A Retrospective on Genomic Processing for Comparative Genomics

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# 1. David Sankoff's Contribution: My Personal Experience

- First heard David's name in 1994.
- First email contact for COCOON'03.
- First switched to computational biology in 2004/5, one of the first problems I worked on was exactly posed by David (exemplar breakpoint distance problem).
- First met David at APBC'07 in HK.
- First collaborated with David in 2010, on the 8/28/2013 scaffold filling problem.

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Jiang, Zheng, Sankoff, B. Zhu. Scaffold filling under the breakpoint and related distances. IEEE/ACM TCBB 9, 2012.

Liu, Jiang, D. Zhu, B. Zhu. An improved approximation algorithm for scaffold filling to maximize the common adjacencies. IEEE/ACM TCBB, 2013 (published on-line<sub>8/28/2013</sub> on Aug 15, 2013).

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For instance, the Sorting Signed Permutations by Reversals problem was shown to be in P (Hannenhalli and Pevzner, 1999); and Sorting by Transpositions problem was shown to be NP-hard recently (Bulteau et al., 2012).

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- In computational genomics, a lot of research has been performed on rearrangement for "perfect" genomes, i.e., permutations.
- However, due to the fast evolution/self-production, duplicated (paralogous) genes are common in some genomes. So it is important to select the ancestral ortholog of a gene family on an evolutionary basis.
- In 1999, David Sankoff first formulated this as the exemplar breakpoint/genomic distance problem 13

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- If ab is a 2-substring in A and either ab or ba is a 2-substring in B, then ab is called an adjacency.

**Example.** A = abcde, B = bcaed, then there are 2 adjacencies in A and B.

 Problem: Given two genomes G' and H' with gene repetitions, compute two exemplar genomes G and H (i.e., exactly one gene in each family is kept) such that the number of breakpoints (resp. adjacencies) between G and H is minimized (resp. maximized).

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- Example: G'=badcbda, H'=abcdab
  optimal: G = bcda, H = bcda
  # breakpoints = 0, # adjacencies = 3

- David Bryant proved that the Exemplar Breakpoint Distance problem is NP-complete in 2000.
- In 2005, I ran a workshop with Zhixiang Chen and Bin Fu and we proved that the Exemplar Breakpoint Distance problem does not admit any polynomialtime approximation, unless P=NP, even when each gene appears at most three times (Chen,Fu,Zhu, AAIM'06). (Improved to 2-times, a few years later by Angibaud et al. 2009; Jiang, 2010).

# 2. The Exemplar Breakpoint **Distance and Related Problems** • 3SAT < ZERO-EBD Example. $\Phi = F_1 \wedge F_2 \wedge F_3 \wedge F_4$ , where $F_1 = x_1 \vee x_2 \vee x_3$ , $F_2 = X_1 V x_2 V X_1 x_4$ , $F_3 = X_2 V X_1 x_3 V x_4$ , $F_4 = X_1 V X_1 x_3 V X_4$ . For each $x_i$ , define $S_i$ (resp. $S_i$ ) as the list of clauses containing $x_i$ (resp. $\neg x_i$ ) followed by the clauses containing $\neg x_i$ (resp. $x_i$ ). Example. $S_1 = F_1 F_4 F_2$ , $S_1' = F_2 F_1 F_4$ .

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#### 2. The Exemplar Breakpoint **Distance and Related Problems** • 3SAT < ZERO-EBD Example. $\Phi = F_1 \wedge F_2 \wedge F_3 \wedge F_4$ , where $F_1 = x_1 \vee V_1 \times V_2 \vee X_3$ , $F_{2} = x_{1}Vx_{2}V_{1} x_{4}, F_{3} = x_{2}V_{1} x_{3}Vx_{4}, F_{4} = x_{1}V_{1} x_{3}V_{1} x_{4}.$ Construct two genomes $G' = S_1 g_1 S_2 g_2 S_3 g_3 S_4$ , $H' = S_1' g_1 S_2' g_2 S_3' g_3 S_4'$ . With this example, we can obtain $G = H = F_4 g_1 F_3 g_2 F_1 g_3 F_2$ (d(G,H)=0) with $x_1 = x_3 = True$ and $x_2 = x_4 = False$ . 8/28/2013 18

- 3SAT < ZERO-EBD
- The construction is simple and can easily produce sequences for NP-hardness proofs in various applications, e.g., computational geometry, protein structure simplification, and multi-channel program downloading.
- The 2-repetition construction by Angibaud et al. and Jiang is still too complex to have extra applications.

- 3SAT < ZERO-EBD
- Implications:

(1)EBD has no polynomial time approximation unless P=NP.

(2)EBD has no FPT algorithm unless P=NP.

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Status: NP-hard and APX-hard, the only known approximation bound is  $\Theta(n)$ .

• For the dual problem of EBD:

Independent Set < Exemplar Adjacency (Chen et al., CPM'07)

The Exemplar Adjacency problem does not admit any polynomial-time factor  $n^{0.5-\epsilon}$  approximation unless NP=ZPP (and, no FPT algorithm unless FPT=W[1]). This holds even when one genome is exemplar and each gene in the other appears at most twice. Moreover, there are matching approximations.

Problem	Inapproximability	FPT Tractability
Exemplar Breakpoint Distance	No poly-time approximation, unless P=NP	No FPT algorithm, unless P=NP
Exemplar Adjacency	Can't have a factor better than $n^{0.5-\varepsilon}$ , unless NP=ZPP	No FPT algorithm, unless FPT=W[1]

- Given two comparative maps, with gene markers, we want to identify noise and redundant markers. Note that in comparative maps only the relative positions of the markers along chromosome are indicated (Bertrand, Blanchette, El-Mabrouk, 2009, ...).
- In 2007, David Sankoff first formalized this as an algorithmic problem (*Zheng*, *Q.Zhu*, *Sankoff*, *TCBB*, 2007).

Example.

G<sub>1</sub>=<1,2,3,4,5,6,7,8,9,10,11,12>

G<sub>2</sub>=<-8,-5,-7,-6,4,1,3,2,-12,-11,-10,9>

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G'<sub>1</sub>=<<u>1,3,6,7,8,10,11,12</u>>

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This can be done by first finding syntenic blocks (strips) with maximum total length.

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G'<sub>1</sub>=<<u>1,3,6,7,8,10,11,12</u>>, 3 <u>syntenic blocks</u>

G'<sub>2</sub>=<<u>-8,-7,-6,1,3,-12,-11,-10</u>>

G<sub>1</sub>=<1,2,3,4,5,6,7,8,9,10,11,12>, redundant

G<sub>2</sub>=<-8,-5,-7,-6,4,1,3,2,-12,-11,-10,9>

 A strip (syntenic block) is a string of distinct markers that appear in two maps, either directly or in reversed and negated form.

Example. 6,7,8 in  $G'_1$ ; -8,-7,-6 in  $G'_2$ .

 MSR (Maximal Strip Recovery): Given two maps G and H, find two subsequences G' and H' of G and H, such that the total length of disjoint strips in G' and H' is maximized.

CMSR --- complement of MSR.

- After some struggle, both MSR and CMSR were shown to be NP-complete (Wang, Zhu, JCB 2010).
- The approximation and FPT algorithm research attracted 2 additional groups in Canada and France.

Problem	Approximability	FPT Tractability
MSR	Factor 4	?
CMSR	Factor 3, then 2.33	O <sup>*</sup> (2.36 <sup>k</sup> ) Best kernel: 78k

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Problem: Insert the missing genes into H to obtain H' s.t. d(G,H') is small or some similarity between G and H' is large.

Status:

- When there is no gene repetition, the one-sided problem under the DCJ distance (for multichromosome genomes) is in P (Munoz et al. 2010).
- When there is no gene repetition, even the twosided problems are in P (Jiang et al. 2012).
- When there are gene repetitions, the one-sided problem is NP-hard, using string adjacencies as a similarity measure (Jiang et al. 2012).

Definition:

- Given a scaffold/sequence A, P<sub>A</sub> represents all A's 2-substrings.
- Given scaffolds/sequences A and B, if a 2substring of A, a<sub>i</sub>a<sub>i+1</sub>, is equal to b<sub>j</sub>b<sub>j+1</sub> or b<sub>j+1</sub>b<sub>j</sub> then we say that they are matched to each other. In a maximum matching of the pairs in P<sub>A</sub> and P<sub>B</sub>, a matched pair is called an adjacency, and any unmatched pair is called a breakpoint.

Definition:

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Example: A=#13245#,

B=#12356#, adjacencies: #1, 23 breakpoints in A: 13, 24, 45, 5#; breakpoints in B: 12, 35, 56, 6#

Note that (string) adjacencies and breakpoints are completely different from those for permutations.

For instance, even if there is no breakpoint we could have  $A \neq B$  or its reversal.

Example: A=#bcdabc#,

B=#bcbadc#, there is no breakpoint.

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But  $A \neq B$  or its reversal.

Nevertheless, biologically and intuitively, more adjacencies would be good.

Approximation Results:

(1)For the one-sided scaffold filling to maximize the number of common adjacencies problem, there is a factor 1.33 approximation (Jiang, Zheng, Sankoff, B. Zhu, IEEE/ACM TCBB 9, 2012.)

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(1)For the one-sided scaffold filling to maximize the number of common adjacencies problem, there is a factor 1.33 approximation (Jiang, Zheng, Sankoff, B. Zhu, IEEE/ACM TCBB 9, 2012).

(2)There is a factor 1.25 approximation (Liu, Jiang, D. Zhu, B. Zhu, IEEE/ACM TCBB, 2013).

Method: maximum matching, local search and greedy search.

# (\*) My Collaborators

- Zhixiang Chen, Richard Fowler, Bin Fu (U. of Texas-Pan American).
- Haitao Jiang, Nan Liu, Daming Zhu (Shandong University, China).
- Zhong Li, Guohui Lin, Weitian Tong (University of Alberta).
- David Sankoff, Chunfang Zheng (University of Ottawa).
- Minghui Jiang, Lusheng Wang, Boting Yang<sub>8/28/2</sub>

#### (\*\*) Tenure Track Openings at Montana State University

- At least 2 openings in CS, ad will be out soon.
- Bioinformatics is one of the targeted areas.
- Female candidates are especially welcome.
- City Bozeman: 40K population, near Yellowstone, constantly ranked "best town to live" in US, especially suitable for people loving outdoor activities ...