} = 11.527, b_{1,2} = 11.745, b_{2,1} = -6.697, b_{2,2} = -6.264.$

#### B.4. Solutions for the M-model

M-model, objective function  $F_S^1$ , case 1:

**P:**  $\mathcal{G}_1 = (\{1\}, \{3\}), \mathcal{G}_2 = (\{2\}, \{3\}), \mathcal{L}_3 = (\{2\}, \{1\}).$

**PD:**  $\mathcal{G}_1 = (\{1, 3\}), \mathcal{G}_2 = (\{2\}, \{3\}), \mathcal{L}_3 = (\{1\}, \{2\}),$   $d_{1,3} = 18.199, d_{2,3} = 19.639.$

**PT:**  $\mathcal{G}_1 = (\{1\}, \{3\}), \mathcal{G}_2 = (\{2\}, \{3\}), \mathcal{L}_3 = (\{2\}, \{1\}),$  No threshold.

**PDT:**  $\mathcal{G}_1 = (\{1, 3\}), \mathcal{G}_2 = (\{3\}, \{2\}), \mathcal{L}_3 = (\{1\}, \{2\}),$   $d_{1,3} = 18.242, d_{2,3} = 17.161,$  No threshold.

**WR:**  $q_{1,1} = 13.131, q_{1,3} = 1.327, q_{2,2} = 12.131, q_{2,3} = 1.564, a_{1,1} = 1475.324, a_{1,3} = -49.268, a_{2,2} = 1465.317, a_{2,3} = -61.802, b_{1,1} = 2007.310, b_{1,3} = 59.652, b_{2,2} = 2007.811, b_{2,3} = 69.192.$

M-model, objective function  $F_S^1$ , case 2:

**P:**  $\mathcal{G}_1 = (\{3\}, \{1\}), \mathcal{G}_2 = (\{2\}, \{3\}), \mathcal{L}_3 = (\{1\}, \{2\})$ .

**PD:**  $\mathcal{G}_1 = (\{1, 3\}), \mathcal{G}_2 = (\{2, 3\}), \mathcal{L}_3 = (\{1\}, \{2\}), d_{1,3} = 0.192, d_{2,3} = 19.908$ .

**PT:**  $\mathcal{G}_1 = (\{3\}, \{1\}), \mathcal{G}_2 = (\{2\}, \{3\}), \mathcal{L}_3 = (\{1\}, \{2\}),$  No threshold.

**PDT:**  $\mathcal{G}_1 = (\{1, 3\}), \mathcal{G}_2 = (\{2, 3\}), \mathcal{L}_3 = (\{1\}, \{2\}), d_{1,3} = 0.303, d_{2,3} = 19.467,$  No threshold.

**WR:**  $q_{1,1} = 16.238, q_{1,3} = 1.144, q_{2,2} = 22.508, q_{2,3} = 0.984, a_{1,1} = 91.471, a_{1,3} = -30.483, a_{2,2} = 46.012, a_{2,3} = -128.674, b_{1,1} = 129.543, b_{1,3} = 60.358, b_{2,2} = 490.729, b_{2,3} = 75.451$ .

M-model, objective function  $F_S^1$ , case 3:

**P:**  $\mathcal{G}_1 = (\{1\}, \{3\}), \mathcal{G}_2 = (\{2, 3\}), \mathcal{L}_3 = (\{2\}, \{1\})$ .

**PD:**  $\mathcal{G}_1 = (\{1\}, \{3\}), \mathcal{G}_2 = (\{2, 3\}), \mathcal{L}_3 = (\{2\}, \{1\}), d_{1,3} = 19.988, d_{2,3} = 0.007$ .

**PT:**  $\mathcal{G}_1 = (\{1\}, \{3\}), \mathcal{G}_2 = (\{2, 3\}), \mathcal{L}_3 = (\{2\}, \{1\}),$  No threshold.

**PDT:**  $\mathcal{G}_1 = (\{1\}, \{3\}), \mathcal{G}_2 = (\{2, 3\}), \mathcal{L}_3 = (\{2\}, \{1\}), d_{1,1} = 1.669, d_{1,3} = 19.962,$  No threshold.

**WR:**  $q_{1,1} = 11.848, q_{1,3} = 0.463, q_{2,2} = 14.610, q_{2,3} = 2.571, a_{1,1} = 1461.065, a_{1,3} = -80.393, a_{2,2} = 1743.424, a_{2,3} = -28.859, b_{1,1} = 2126.878, b_{1,3} = 25.268, b_{2,2} = 2358.688, b_{2,3} = 143.727$ .

M-model, objective function  $F_{SA}$ , case 1:

**P:**  $\mathcal{G}_1 = (\{1\}, \{3\}), \mathcal{G}_2 = (\{2\}, \{3\}), \mathcal{L}_3 = (\{2\}, \{1\})$ .

**PD:**  $\mathcal{G}_1 = (\{1, 3\}), \mathcal{G}_2 = (\{2\}, \{3\}), \mathcal{L}_3 = (\{1\}, \{2\}), d_{1,3} = 6.564$ .

**PT:**  $\mathcal{G}_1 = (\{1\}, \{3\}), \mathcal{G}_2 = (\{2\}, \{3\}), \mathcal{L}_3 = (\{2\}, \{1\}),$  No threshold.

**PDT:**  $\mathcal{G}_1 = (\{1, 3\}), \mathcal{G}_2 = (\{2, 3\}), \mathcal{L}_3 = (\{1\}, \{2\}), d_{1,3} = 7.701, d_{2,3} = 0.781,$  No threshold.

**WR:**  $q_{1,1} = 21.483, q_{1,3} = 5.268, q_{2,2} = 18.599, q_{2,3} = 5.415, a_{1,1} = 6.311, a_{1,3} = -36.723, a_{2,2} = 6.194, a_{2,3} = -57.558, b_{1,1} = 185.072, b_{1,3} = 21.683, b_{2,2} = 108.922, b_{2,3} = 30.099$ .

M-model, objective function  $F_{SA}$ , case 2:

**P:**  $\mathcal{G}_1 = (\{3\}, \{1\}), \mathcal{G}_2 = (\{2\}, \{3\}), \mathcal{L}_3 = (\{1\}, \{2\})$ .

**PD:**  $\mathcal{G}_1 = (\{1, 3\}), \mathcal{G}_2 = (\{2, 3\}), \mathcal{L}_3 = (\{1\}, \{2\}), d_{2,3} = 19.803$ .

**PT:**  $\mathcal{G}_1 = (\{3\}, \{1\}), \mathcal{G}_2 = (\{2\}, \{3\}), \mathcal{L}_3 = (\{1\}, \{2\}),$  No threshold.

**PDT:**  $\mathcal{G}_1 = (\{1\}, \{3\}), \mathcal{G}_2 = (\{2, 3\}), \mathcal{L}_3 = (\{1\}, \{2\}), d_{2,3} = 19.795,$  No threshold.

**WR:**  $q_{1,1} = 0.347, q_{1,3} = 0.796, q_{2,2} = 1.808, q_{2,3} = 0.0431, a_{1,1} = -3.011, a_{1,3} = -6.726, a_{2,2} = -11.046, a_{2,3} = -7.689, b_{1,1} = 145.885, b_{1,3} = 58.745, b_{2,2} = 397.704, b_{2,3} = 10.761$ .

M-model, objective function  $F_{SA}$ , case 3:

**P:**  $\mathcal{G}_1 = (\{1\}, \{3\}), \mathcal{G}_2 = (\{2, 3\}), \mathcal{L}_3 = (\{2\}, \{1\})$ .

**PD:**  $\mathcal{G}_1 = (\{3\}, \{1\}), \mathcal{G}_2 = (\{2\}, \{3\}), \mathcal{L}_3 = (\{2\}, \{1\}), d_{1,1} = 0.062, d_{1,3} = 16.950$ .

**PT:**  $\mathcal{G}_1 = (\{1\}, \{3\}), \mathcal{G}_2 = (\{2, 3\}), \mathcal{L}_3 = (\{2\}, \{1\}),$  No threshold.

**PDT:**  $\mathcal{G}_1 = (\{1\}, \{3\}), \mathcal{G}_2 = (\{2, 3\}), \mathcal{L}_3 = (\{2\}, \{1\}), d_{1,3} = 19.915, d_{2,3} = 0.027,$  No threshold.

**WR:**  $q_{1,1} = 941.507, q_{1,3} = 22.525, q_{2,2} = -0.991, q_{2,3} = 198.852, a_{1,1} = 191487.257, a_{1,3} = -4648.158, a_{2,2} = 210780.036, a_{2,3} = 5046.353, b_{1,1} = 266450.291, b_{1,3} = 7087.502, b_{2,2} = 289914.998, b_{2,3} = 9286.888$ .

M-model, objective function  $F_{SO}$ , case 1:

**P:**  $\mathcal{G}_1 = (\{1, 3\}), \mathcal{G}_2 = (\{2\}, \{3\}), \mathcal{L}_3 = (\{2\}, \{1\})$ .

**PD:**  $\mathcal{G}_1 = (\{1, 3\}), \mathcal{G}_2 = (\{2, 3\}), \mathcal{L}_3 = (\{1\}, \{2\}), d_{1,1} = 7.159, d_{1,3} = 10.989$ .

**PT:**  $\mathcal{G}_1 = (\{1, 3\}), \mathcal{G}_2 = (\{2\}, \{3\}), \mathcal{L}_3 = (\{2\}, \{1\}),$  No threshold.

**PDT:**  $\mathcal{G}_1 = (\{1, 3\}), \mathcal{G}_2 = (\{2\}, \{3\}), \mathcal{L}_3 = (\{2\}, \{1\}), d_{2,2} = 11.545, d_{2,3} = 12.594,$  No threshold.

**WR:**  $q_{1,1} = 0.0537, q_{1,3} = 0.00319, q_{2,2} = 0.0476, q_{2,3} = 0.00493, a_{1,1} = -1.546, a_{1,3} = -0.454, a_{2,2} = 7.753, a_{2,3} = -0.902, b_{1,1} = 34.110, b_{1,3} = 8.102, b_{2,2} = 33.845, b_{2,3} = 11.183$ .

M-model, objective function  $F_{SO}$ , case 2:

**P:**  $\mathcal{G}_1 = (\{1\}, \{3\}), \mathcal{G}_2 = (\{2, 3\}), \mathcal{L}_3 = (\{2\}, \{1\})$ .

**PD:**  $\mathcal{G}_1 = (\{1\}, \{3\}), \mathcal{G}_2 = (\{2, 3\}), \mathcal{L}_3 = (\{2\}, \{1\}), d_{1,3} = 19.567$ .

**PT:**  $\mathcal{G}_1 = (\{1\}, \{3\}), \mathcal{G}_2 = (\{2\}, \{3\}), \mathcal{L}_3 = (\{2\}, \{1\}), m_{2,2} = 1$ .

**PDT:**  $\mathcal{G}_1 = (\{1, 3\}), \mathcal{G}_2 = (\{2, 3\}), \mathcal{L}_3 = (\{2\}, \{1\}), d_{1,1} = 3.725, d_{1,3} = 19.71, d_{2,2} = 11.954, m_{2,3} = 1$ .

**WR:**  $q_{1,1} = 2.104, q_{1,3} = 0.142, q_{2,2} = 0.193, q_{2,3} = 0.353, a_{1,1} = 113.046, a_{1,3} = -5.762, a_{2,2} = -32.384, a_{2,3} = -37.071, b_{1,1} = 11.126, b_{1,3} = 5.959, b_{2,2} = 44.645, b_{2,3} = 24.097$ .

M-model, objective function  $F_{SO}$ , case 3:

**P:**  $\mathcal{G}_1 = (\{1\}, \{3\}), \mathcal{G}_2 = (\{2, 3\}), \mathcal{L}_3 = (\{1\}, \{2\})$ .

**PD:**  $\mathcal{G}_1 = (\{1\}, \{3\}), \mathcal{G}_2 = (\{2, 3\}), \mathcal{L}_3 = (\{1\}, \{2\}), d_{1,1} = 12.129, d_{1,3} = 19.758$ .

**PT:**  $\mathcal{G}_1 = (\{3\}, \{1\}), \mathcal{G}_2 = (\{2\}, \{3\}), \mathcal{L}_3 = (\{1\}, \{2\}), m_{1,3} = 1$ .

**PDT:**  $\mathcal{G}_1 = (\{1, 3\}), \mathcal{G}_2 = (\{2, 3\}), \mathcal{L}_3 = (\{1\}, \{2\}), d_{1,1} = 10.924, d_{1,3} = 1.213, d_{2,2} = 3.795, d_{2,3} = 4.883, m_{1,3} =$



1.

**WR:**  $q_{1,1} = 7.125, q_{1,3} = 1.109, q_{2,2} = 8.743, q_{2,3} = 6.805, a_{1,1} = 142.029, a_{1,3} = -141.555, a_{2,2} = -26.210, a_{2,3} = -16.151, b_{1,1} = 87.176, b_{1,3} = 170.957, b_{2,2} = 158.573, b_{2,3} = 109.214.$

### B.5. Solutions for the large example

Large example, objective function  $F_S^1$ :

**P:**  $\mathcal{G}_1 = (\{1, 7, 9\}), \mathcal{G}_2 = (\{2, 5, 8, 10\}), \mathcal{G}_3 = (\{3, 7, 9\}), \mathcal{G}_4 = (\{4\}, \{8\}, \{1, 3, 9\}), \mathcal{G}_5 = (\{5\}, \{2\}), \mathcal{G}_6 = (\{4, 6\}), \mathcal{G}_7 = (\{6, 7, 10\}, \{3\}), \mathcal{G}_8 = (\{4, 6, 8, 9\}, \{10\}), \mathcal{L}_1 = (\{1, 4\}), \mathcal{L}_2 = (\{5\}, \{2\}), \mathcal{L}_3 = (\{3, 7\}, \{4\}), \mathcal{L}_4 = (\{4, 6, 8\}), \mathcal{L}_5 = (\{5\}, \{2\}), \mathcal{L}_6 = (\{6, 7, 8\}), \mathcal{L}_7 = (\{3\}, \{1, 7\}), \mathcal{L}_8 = (\{2\}, \{4, 8\}), \mathcal{L}_9 = (\{1\}, \{4, 8\}, \{3\}), \mathcal{L}_{10} = (\{2\}, \{7\}, \{8\}).$

**PD:**  $\mathcal{G}_1 = (\{1, 7, 9\}), \mathcal{G}_2 = (\{8, 10\}, \{2\}, \{5\}), \mathcal{G}_3 = (\{3, 7, 9\}), \mathcal{G}_4 = (\{4, 9\}, \{1, 3, 8\}), \mathcal{G}_5 = (\{5\}, \{2\}), \mathcal{G}_6 = (\{4, 6\}), \mathcal{G}_7 = (\{3, 10\}, \{6, 7\}), \mathcal{G}_8 = (\{4\}, \{10\}, \{6, 8\}, \{9\}), \mathcal{L}_1 = (\{1\}, \{4\}), \mathcal{L}_2 = (\{5\}, \{2\}), \mathcal{L}_3 = (\{3, 4, 7\}), \mathcal{L}_4 = (\{6, 8\}, \{4\}), \mathcal{L}_5 = (\{5\}, \{2\}), \mathcal{L}_6 = (\{6\}, \{7\}, \{8\}), \mathcal{L}_7 = (\{3, 7\}, \{1\}), \mathcal{L}_8 = (\{4, 8\}, \{2\}), \mathcal{L}_9 = (\{1\}, \{4\}, \{3, 8\}), \mathcal{L}_{10} = (\{2, 7\}, \{8\}), d_{1,9} = 10.010, d_{2,2} = 11.413, d_{2,5} = 4.899, d_{2,8} = 7.565, d_{3,7} = 39.797, d_{3,9} = 5.726, d_{4,3} = 35.883, d_{4,9} = 32.298, d_{5,2} = 2.111, d_{5,5} = 6.624, d_{6,4} = 15.494, d_{6,6} = 17.231, d_{7,6} = 18.753, d_{7,7} = 27.404, d_{7,10} = 22.699, d_{8,4} = 15.247, d_{8,8} = 14.324, d_{8,9} = 41.639.$

**PD\*:** Priority rules identical to the solution of the policy P,  $d_{1,7} = 0.538, d_{1,9} = 0.865, d_{2,5} = 0.966, d_{2,8} = 0.063, d_{3,7} = 0.027, d_{3,9} = 0.005, d_{4,3} = 5.815, d_{4,9} = 0.643, d_{5,2} = 0.001, d_{6,4} = 19.557, d_{6,6} = 0.005, d_{7,3} = 21.128, d_{7,7} = 0.510, d_{7,10} = 6.358, d_{8,8} = 5.784, d_{8,9} = 0.609.$

**PT:**  $\mathcal{G}_1 = (\{7\}, \{9\}, \{1\}), \mathcal{G}_2 = (\{2, 5, 8, 10\}), \mathcal{G}_3 = (\{3, 7\}, \{9\}), \mathcal{G}_4 = (\{1, 3, 4, 8, 9\}), \mathcal{G}_5 = (\{5\}, \{2\}), \mathcal{G}_6 = (\{4, 6\}), \mathcal{G}_7 = (\{6, 7\}, \{3\}, \{10\}), \mathcal{G}_8 = (\{4, 6, 9\}, \{8, 10\}), \mathcal{L}_1 = (\{1, 4\}), \mathcal{L}_2 = (\{2, 5\}), \mathcal{L}_3 = (\{3, 4, 7\}), \mathcal{L}_4 = (\{4, 6, 8\}), \mathcal{L}_5 = (\{2\}, \{5\}), \mathcal{L}_6 = (\{6\}, \{8\}, \{7\}), \mathcal{L}_7 = (\{7\}, \{3\}, \{1\}), \mathcal{L}_8 = (\{2\}, \{4, 8\}), \mathcal{L}_9 = (\{3, 4\}, \{1\}, \{8\}), \mathcal{L}_{10} = (\{7\}, \{2, 8\}), m_{2,5} = 1, m_{3,7} = 3, m_{4,1} = 4, m_{4,8} = 14, m_{4,9} = 10, m_{7,7} = 6, m_{7,10} = 4, m_{8,6} = 4, m_{8,8} = 2, m_{8,10} = 2.$

**PDT:**  $\mathcal{G}_1 = (\{1\}, \{9\}, \{7\}), \mathcal{G}_2 = (\{2, 5, 8, 10\}), \mathcal{G}_3 = (\{3, 7\}, \{9\}), \mathcal{G}_4 = (\{4\}, \{8, 9\}, \{1, 3\}), \mathcal{G}_5 = (\{2\}, \{5\}), \mathcal{G}_6 = (\{4, 6\}), \mathcal{G}_7 = (\{6, 7\}, \{3, 10\}), \mathcal{G}_8 = (\{4\}, \{10\}, \{8\}, \{6, 9\}), \mathcal{L}_1 = (\{1, 4\}), \mathcal{L}_2 = (\{5\}, \{2\}), \mathcal{L}_3 = (\{3, 4\}, \{7\}), \mathcal{L}_4 = (\{8\}, \{6\}, \{4\}), \mathcal{L}_5 = (\{5\}, \{2\}), \mathcal{L}_6 = (\{6, 7, 8\}), \mathcal{L}_7 = (\{3\}, \{1, 7\}), \mathcal{L}_8 = (\{2, 4, 8\}), \mathcal{L}_9 = (\{1\}, \{3, 8\}, \{4\}), \mathcal{L}_{10} = (\{8\}, \{7\}, \{2\}), No delay, No threshold.$

**WR:**  $q_{1,1} = 4.447, q_{1,7} = 0.208, q_{1,9} = 0.691, q_{2,2} = 0.127, q_{2,5} = 0.616, q_{2,8} = 2.402, q_{2,10} = 1.840, q_{3,3} = 3.874, q_{3,7} = 0.498, q_{3,9} = 0.601, q_{4,1} = -0.311, q_{4,3} = -0.0986, q_{4,4} = 0.634, q_{4,8} = -0.172, q_{4,9} = 0.0934, q_{5,2} = 0.639, q_{5,5} = 1.358, q_{6,4} = -0.0379, q_{6,6} = -0.135, q_{7,3} = -0.207, q_{7,6} = 0.214, q_{7,7} = -0.0904, q_{7,10} = 0.106, q_{8,4} = 0.164, q_{8,6} = 0.105, q_{8,8} = -0.0397, q_{8,9} = -0.0434, q_{8,10} = -0.0373, a_{1,1} = 438.025, a_{1,7} = 7.365, a_{1,9} = -10.521, a_{2,2} = -23.123, a_{2,5} = -9.701, a_{2,8} = 46.672, a_{2,10} = 52.065, a_{3,3} = 69.520, a_{3,7} = -20.382, a_{3,9} = -3.823, a_{4,1} = -4.943, a_{4,3} = -13.729, a_{4,4} = -5.992, a_{4,8} = -12.393, a_{4,9} = -25.652, a_{5,2} = -14.617, a_{5,5} = -100.206, a_{6,4} = -15.991, a_{6,6} = -0.938, a_{7,3} = -4.167, a_{7,6} = -11.838, a_{7,7} = -14.413, a_{7,10} = -22.004, a_{8,4} = -11.705, a_{8,6} = -8.790, a_{8,8} = -10.480, a_{8,9} = -9.341, a_{8,10} = -15.791, b_{1,1} = -17.201, b_{1,7} = 15.318, b_{1,9} = 48.553, b_{2,2} = 46.694, b_{2,5} = 203.604, b_{2,8} = 202.299, b_{2,10} = 475.297, b_{3,3} = 542.446, b_{3,7} = 3.280, b_{3,9} = 68.666, b_{4,1} = 14.329, b_{4,3} = 14.877, b_{4,4} = 112.830, b_{4,8} = 15.120, b_{4,9} = 27.619, b_{5,2} = 38.537, b_{5,5} = 59.855, b_{6,4} = 70.784, b_{6,6} = 505.953, b_{7,3} = -5.638, b_{7,6} = 115.362, b_{7,7} = 9.591, b_{7,10} = -1.540, b_{8,4} = 7.650, b_{8,6} = 24.303, b_{8,8} = -3.622, b_{8,9} = -19.250, b_{8,10} = -7.710.$

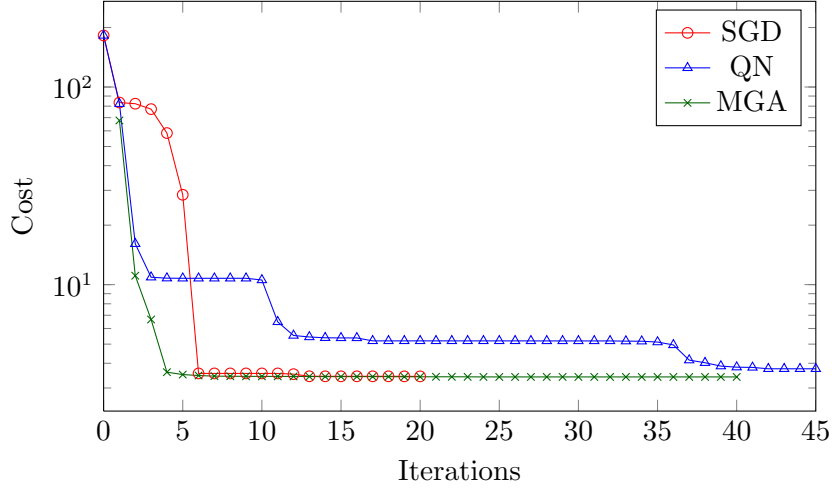
Large example, objective function  $F_{SO}$ :

**P:**  $\mathcal{G}_1 = (\{1, 7, 9\}), \mathcal{G}_2 = (\{2, 5, 8, 10\}), \mathcal{G}_3 = (\{3\}, \{7, 9\}), \mathcal{G}_4 = (\{4\}, \{3, 8\}, \{1\}, \{9\}), \mathcal{G}_5 = (\{5\}, \{2\}), \mathcal{G}_6 = (\{4, 6\}), \mathcal{G}_7 = (\{3, 6, 7, 10\}), \mathcal{G}_8 = (\{6, 10\}, \{4, 8, 9\}), \mathcal{L}_1 = (\{1\}, \{4\}), \mathcal{L}_2 = (\{5\}, \{2\}), \mathcal{L}_3 = (\{3, 7\}, \{4\}), \mathcal{L}_4 = (\{4, 6, 8\}), \mathcal{L}_5 = (\{5\}, \{2\}), \mathcal{L}_6 = (\{6, 7, 8\}), \mathcal{L}_7 = (\{1\}, \{3, 7\}), \mathcal{L}_8 = (\{2\}, \{4, 8\}), \mathcal{L}_9 = (\{3\}, \{1, 8\}, \{4\}), \mathcal{L}_{10} = (\{7\}, \{2, 8\}).$

**PD:**  $\mathcal{G}_1 = (\{1, 7, 9\}), \mathcal{G}_2 = (\{8, 10\}, \{2\}, \{5\}), \mathcal{G}_3 = (\{3, 7, 9\}), \mathcal{G}_4 = (\{4, 9\}, \{1, 3, 8\}), \mathcal{G}_5 = (\{5\}, \{2\}), \mathcal{G}_6 = (\{4, 6\}), \mathcal{G}_7 = (\{3, 10\}, \{6, 7\}), \mathcal{G}_8 = (\{4\}, \{10\}, \{6, 8\}, \{9\}), \mathcal{L}_1 = (\{1\}, \{4\}), \mathcal{L}_2 = (\{5\}, \{2\}), \mathcal{L}_3 = (\{3, 4, 7\}), \mathcal{L}_4 = (\{6, 8\}, \{4\}), \mathcal{L}_5 = (\{5\}, \{2\}), \mathcal{L}_6 = (\{6\}, \{7\}, \{8\}), \mathcal{L}_7 = (\{3, 7\}, \{1\}), \mathcal{L}_8 = (\{4, 8\}, \{2\}), \mathcal{L}_9 = (\{1\}, \{4\}, \{3, 8\}), \mathcal{L}_{10} = (\{2, 7\}, \{8\}), d_{1,9} = 10.010, d_{2,2} = 11.413, d_{2,5} = 4.899, d_{2,8} = 7.565, d_{3,7} = 39.797, d_{3,9} = 5.726, d_{4,3} = 35.883, d_{4,9} = 32.298, d_{5,2} = 2.111, d_{5,5} = 6.624, d_{6,4} = 15.494, d_{6,6} = 17.231, d_{7,6} = 18.753, d_{7,7} = 27.404, d_{7,10} = 22.699, d_{8,4} = 15.247, d_{8,8} = 14.324, d_{8,9} = 41.639.$

**PD\*:** Priority rules identical to the solution of the policy P,  $d_{2,5} = 1.540, d_{2,8} = 1.082, d_{3,7} = 17.280, d_{3,9} = 0.765, d_{4,1} = 28.506, d_{4,9} = 5.961, d_{5,2} = 0.552, d_{6,4} = 2.287, d_{7,3} = 15.51, d_{7,7} = 7.844, d_{7,10} = 10.891, d_{8,8} = 9.679, d_{8,9} = 2.296, d_{8,10} = 14.734.$

**PT:**  $\mathcal{G}_1 = (\{7\}, \{9\}, \{1\}), \mathcal{G}_2 = (\{2, 5\}, \{8, 10\}), \mathcal{G}_3 = (\{3, 7\}, \{9\}), \mathcal{G}_4 = (\{1, 3, 4, 8, 9\}), \mathcal{G}_5 = (\{5\}, \{2\}), \mathcal{G}_6 =$

**Figure 1** V-model case 2 with objective function  $F_S^1$ , WR policy.

$(\{4, 6\})$ ,  $\mathcal{G}_7 = (\{6, 7\}, \{3, 10\})$ ,  $\mathcal{G}_8 = (\{4, 6\}, \{9\}, \{8, 10\})$ ,  $\mathcal{L}_1 = (\{1\}, \{4\})$ ,  $\mathcal{L}_2 = (\{2, 5\})$ ,  $\mathcal{L}_3 = (\{3, 4, 7\})$ ,  $\mathcal{L}_4 = (\{4, 6, 8\})$ ,  $\mathcal{L}_5 = (\{2\}, \{5\})$ ,  $\mathcal{L}_6 = (\{6\}, \{8\}, \{7\})$ ,  $\mathcal{L}_7 = (\{7\}, \{3\}, \{1\})$ ,  $\mathcal{L}_8 = (\{2\}, \{4, 8\})$ ,  $\mathcal{L}_9 = (\{3, 4\}, \{8\}, \{1\})$ ,  $\mathcal{L}_{10} = (\{7\}, \{2, 8\})$ ,  $m_{2,5} = 1$ ,  $m_{3,7} = 4$ ,  $m_{4,1} = 4$ ,  $m_{4,8} = 14$ ,  $m_{4,9} = 9$ ,  $m_{7,7} = 5$ ,  $m_{7,10} = 4$ ,  $m_{8,6} = 3$ ,  $m_{8,8} = 2$ ,  $m_{8,9} = 1$ ,  $m_{8,10} = 3$ .

**PDT:**  $\mathcal{G}_1 = (\{1, 7, 9\})$ ,  $\mathcal{G}_2 = (\{2, 8, 10\}, \{5\})$ ,  $\mathcal{G}_3 = (\{3\}, \{7, 9\})$ ,  $\mathcal{G}_4 = (\{4\}, \{8\}, \{1\}, \{9\}, \{3\})$ ,  $\mathcal{G}_5 = (\{5\}, \{2\})$ ,  $\mathcal{G}_6 = (\{6\}, \{4\})$ ,  $\mathcal{G}_7 = (\{6\}, \{7\}, \{10\}, \{3\})$ ,  $\mathcal{G}_8 = (\{6\}, \{10\}, \{8, 9\}, \{4\})$ ,  $\mathcal{L}_1 = (\{1\}, \{4\})$ ,  $\mathcal{L}_2 = (\{5\}, \{2\})$ ,  $\mathcal{L}_3 = (\{4, 7\}, \{3\})$ ,  $\mathcal{L}_4 = (\{4, 6, 8\})$ ,  $\mathcal{L}_5 = (\{5\}, \{2\})$ ,  $\mathcal{L}_6 = (\{6, 7, 8\})$ ,  $\mathcal{L}_7 = (\{1\}, \{7\}, \{3\})$ ,  $\mathcal{L}_8 = (\{2, 4, 8\})$ ,  $\mathcal{L}_9 = (\{3\}, \{8\}, \{1\}, \{4\})$ ,  $\mathcal{L}_{10} = (\{2\}, \{7, 8\})$ , No delay, No threshold.

**WR:**  $q_{1,1} = 4.190$ ,  $q_{1,7} = 0.164$ ,  $q_{1,9} = 0.584$ ,  $q_{2,2} = 0.195$ ,  $q_{2,5} = 0.520$ ,  $q_{2,8} = 1.984$ ,  $q_{2,10} = 1.594$ ,  $q_{3,3} = 3.449$ ,  $q_{3,7} = 0.282$ ,  $q_{3,9} = 0.574$ ,  $q_{4,1} = -0.215$ ,  $q_{4,3} = -0.105$ ,  $q_{4,4} = 0.504$ ,  $q_{4,8} = -0.163$ ,  $q_{4,9} = -0.244$ ,  $q_{5,2} = 0.592$ ,  $q_{5,5} = 1.162$ ,  $q_{6,4} = 0.202$ ,  $q_{6,6} = 0.110$ ,  $q_{7,3} = 0.0387$ ,  $q_{7,6} = 0.150$ ,  $q_{7,7} = -0.047$ ,  $q_{7,10} = -0.005$ ,  $q_{8,4} = 0.007$ ,  $q_{8,6} = 0.040$ ,  $q_{8,8} = 7.548E-4$ ,  $q_{8,9} = -0.0399$ ,  $q_{8,10} = -1.016$ ,  $a_{1,1} = 619.386$ ,  $a_{1,7} = -7.898$ ,  $a_{1,9} = -8.086$ ,  $a_{2,2} = 1.546$ ,  $a_{2,5} = 4.254$ ,  $a_{2,8} = -9.638$ ,  $a_{2,10} = -6.399$ ,  $a_{3,3} = 77.074$ ,  $a_{3,7} = -14.814$ ,  $a_{3,9} = -11.617$ ,  $a_{4,1} = 1.680$ ,  $a_{4,3} = -2.187$ ,  $a_{4,4} = -15.449$ ,  $a_{4,8} = 16.457$ ,  $a_{4,9} = 6.891$ ,  $a_{5,2} = -9.653$ ,  $a_{5,5} = -16.552$ ,  $a_{6,4} = 12.549$ ,  $a_{6,6} = 12.936$ ,  $a_{7,3} = 8.165$ ,  $a_{7,6} = 13.246$ ,  $a_{7,7} = 9.230$ ,  $a_{7,10} = 11.799$ ,  $a_{8,4} = 11.018$ ,  $a_{8,6} = 10.454$ ,  $a_{8,8} = 9.655$ ,  $a_{8,9} = 9.521$ ,  $a_{8,10} = -2.056$ ,  $b_{1,1} = 653.285$ ,  $b_{1,7} = 25.759$ ,  $b_{1,9} = 54.625$ ,  $b_{2,2} = 50.780$ ,  $b_{2,5} = 185.740$ ,  $b_{2,8} = 168.865$ ,  $b_{2,10} = 418.720$ ,  $b_{3,3} = 532.827$ ,  $b_{3,7} = 32.371$ ,  $b_{3,9} = 52.263$ ,  $b_{4,1} = 14.174$ ,  $b_{4,3} = 10.280$ ,  $b_{4,4} = 103.700$ ,  $b_{4,8} = 7.374$ ,  $b_{4,9} = 13.644$ ,  $b_{5,2} = 46.302$ ,  $b_{5,5} = -8.859$ ,  $b_{6,4} = 52.229$ ,  $b_{6,6} = 326.916$ ,  $b_{7,3} = 2.604$ ,  $b_{7,6} = 74.914$ ,  $b_{7,7} = 5.020$ ,  $b_{7,10} = 10.702$ ,  $b_{8,4} = 9.204$ ,  $b_{8,6} = 19.331$ ,  $b_{8,8} = 9.277$ ,  $b_{8,9} = 7.271$ ,  $b_{8,10} = 8.573$ .

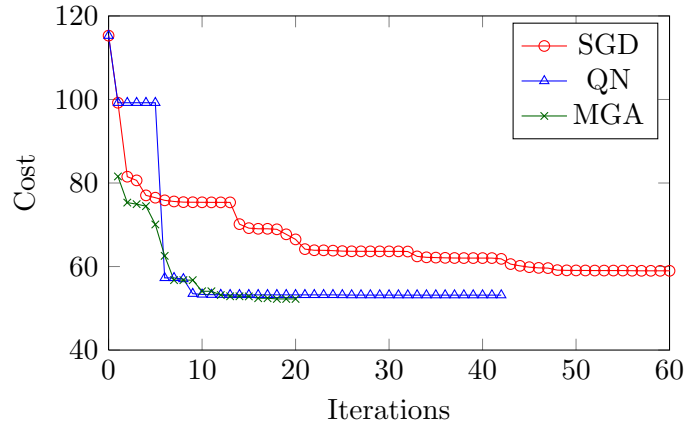
### Appendix C: Convergence of the optimization algorithms

Simulation-based optimization methods are usually costly in execution time. We compare the evolution of the costs, in function of the number of iterations, of the stochastic gradient descent (SGD), the Quasi-Newton (QN) method and the modified genetic algorithm (MGA) for the optimization of the weight routing (WR) policy. The number of iterations is not necessarily representative of the execution length. The SGD and the QN start with the same initial solution at iteration 0. Since the MGA does not start with an initial solution but some initial distribution parameters, it starts at iteration 1. The central finite difference is used to estimate the gradients for the SGD and the QN.

Figure 1 shows the convergence for the simple V-model case 2 example with the objective function  $F_S^1$ . All three algorithms find solutions with a cost of 3.4, but the MGA converges faster to the empirical optimal cost. Most of the optimization is done in the first 2 iterations for the QN and MGA. The SGD converges slowly in the first 6 iterations. It executes 20 iterations in 65 minutes, but the cost improves less than 4% after the 6th iteration.

The QN is blocked at a few local optima (from iterations 5 to 9, and 18 to 26). Restarting the QN, by increasing the step size, allows to get away from those local optima. However, it takes a few iterations to

**Figure 2** X-model case 1 with objective function  $F_{SA}$ , WR policy.



reduce sufficiently the step size in order to detect a local optimum and to activate a restart. The QN executes a total of 86 iterations in 6.7 hours, but the cost reduces less than 3% after the 64th iteration.

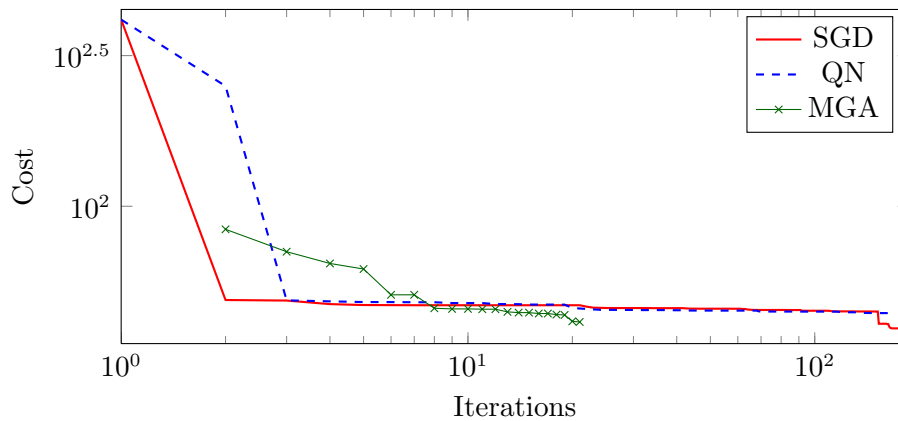
The parameters of the MGA for this example are:  $P = 200$ ,  $\hat{P} = 10$  and  $\text{maxIt} = 40$ . As stated in the paper, the  $P$  solutions are simulated with less replications (5 replications instead of 20) and the  $2\hat{P}$  best solution candidates are simulated again with 20 replications. The simulation time is potentially reduced to  $(200 \times 5 + 20 \times 20) / (200 \times 20) = 35\%$ , but the time saved may be less because of the additional simulation overheads. Using a simulation-based genetic algorithm is very time-consuming. This execution of the MGA requires a total of 14 hours. However, the cost improves less than 3% after the fifth iteration. If we need a good solution but not necessarily the optimum, the MGA could be stopped after 6 iterations (or 2 hours).

Figure 2 shows the convergence of the algorithms for the X-model example, case 1 and the objective function  $F_{SA}$ . This problem has more variables than the V-model and the algorithms need more iterations to find good solutions. The empirical optimal cost of 52.2 is found by the MGA, followed closely by the QN with 53.2. The parameters of the MGA are:  $P = 100$ ,  $\hat{P} = 10$  and  $\text{maxIt} = 20$ . The 20 iterations of the MGA are executed in 10.5 hours and the cost improves less than 2% after the 10th iteration or 5.26 hours. The QN executes 42 iterations in 8.2 hours, but the cost improves less than 0.7% after the 9th iteration. The SGD converges very slowly and ends at 58.8 or 12.5% above the empirical optimal cost after 89 iterations in 18 hours.

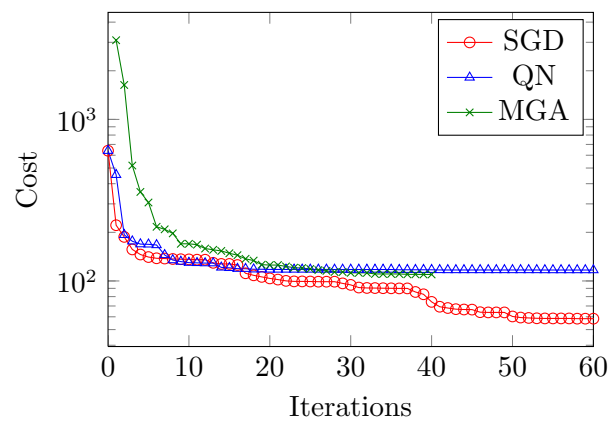
Figure 3 shows the convergence of the algorithms for the M-model example, case 1 with the objective function  $F_{SO}$ . Again, most of the optimization work occurs in the first few iterations. The starting solution of the SGD and QN has a cost of 416.4 and the empirical optimal cost is 39.3. To find a solution with a cost below 50, the SGD needs only 1 iteration, the QN requires 2 iterations and 7 iterations for the MGA. The MGA reaches the stopping criterion  $\text{maxIt}$  of 20 iterations with a cost of 41.3 in 8.9 hours. The SGD and QN converge very slowly, requiring 187 iterations in 33.7 hours and 163 iterations in 36.1 hours respectively. The QN uses more time per iteration than the SGD, but the MGA uses even more time per iteration than the QN. The SGD finds the best empirical solution after many iterations, but the QN fails to do so.

Figures 4 and 5 show the convergence of the algorithms for the large model example with the objective functions  $F_S^1$  and  $F_{SA}$  respectively. The comparisons are similar with both objective functions. This large model has many parameters and each parameter increases the number of solutions exponentially. It is harder for the MGA to generate good solutions in such a large domain region in the early iterations. In contrast to the smaller models, the first 2 or 3 iterations of the MGA have very expensive solutions, much higher than the starting solutions of the SGD and QN. However, the MGA does find competitive solutions to the SGD and NQ with a few iteration lag. For all three algorithms, most of the optimization work is done in the first 20 iterations, except for the SGD in Figure 4 that reduces the cost by an additional 43% until the 51st iteration. The SGD clearly performs better than the QN and the MGA for this large model. The QN has the worst performance by converging too quickly to a local minimum. The MGA has a slow start, but it eventually finds better solutions than the QN. With the objective function  $F_S^1$ , the SGD executes 67 iterations in 196.6 hours (2.9h/it.), 87 iterations in 234.6 hours (2.7h/it.) for the QN and 40 iterations in 54.2 hours (1.4h/it.) for the MGA. With the objective function  $F_{SA}$ , the SGD executes 56 iterations in 173.1

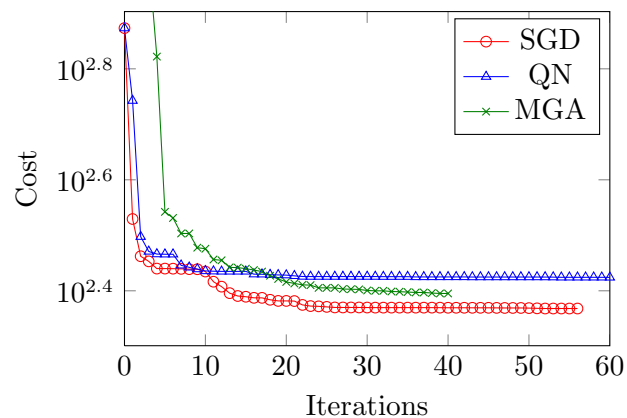
**Figure 3** M-model case 1 with objective function  $F_{SO}$ , WR policy. The SGD and QN require a lot more iterations than the MGA. All iteration numbers have been shifted by +1 because the iteration 0 cannot be plotted on the log scale.



**Figure 4** Large example with objective function  $F_{S1}^1$ , WR policy.



**Figure 5** Large example with objective function  $F_{SA}$ , WR policy.



hours (3.1h/it.), 69 iterations in 191.7 hours (2.8h/it.) for the QN and 40 iterations in 51.8 hours (1.3h/it.) for the MGA. The parameters of the MGA for both objective functions are:  $P = 100$ ,  $\hat{P} = 10$  and  $\max It = 40$ .