# Wavelet transform for dimensionality reduction.

Intro

(4回) (4回) (4回)

### Wavelet transform for dimensionality reduction.

Intro

- 4 回 2 - 4 回 2 - 4 回 2 - 4

What is a wavelet? Scale and translation invariant The continuous wavelet transform Example using Mexican hat wavelet

#### A function $\psi(t)$ , to be a wavelet must satisfy :

・ロン ・回 と ・ ヨ と ・ ヨ と

What is a wavelet? Scale and translation invariant The continuous wavelet transform Example using Mexican hat wavelet

A function  $\psi(t)$ , to be a wavelet must satisfy :

1. The wavelet must be centered at 0 amplitude.

$$\int_{-\infty}^{\infty}\psi(t)\,dt=0$$

・ロン ・回と ・ヨン・

What is a wavelet? Scale and translation invariant The continuous wavelet transform Example using Mexican hat wavelet

A function  $\psi(t)$ , to be a wavelet must satisfy :

1. The wavelet must be centered at 0 amplitude.

$$\int_{-\infty}^{\infty}\psi\left(t\right)dt=0$$

2. The wavelet must have a finite energy. Therefor it is localized in time (or space).

$$\int_{-\infty}^{\infty} |\psi(t)|^2 \, dt < \infty$$

・ロン ・回と ・ヨン・

What is a wavelet? Scale and translation invariant The continuous wavelet transform Example using Mexican hat wavelet

A function  $\psi(t)$ , to be a wavelet must satisfy :

1. The wavelet must be centered at 0 amplitude.

٠

$$\int_{-\infty}^{\infty}\psi(t)\,dt=0$$

2. The wavelet must have a finite energy. Therefor it is localized in time (or space).

$$\int_{-\infty}^{\infty} |\psi(t)|^2 \, dt < \infty$$

3. Sufficient condition for inverse wavelet transform

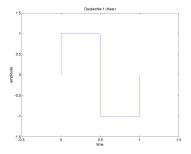
$$c \equiv \int_{-\infty}^{\infty} \frac{\left|\Psi\left(w\right)\right|^{2}}{\left|w\right|}$$

 $0 < c < \infty$ 

・ロト ・回ト ・ヨト ・ヨト

What is a wavelet? Scale and translation invariant The continuous wavelet transform Example using Mexican hat wavelet

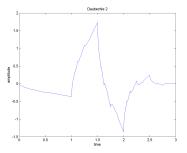
#### Popular wavelets which satisfy the previous conditions



・ 同 ト ・ ヨ ト ・ ヨ ト

What is a wavelet? Scale and translation invariant The continuous wavelet transform Example using Mexican hat wavelet

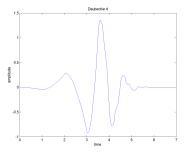
#### Popular wavelets which satisfy the previous conditions



イロト イヨト イヨト イヨト

What is a wavelet? Scale and translation invariant The continuous wavelet transform Example using Mexican hat wavelet

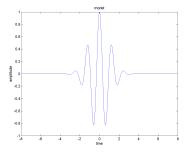
#### Popular wavelets which satisfy the previous conditions



- - 4 回 ト - 4 回 ト

What is a wavelet? Scale and translation invariant The continuous wavelet transform Example using Mexican hat wavelet

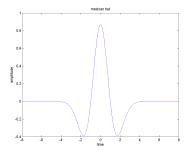
#### Popular wavelets which satisfy the previous conditions



・ 同 ト ・ ヨ ト ・ ヨ ト

What is a wavelet? Scale and translation invariant The continuous wavelet transform Example using Mexican hat wavelet

#### Popular wavelets which satisfy the previous conditions



- - 4 回 ト - 4 回 ト

What is a wavelet? Scale and translation invariant The continuous wavelet transform Example using Mexican hat wavelet

we can move and stretch the mother wavelet

$$\psi_{a,b}\left(t
ight)\equivrac{1}{\sqrt{\left|a
ight|}}\psi\left(rac{t-a}{b}
ight)$$

*a* is scale factor while *b* is a translation factor.  $\frac{1}{\sqrt{|a|}}$  is a normalization factor to make sure the energy stay the same.

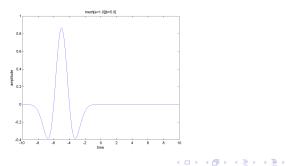
・ロト ・回ト ・ヨト ・ヨト

What is a wavelet? Scale and translation invariant The continuous wavelet transform Example using Mexican hat wavelet

we can move and stretch the mother wavelet

$$\psi_{a,b}(t) \equiv \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-a}{b}\right)$$

*a* is scale factor while *b* is a translation factor.  $\frac{1}{\sqrt{|a|}}$  is a normalization factor to make sure the energy stay the same.

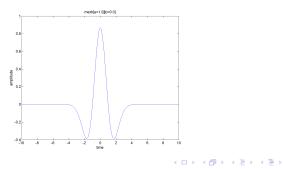


What is a wavelet? Scale and translation invariant The continuous wavelet transform Example using Mexican hat wavelet

we can move and stretch the mother wavelet

$$\psi_{a,b}(t) \equiv \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-a}{b}\right)$$

*a* is scale factor while *b* is a translation factor.  $\frac{1}{\sqrt{|a|}}$  is a normalization factor to make sure the energy stay the same.

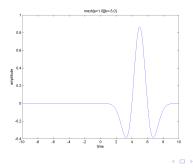


What is a wavelet? Scale and translation invariant The continuous wavelet transform Example using Mexican hat wavelet

we can move and stretch the mother wavelet

$$\psi_{a,b}(t) \equiv \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-a}{b}\right)$$

*a* is scale factor while *b* is a translation factor.  $\frac{1}{\sqrt{|a|}}$  is a normalization factor to make sure the energy stay the same.



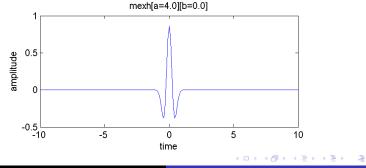
< □ > < □ > < □ >

What is a wavelet? Scale and translation invariant The continuous wavelet transform Example using Mexican hat wavelet

we can move and stretch the mother wavelet

$$\psi_{a,b}(t) \equiv \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-a}{b}\right)$$

*a* is scale factor while *b* is a translation factor.  $\frac{1}{\sqrt{|a|}}$  is a normalization factor to make sure the energy stay the same.



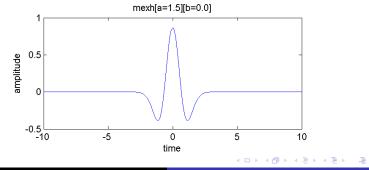
Alexandre Lacoste wavelet transform for dimensionality reduction

Scale and translation invariant The continuous wavelet transform Example using Mexican hat wavelet

we can move and stretch the mother wavelet

$$\psi_{a,b}(t) \equiv \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-a}{b}\right)$$

*a* is scale factor while *b* is a translation factor.  $\frac{1}{\sqrt{|a|}}$  is a normalization factor to make sure the energy stay the same.

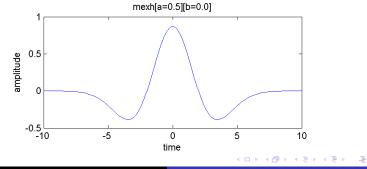


What is a wavelet? Scale and translation invariant The continuous wavelet transform Example using Mexican hat wavelet

we can move and stretch the mother wavelet

$$\psi_{a,b}(t) \equiv \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-a}{b}\right)$$

*a* is scale factor while *b* is a translation factor.  $\frac{1}{\sqrt{|a|}}$  is a normalization factor to make sure the energy stay the same.



Introduction
Continuous wavelet transform
CWT vs STFT
Incertitude principle
Discrete wavelet transform (DWT)
Conclusion
What is a wavelet?
Scale and translation invariant
The continuous wavelet transform
Example using Mexican hat wavelet

The dot product for continuous function is defined as this :

$$\langle f,g\rangle\equiv\int_{-\infty}^{\infty}f\left(t
ight)g\left(t
ight)^{*}dt$$

・ロン ・回 と ・ヨン ・ヨン

The dot product for continuous function is defined as this :

$$\langle f,g \rangle \equiv \int_{-\infty}^{\infty} f(t) g(t)^* dt$$

which is similar to the discrete dot product :

$$u \cdot v \equiv \sum_{i=1}^{n} u_i v_i$$

・ロン ・回と ・ヨン・

The dot product for continuous function is defined as this :

$$\langle f,g \rangle \equiv \int_{-\infty}^{\infty} f(t) g(t)^{*} dt$$

which is similar to the discrete dot product :

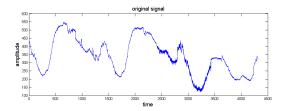
$$u\cdot v\equiv \sum_{i=1}^n u_iv_i$$

The wavelet transform is simply the dot product between the signal and the wavelet at each translation and each scale.

$$W_{\mathsf{a},b} \equiv \int_{-\infty}^{\infty} f(t) \psi_{\mathsf{a},b}^{*}(t) dt$$

イロト イポト イヨト イヨト

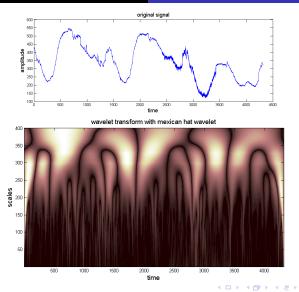
What is a wavelet? Scale and translation invariant The continuous wavelet transform Example using Mexican hat wavelet



<ロ> <同> <同> < 同> < 同> < 同> :

æ

What is a wavelet? Scale and translation invariant The continuous wavelet transform Example using Mexican hat wavelet



Alexandre Lacoste wavelet transform for dimensionality reduction

- ∢ .≜ ▶

Short time fourier transform, a review Scales vs Frequencies Spectrogram of a sound using STFT Wavelet spectrogram

 The STFT is the fourier transform computed for every time step.

・ロト ・回ト ・ヨト ・ヨト

Short time fourier transform, a review Scales vs Frequencies Spectrogram of a sound using STFT Wavelet spectrogram

- The STFT is the fourier transform computed for every time step.
- ► To isolate a particular time step, a window function is used.

・ロン ・回と ・ヨン・

- The STFT is the fourier transform computed for every time step.
- ▶ To isolate a particular time step, a window function is used.

►

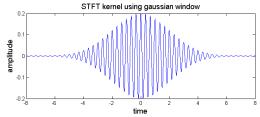
$$STFT\left(t,\omega
ight)=\int_{-\infty}^{\infty}x\left( au
ight)W\left( au-t
ight)e^{-j\omega au}d au$$

・ロン ・回と ・ヨン・

- The STFT is the fourier transform computed for every time step.
- ► To isolate a particular time step, a window function is used.

$$STFT\left(t,\omega
ight)=\int_{-\infty}^{\infty}x\left( au
ight)W\left( au-t
ight)e^{-j\omega au}d au$$

The kernel have all the characteristics to be a wavelet



・ 同・ ・ ヨ・

- - E

If we choose a gaussian window for the SFTF, it is exactly the Morlet wavelet.

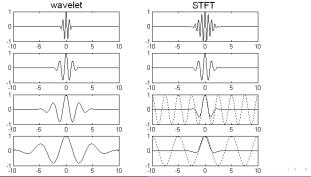
・ロン ・回 と ・ ヨ と ・ ヨ と

Introduction
Continuous wavelet transform
CWT vs STFT
Incertitude principle
Discrete wavelet transform (DWT)
Conclusion
Short time fourier transform, a review
Scales vs Frequencies
Spectrogram of a sound using STFT
Wavelet spectrogram

- If we choose a gaussian window for the SFTF, it is exactly the Morlet wavelet.
- ▶ The only difference, is when the frequency is changing.

イロト イポト イヨト イヨト

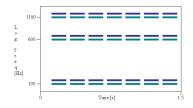
- If we choose a gaussian window for the SFTF, it is exactly the Morlet wavelet.
- ► The only difference, is when the frequency is changing.
- In the case of the wavelet, the widow width is adapting with the frequency, keeping the number of cycles constant inside the window.



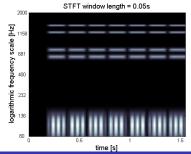
Alexandre Lacoste

wavelet transform for dimensionality reduction

Short time fourier transform, a review Scales vs Frequencies Spectrogram of a sound using STFT Wavelet spectrogram

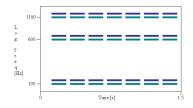


Theoretical spectrogram of 2 notes repeated 7 times with 200ms of duration.



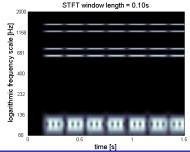
STFT with gaussian window of width 0.05ms.

Short time fourier transform, a review Scales vs Frequencies Spectrogram of a sound using STFT Wavelet spectrogram



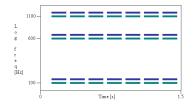
Theoretical spectrogram of 2 notes repeated 7 times with 200ms of duration.

## STFT with gaussian window of width 0.1ms.

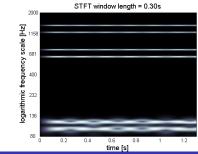


Alexandre Lacoste wavelet transform for dimensionality reduction

Short time fourier transform, a review Scales vs Frequencies Spectrogram of a sound using STFT Wavelet spectrogram



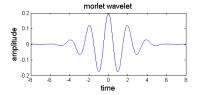
Theoretical spectrogram of 2 notes repeated 7 times with 200ms of duration.



### STFT with gaussian window of width 0.3ms.

Short time fourier transform, a review Scales vs Frequencies Spectrogram of a sound using STFT Wavelet spectrogram

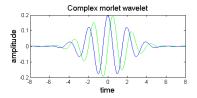
 Morlet wavelet is similar to the STFT kernel.



・ロン ・回と ・ヨン・

Short time fourier transform, a review Scales vs Frequencies Spectrogram of a sound using STFT Wavelet spectrogram

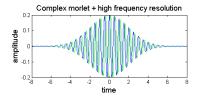
- Morlet wavelet is similar to the STFT kernel.
- To build a spectrogram, we need a complex wavelet.



イロン 不同と 不同と 不同と

Short time fourier transform, a review Scales vs Frequencies Spectrogram of a sound using STFT Wavelet spectrogram

- Morlet wavelet is similar to the STFT kernel.
- To build a spectrogram, we need a complex wavelet.
- We also need to boost the frequency resolution.

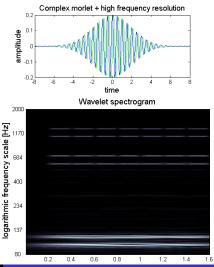


イロン イヨン イヨン イヨン

Short time fourier transform, a review Scales vs Frequencies Spectrogram of a sound using STFT Wavelet spectrogram

- Morlet wavelet is similar to the STFT kernel.
- To build a spectrogram, we need a complex wavelet.
- We also need to boost the frequency resolution.

$$\psi(t) = \left(\frac{1}{\sqrt{\pi\sigma^2}}\right) e^{2i\pi f} e^{-\frac{x^2}{\sigma^2}}$$



Time and frequency resolution Maximum resolution STFT resolution VS CWT resolution

► Wavelet are like bandpass filters.

(日) (同) (目) (日) (日) (日)

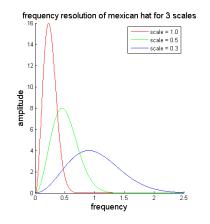
Time and frequency resolution Maximum resolution STFT resolution VS CWT resolution

- ► Wavelet are like bandpass filters.
- By computing their fourier transform, we find their frequency response.

・ロン ・回と ・ヨン・

Time and frequency resolution Maximum resolution STFT resolution VS CWT resolution

- ► Wavelet are like bandpass filters.
- By computing their fourier transform, we find their frequency response.

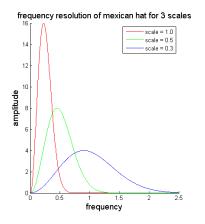


イロト イヨト イヨト イヨト

Time and frequency resolution Maximum resolution STFT resolution VS CWT resolution

- Wavelet are like bandpass filters.
- By computing their fourier transform, we find their frequency response.
- The standard deviation of the frequency response gives the frequency resolution.

$$\Delta \omega = \sqrt{\frac{\int_{-\infty}^{\infty} (\omega - \omega_0)^2 |\Psi(\omega)|^2 d\omega}{\int_{-\infty}^{\infty} |\Psi(\omega)|^2 d\omega}}$$



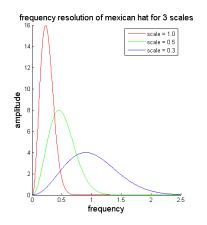
イロト イヨト イヨト イヨト

Time and frequency resolution Maximum resolution STFT resolution VS CWT resolution

- Wavelet are like bandpass filters.
- By computing their fourier transform, we find their frequency response.
- The standard deviation of the frequency response gives the frequency resolution.

$$\Delta \omega = \sqrt{\frac{\int_{-\infty}^{\infty} (\omega - \omega_0)^2 |\Psi(\omega)|^2 d\omega}{\int_{-\infty}^{\infty} |\Psi(\omega)|^2 d\omega}}$$

 similarly, we can find the time resolution.



イロト イヨト イヨト イヨト

Time and frequency resolution Maximum resolution STFT resolution VS CWT resolution

 But the resolution depends of the mother wavelet and the scale.

・ロン ・回 と ・ヨン ・ヨン

Time and frequency resolution Maximum resolution STFT resolution VS CWT resolution

- But the resolution depends of the mother wavelet and the scale.
- Time resolution

 $\Delta t 
ightarrow \Delta t_{\psi}\left( a
ight) =\left| a
ight| \Delta t_{\psi}$ 

・ロト ・回ト ・ヨト ・ヨト

Time and frequency resolution Maximum resolution STFT resolution VS CWT resolution

- But the resolution depends of the mother wavelet and the scale.
- Time resolution

$$\Delta t 
ightarrow \Delta t_{\psi}\left(a
ight) = \left|a
ight|\Delta t_{\psi}$$

Frequency resolution

$$\Delta \omega 
ightarrow \Delta \omega_{\psi}\left( a
ight) =\Delta \omega_{\psi}/|a|$$

・ロン ・回と ・ヨン・

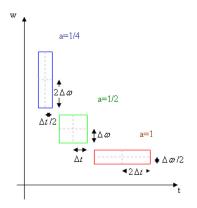
Time and frequency resolution Maximum resolution STFT resolution VS CWT resolution

- But the resolution depends of the mother wavelet and the scale.
- Time resolution

$$\Delta t 
ightarrow \Delta t_{\psi}\left( a
ight) =\left| a
ight| \Delta t_{\psi}$$

Frequency resolution

$$\Delta \omega 
ightarrow \Delta \omega_\psi \left( a 
ight) = \Delta \omega_\psi / \left| a 
ight|$$



イロト イヨト イヨト イヨト

Time and frequency resolution Maximum resolution STFT resolution VS CWT resolution

- But the resolution depends of the mother wavelet and the scale.
- Time resolution

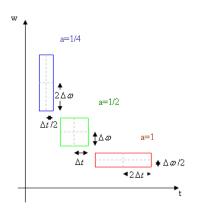
$$\Delta t 
ightarrow \Delta t_{\psi}\left(a
ight) = \left|a
ight|\Delta t_{\psi}$$

Frequency resolution

$$\Delta \omega 
ightarrow \Delta \omega_{\psi} \left( a 
ight) = \Delta \omega_{\psi} / \left| a 
ight|$$

 Thus the area of the square is constant

$$\Delta t_{\psi}\left(a
ight)\Delta\omega_{\psi}\left(a
ight)=\Delta t_{\psi}\Delta\omega_{\psi}=c_{\psi}$$



- 4 回 2 - 4 □ 2 - 4 □

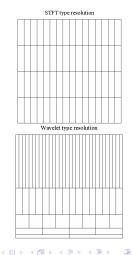
Time and frequency resolution Maximum resolution STFT resolution VS CWT resolution

 Thus, the main difference between STFT and CWT is the tiling of the resolution.

イロン 不同 とくほど 不良 とうほ

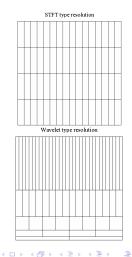
Time and frequency resolution Maximum resolution STFT resolution VS CWT resolution

 Thus, the main difference between STFT and CWT is the tiling of the resolution.



Time and frequency resolution Maximum resolution STFT resolution VS CWT resolution

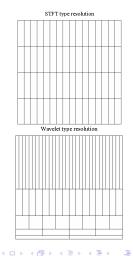
- Thus, the main difference between STFT and CWT is the tiling of the resolution.
- One can tile the time-frequency space with any shape of rectangle, even with overlap.



Time and frequency resolution Maximum resolution STFT resolution VS CWT resolution

- Thus, the main difference between STFT and CWT is the tiling of the resolution.
- One can tile the time-frequency space with any shape of rectangle, even with overlap.
- The rectangle can have a smaller area with a different filter, but there is a minimal area.

$$\Delta t_{\psi} \Delta \omega_{\psi} \geqslant 1/2$$



continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

 It took about 10 minutes to generate the previous wavelet spectrogram (P4 1.8GHz with Matlab wavelet toolbox).

・ロン ・回と ・ヨン・

continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

- It took about 10 minutes to generate the previous wavelet spectrogram (P4 1.8GHz with Matlab wavelet toolbox).
- For each of the m scales, the CWT perform a convolution on the raw signal of length n.

イロト イポト イヨト イヨト

continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

- It took about 10 minutes to generate the previous wavelet spectrogram (P4 1.8GHz with Matlab wavelet toolbox).
- For each of the m scales, the CWT perform a convolution on the raw signal of length n.
- ▶ The CWT return  $m \cdot n$  coefficients in time  $O(m \cdot n \log(n))$ .

イロト イポト イヨト イヨト

continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

- It took about 10 minutes to generate the previous wavelet spectrogram (P4 1.8GHz with Matlab wavelet toolbox).
- For each of the m scales, the CWT perform a convolution on the raw signal of length n.
- ▶ The CWT return  $m \cdot n$  coefficients in time  $O(m \cdot n \log(n))$ .
- There is a huge amount of redundancy and for higher scales, we could use a smaller sampling rate.

continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

To get a speed up in DWT, instead of stretching the wavelet to get to a bigger scale, we will compress the original signal.

・ロト ・回ト ・ヨト ・ヨト

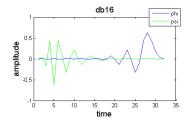
continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

- To get a speed up in DWT, instead of stretching the wavelet to get to a bigger scale, we will compress the original signal.
- For that we need a second wavelet, called the scaling function.

- 4 同 6 4 日 6 4 日 6

continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

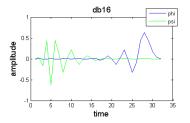
- To get a speed up in DWT, instead of stretching the wavelet to get to a bigger scale, we will compress the original signal.
- For that we need a second wavelet, called the scaling function.



(4月) (4日) (4日)

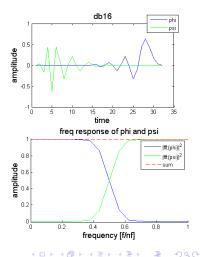
continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

- To get a speed up in DWT, instead of stretching the wavelet to get to a bigger scale, we will compress the original signal.
- For that we need a second wavelet, called the scaling function.
- This function is a low-pass filter with frequency cut half of Nyquist.



continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

- To get a speed up in DWT, instead of stretching the wavelet to get to a bigger scale, we will compress the original signal.
- For that we need a second wavelet, called the scaling function.
- This function is a low-pass filter with frequency cut half of Nyquist.
- The wavelet is a complementary filter in that it is sensible for the rest of the frequency.



continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

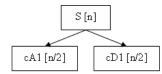
 To perform the DWT, we start from the signal.



・ロト ・回ト ・ヨト ・ヨト

continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

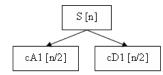
- To perform the DWT, we start from the signal.
- ▶ Then we split the signal in tow part.



・ロン ・回と ・ヨン・

continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

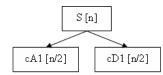
- To perform the DWT, we start from the signal.
- Then we split the signal in tow part.
- Details, using the wavelet.



・ロン ・回と ・ヨン・

continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

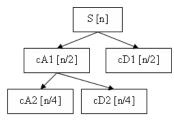
- To perform the DWT, we start from the signal.
- Then we split the signal in tow part.
- Details, using the wavelet.
- Approximation, using the scaling function.



イロン イヨン イヨン イヨン

continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

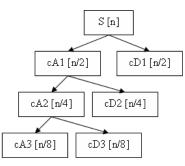
- To perform the DWT, we start from the signal.
- Then we split the signal in tow part.
- Details, using the wavelet.
- Approximation, using the scaling function.
- We can start back the decomposition from the approximated signal.



イロン イヨン イヨン イヨン

continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

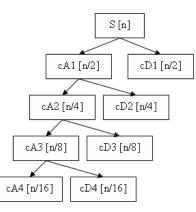
- To perform the DWT, we start from the signal.
- Then we split the signal in tow part.
- Details, using the wavelet.
- Approximation, using the scaling function.
- We can start back the decomposition from the approximated signal.
- And again...



・ロン ・回と ・ヨン・

continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

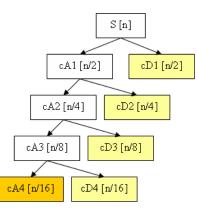
- To perform the DWT, we start from the signal.
- Then we split the signal in tow part.
- Details, using the wavelet.
- Approximation, using the scaling function.
- We can start back the decomposition from the approximated signal.
- And again...
- All the details is our wavelet transform. But we need to keep the last approximation for the inverse transform.



・ロン ・回 と ・ ヨ と ・ ヨ と

continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

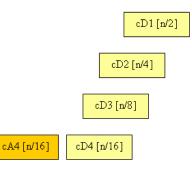
- To perform the DWT, we start from the signal.
- Then we split the signal in tow part.
- Details, using the wavelet.
- Approximation, using the scaling function.
- We can start back the decomposition from the approximated signal.
- And again...
- All the details is our wavelet transform. But we need to keep the last approximation for the inverse transform.



・ロン ・回 と ・ ヨ と ・ ヨ と

continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

 To perform the inverse DWT, we start from the details coefficients and the last approximation.



・ロン ・回と ・ヨン・

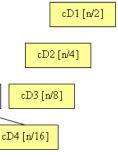
continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

cA3[n/8]

イロト イヨト イヨト イヨト

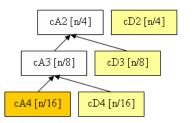
cA4 [n/16]

- To perform the inverse DWT, we start from the details coefficients and the last approximation.
- Then we combine the last approximation with the last details, to find the second last approximation.



continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

- To perform the inverse DWT, we start from the details coefficients and the last approximation.
- Then we combine the last approximation with the last details, to find the second last approximation.
- And we repeat...

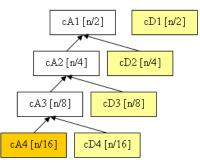


イロト イヨト イヨト イヨト

cD1[n/2]

continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

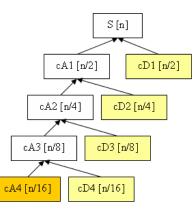
- To perform the inverse DWT, we start from the details coefficients and the last approximation.
- Then we combine the last approximation with the last details, to find the second last approximation.
- And we repeat...



・ロン ・回 と ・ ヨ と ・ ヨ と

continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

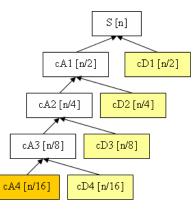
- To perform the inverse DWT, we start from the details coefficients and the last approximation.
- Then we combine the last approximation with the last details, to find the second last approximation.
- And we repeat...



・ロン ・回と ・ヨン・

continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

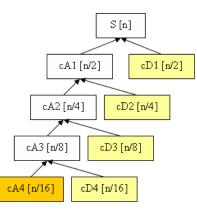
- To perform the inverse DWT, we start from the details coefficients and the last approximation.
- Then we combine the last approximation with the last details, to find the second last approximation.
- And we repeat...
- Both inverse and forward take O(n)



・ロン ・回 と ・ ヨ と ・ ヨ と

continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

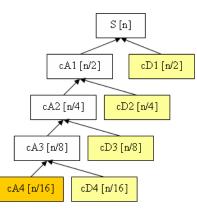
- To perform the inverse DWT, we start from the details coefficients and the last approximation.
- Then we combine the last approximation with the last details, to find the second last approximation.
- And we repeat...
- ▶ Both inverse and forward take *O*(*n*)
- It is thus faster than the fourier transform.



・ロン ・回 と ・ ヨ と ・ ヨ と

continuous wavelet transform is very slow scaling function DWT, a dyadic decomposition The inverse discrete wavelet transform

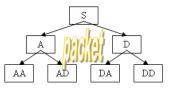
- To perform the inverse DWT, we start from the details coefficients and the last approximation.
- Then we combine the last approximation with the last details, to find the second last approximation.
- And we repeat...
- ▶ Both inverse and forward take *O*(*n*)
- It is thus faster than the fourier transform.
- But DWT restrict us to an octave of frequency resolution.



イロト イヨト イヨト イヨト

Many other things to talk Texture analysis End

 We can compute the complete tree or choose a particular path (wavelet packet).



・ロン ・回と ・ヨン・

Many other things to talk Texture analysis End

- We can compute the complete tree or choose a particular path (wavelet packet).
- We can compute the wavelet transform for n dimensions signals.



イロト イヨト イヨト イヨト

Many other things to talk Texture analysis End

- We can compute the complete tree or choose a particular path (wavelet packet).
- We can compute the wavelet transform for n dimensions signals.
- By selecting the highest coefficients on each scales, we can keep only the most important details (image compression).

・ 同 ト ・ ヨ ト ・ ヨ ト

Many other things to talk Texture analysis End

- We can compute the complete tree or choose a particular path (wavelet packet).
- We can compute the wavelet transform for n dimensions signals.
- By selecting the highest coefficients on each scales, we can keep only the most important details (image compression).
- By making statistics on wavelet coefficients, we can extract the global structure and use it as feature extraction for classification algorithms.

・ 同 ト ・ ヨ ト ・ ヨ ト

Many other things to talk Texture analysis End

 Wavelet statistics are mainly used for texture classification in image processing.



<ロ> (日) (日) (日) (日) (日)

Many other things to talk Texture analysis End

- Wavelet statistics are mainly used for texture classification in image processing.
- But with a little of imagination, we can see texture everywhere.



Many other things to talk Texture analysis End

- Wavelet statistics are mainly used for texture classification in image processing.
- But with a little of imagination, we can see texture everywhere.
- We have used wavelet statistics for genre classification.



Many other things to talk Texture analysis End

- Wavelet statistics are mainly used for texture classification in image processing.
- But with a little of imagination, we can see texture everywhere.
- We have used wavelet statistics for genre classification.
- The texture of a small frame of sound represent the timbre.



Many other things to talk Texture analysis End

- Wavelet statistics are mainly used for texture classification in image processing.
- But with a little of imagination, we can see texture everywhere.
- We have used wavelet statistics for genre classification.
- The texture of a small frame of sound represent the timbre.
- The texture of the temporal structure represent the rhythm...



Many other things to talk Texture analysis End

## that is all !!! questions ?