Wavelet transform for dimensionality reduction.
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1. The wavelet must be centered at 0 amplitude:
   $$\int_{-\infty}^{\infty} \psi(t) \, dt = 0$$

2. The wavelet must have a finite energy. Therefore it is localized in time (or space):
   $$\int_{-\infty}^{\infty} |\psi(t)|^2 \, dt < \infty$$

3. Sufficient condition for inverse wavelet transform:
   $$c \equiv \int_{-\infty}^{\infty} |\Psi(w)|^2 \, |w| \, dw < c < \infty$$
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   0 < c < \infty
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Popular wavelets which satisfy the previous conditions
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we can move and stretch the mother wavelet

\[ \psi_{a,b}(t) \equiv \frac{1}{\sqrt{|a|}} \psi \left( \frac{t - a}{b} \right) \]

\(a\) is scale factor while \(b\) is a translation factor. \(\frac{1}{\sqrt{|a|}}\) is a normalization factor to make sure the energy stay the same.
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The dot product for continuous function is defined as this : 

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The wavelet transform is simply the dot product between the signal and the wavelet at each translation and each scale.

$$W_{a,b} \equiv \int_{-\infty}^{\infty} f(t) \psi_{a,b}^*(t) \, dt$$
Introduction
Continuous wavelet transform
CWT vs STFT
Incertitude principle
Discrete wavelet transform (DWT)
Conclusion

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Scale and translation invariant
The continuous wavelet transform
Example using Mexican hat wavelet
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wavelet transform for dimensionality reduction
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\[ STFT(t, \omega) = \int_{-\infty}^{\infty} x(\tau) W(\tau - t) e^{-j\omega \tau} d\tau \]

The kernel have all the characteristics to be a wavelet.
If we choose a gaussian window for the SFTF, it is exactly the Morlet wavelet.
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▶ The only difference, is when the frequency is changing.
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In the case of the wavelet, the widow width is adapting with the frequency, keeping the number of cycles constant inside the window.

![Wavelet and STFT comparisons]
Theoretical spectrogram of 2 notes repeated 7 times with 200ms of duration.

STFT with gaussian window of width 0.05ms.
Theoretical spectrogram of 2 notes repeated 7 times with 200ms of duration.

STFT with gaussian window of width 0.1ms.
Theoretical spectrogram of 2 notes repeated 7 times with 200ms of duration.

STFT with gaussian window of width 0.3ms.
- Morlet wavelet is similar to the STFT kernel.
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- We also need to boost the frequency resolution.
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▶ We also need to boost the frequency resolution.

$$\psi(t) = \left(\frac{1}{\sqrt{\pi \sigma^2}}\right) e^{2i\pi f} e^{-\frac{x^2}{\sigma^2}}$$
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Similarly, we can find the time resolution.
- But the resolution depends of the mother wavelet and the scale.
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- **Frequency resolution**
  \[ \Delta \omega \rightarrow \Delta \omega_\psi (a) = \Delta \omega_\psi / |a| \]

- Thus the area of the square is constant
  \[ \Delta t_\psi (a) \Delta \omega_\psi (a) = \Delta t_\psi \Delta \omega_\psi = c_\psi \]
Thus, the main difference between STFT and CWT is the tiling of the resolution.
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One can tile the time-frequency space with any shape of rectangle, even with overlap.

The rectangle can have a smaller area with a different filter, but there is a minimal area.

\[ \Delta t_\psi \Delta \omega_\psi \geq 1/2 \]
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- For each of the m scales, the CWT perform a convolution on the raw signal of length n.
- The CWT return $m \cdot n$ coefficients in time $O(m \cdot n \log(n))$.
- There is a huge amount of redundancy and for higher scales, we could use a smaller sampling rate.
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▶ For that we need a second wavelet, called the scaling function.
▶ This function is a low-pass filter with frequency cut half of Nyquist.
▶ The wavelet is a complementary filter in that it is sensible for the rest of the frequency.
To perform the DWT, we start from the signal.
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Then we split the signal in two parts.

Details, using the wavelet.

Approximation, using the scaling function.

We can start back the decomposition from the approximated signal.

And again...

All the details is our wavelet transform. But we need to keep the last approximation for the inverse transform.
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But DWT restrict us to an octave of frequency resolution.
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We can compute the wavelet transform for n dimensions signals.

By selecting the highest coefficients on each scales, we can keep only the most important details (image compression).

By making statistics on wavelet coefficients, we can extract the global structure and use it as feature extraction for classification algorithms.
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The texture of the temporal structure represent the rhythm...
that is all !!!
questions ?