(Lack of) Deep Learning Theory

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The theorem statement we want

Conjecture: ∃ a set of classification problems \( \{D(x, y)\} \) such that:

1. ∀ “shallow” classification algorithms ∃ classification problem \( D \) such that when trained on samples from \( D \), the error rate of the learned classifier is large.

2. ∃ an efficient “deep” classification algorithm \( A \) such that ∀ classification problems \( D \), when trained on samples from \( D \), \( A \) efficiently produces a high accuracy classifier.
Some Theorems To Guide our Intuition

1. ∀ learning problems $D$, ∃ a two layer neural network with near optimal performance. (But it may be very inefficient to represent and learn.)

2. ∃ Monotone functions computable by small depth-$k$ circuits but which require very large depth $k - 1$ circuits. (Monotonicity is a very strong constraint.)

3. XOR is not representable by small constant depth circuits. (But hard to learn—some people use noisy XOR for crypto because it’s so hard to learn.)
A Theorem Statement that we can make.

Conjecture: \( \exists \) a set of classification problems \( \{D(x, y)\} \) such that:

1. \( \forall \) linear classification algorithms \( \exists \) classification problem \( D \) such that when applied to samples from \( D \), the error rate of the learned classifier is large.

2. \( \exists \) an efficient “deep” classification algorithm which produces a small error rate classifier with high probability for all \( D \).
A Tricky Classification Distribution Over \((x, y)\):

\[ y \sim U(\{0, 1\}) \]

\( k \) hidden bits
\( k/2+y \) set to 1 and the rest 0

4 visible bits per hidden bit
hidden bit = 0 \( \Rightarrow \) 1 or fewer visible bits = 1
hidden bit = 1 \( \Rightarrow \) 3 or more visible bits = 1

(We could recurse this construction to greater depth, if desired.)
Lower Bound

By symmetry, the best linear predictor uses uniform weights.

...but this predictor has a high loss because the influence of $y$ on the number of visible bits with value 1 is much smaller than the influence of noise.

(The negative statement also applies to kernels and many decision trees.)
Upper Bound

Consider the algorithm which first predicts each bit of $x$ from the other bits, then make a linear prediction on $x$.

The only useful bits for predicting bits of $x$ are siblings.

The optimal prediction for visible bits given sibling visible bits is the hidden bit.

Given the hidden bits, you can predict $y$ perfectly with a linear predictor.
Sideways Progress

Given the difficulty of even representational lower bounds, sideways progress seems the best hope.

1. What is the set of all learning algorithms on deep structures? (Backprop + ??)

2. What computational tricks are useful for these algorithms?

3. How do we avoid overfitting with deeply conditional prediction problems?
Further Reading
