Etude d'un problème de tarification en télécommunication

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6. Focus on the F. T. Context Joint Design and Pricing

Companies must decide on **prices** for services

⇒ Maximize revenue

Design choices capture many of the most important features of companies

⇒ Cost of providing service and attractiveness

At the present time, design and pricing issues are not treated jointly

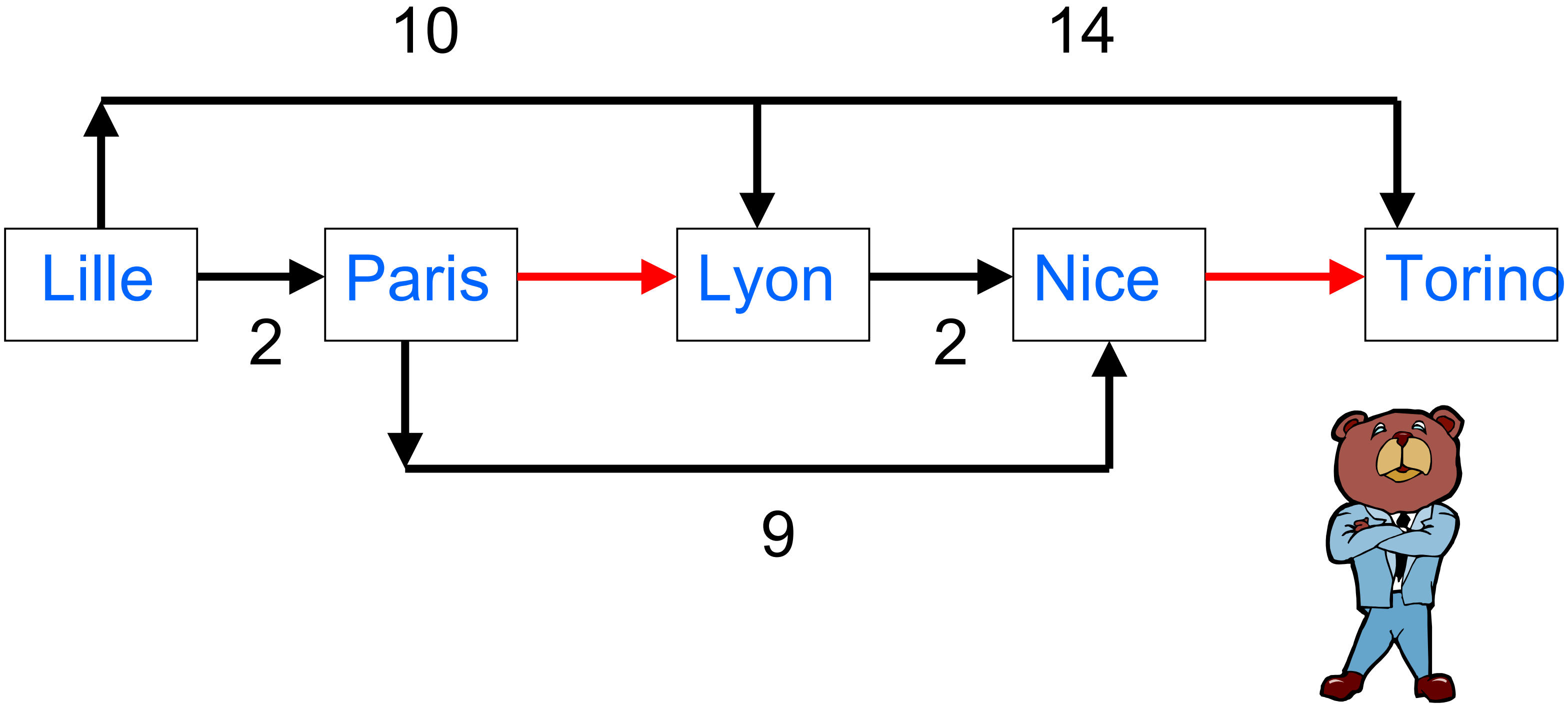
Tradeoff between

the cost of service
the revenue generated

when choosing its system **design** and **prices**

Problem

- An operator strives to maximize its revenue
  - Open connections (interconnection or connection building)
  - Determine the associated tariffs
- Customers are set to send prescribed amounts of flows from origins to destinations at minimun cost. Choice among several operators



No fixed opening cost:

$T_{PL} = 7$

$T_{NT} = 12$

Fixed opening cost (1):

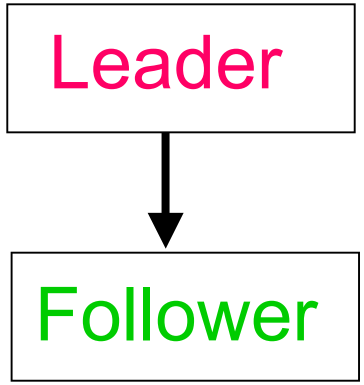
$0 \leq f_{PL} \leq 7 \Rightarrow T_{PL} = 7$

$0 \leq f_{NT} \leq 12 \Rightarrow T_{NT} = 12$

Hierarchichal process with opposite interests

Bilevel Programming

7. Bilevel Programming



$$
\begin{aligned}
\min_{x} \quad & f(x,y) \\
\text{s.c.} \quad & (x,y) \in X, \\
& y \text{ solution de } \min_{y} \quad g(x,y), \\
& \qquad\qquad\qquad\quad \text{s.c.} \quad (x,y) \in Y.
\end{aligned}
$$

For fixed x, the second level problem is defined as:

$$\begin{aligned} y \in P(\tilde{x}) = \operatorname{argmin}_y \;\; & g(\tilde{x}, y) \\ \text{s.c.} \;\; & (\tilde{x}, y) \in Y. \end{aligned}$$

Problem parametrized in x.

Non unicity of follower solution

1) Non Cooperation:

$$\begin{aligned} \min_{x} \quad & \max_{y} f(x,y) \\ \text{s.c.} \quad & (x,y) \in X, \\ & y \in P(x). \end{aligned}$$

2) Cooperation:

$$\begin{aligned} \min_{x,y} \quad & f(x,y) \\ \text{s.c.} \quad & (x,y) \in X, \\ & y \in P(x). \end{aligned}$$

$$\begin{aligned} \min_{x,y} \quad & f(x,y) \\ \text{s.c.} \quad & (x,y) \in X, \\ \min_{y} \quad & g(x,y) \\ \text{s.c.} \quad & (x,y) \in Y. \end{aligned}$$

Bilevel Programs

$$\begin{aligned} \min_{x,y} \quad & f(x,y) \\ \text{s.c.} \quad & (x,y) \in X, \\ \min_{y} \quad & g(x,y) \\ \text{s.c.} \quad & (x,y) \in Y. \end{aligned}$$

Difficulties

- Leader's domain is generally non convex and discontinuous

IR=$\{$(x,y) : (x,y) $\in$ X, y $\in$ P(x)$\}$

- y(x) is generally non differentiable

- Properties
  - Non convex problems
  - Optimal solution non pareto-optimal
  - Linear Linear bilevel programs:
    - Sharply NP-Difficult
    - Prove optimality is NP-Difficult
- Algorithms
  - General Case: Penalties methods, Descent methods,..
  - Linear Linear bilevel programs: Branch and Bound, Extremal Points, Descent methods, Penalties methods,..

- Applications
  - Tarification
  - Network Design
  - Planning
  - …
- References

  - Shimizu K., Ishizuka Y. and Bard J.F., *Non Differentiable and two-level mathematical programming*, Kluwer Academic Publisher, 1997.
  - Vicente N. and Calamai P.H., Bilevel and multilevel programming: a bibliography, *Journal of Global Optimization*, 5, PP. 291-306, 1994

8. Bilevel Model for JDP

$k \in K$: Set of origin destination pairs ((o(k),d(k))
n^k : Demand for a commodity k

C_a : Operating cost per unit on $a \in A_1$
T_a : Tariff per unit on $a \in A_1$
f_a : fixed opening cost on $a \in A_1$
v_a : Binary variable for $a \in A_1$

d_a : Transshipment cost per unit on $a \in A_2$

x_a^k : Commodity k flow on $a \in A_1$
y_a^k : Commodity k flow on $a \in A_2$

Bilevel Model

$$\max_{T,v} \boxed{\boxed{\sum_{k\in\mathcal{K}}\sum_{a\in\mathcal{A}_1} n^k T_a x_a^k} - \sum_{a\in\mathcal{A}_1} f_a v_a - \sum_{k\in\mathcal{K}}\sum_{a\in\mathcal{A}_1} n^k c_a x_a^k}$$ Revenue

$$\text{s.t.} \quad v_a \in \{0,1\} \qquad \forall a \in \mathcal{A}_1,$$

$$\min_{x,y} \boxed{\sum_{k\in\mathcal{K}} n^k \Big(\sum_{a\in\mathcal{A}_1} T_a x_a^k + \sum_{a\in\mathcal{A}_2} d_a y_a^k \Big)}$$ Customer cost

$$A_1 x^k + A_2 y^k = e^k \qquad \forall k \in \mathcal{K},$$

$$x_a^k \le v_a \qquad \forall k \in \mathcal{K}\ \forall a \in \mathcal{A}_1,$$

$$x^k, y^k \ge 0 \qquad \forall k \in \mathcal{K}.$$

The operator (leader) $\Rightarrow$ maximizes the net revenue

The customers (followers) $\Rightarrow$ minimize the cost

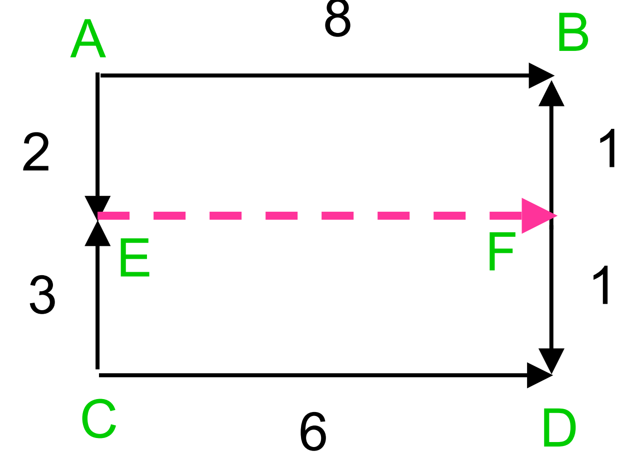
Assumptions

1. If the set of solutions of (FP) not single valued
   $\Rightarrow$ The customers select minimal cost paths generating the highest profit

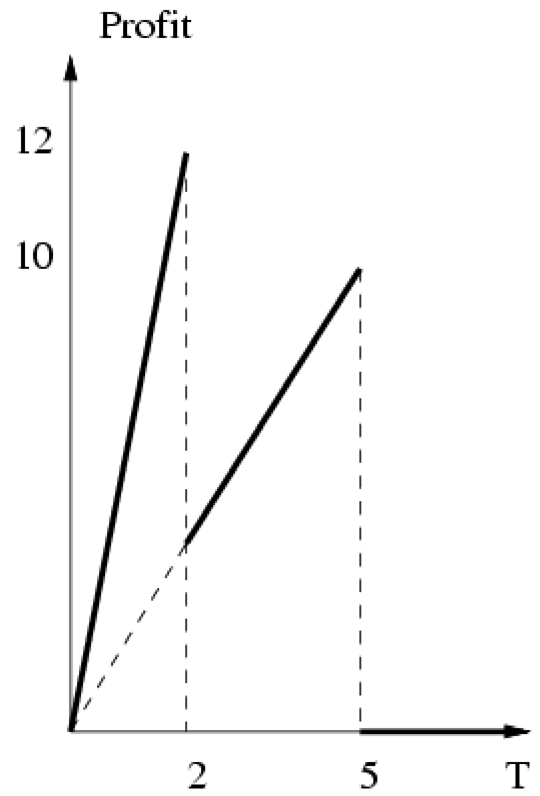
2. $\exists$ one untariff path for each O-D pair

3. $\exists$ a tariff setting scheme generating profit and creating negative cost cycles

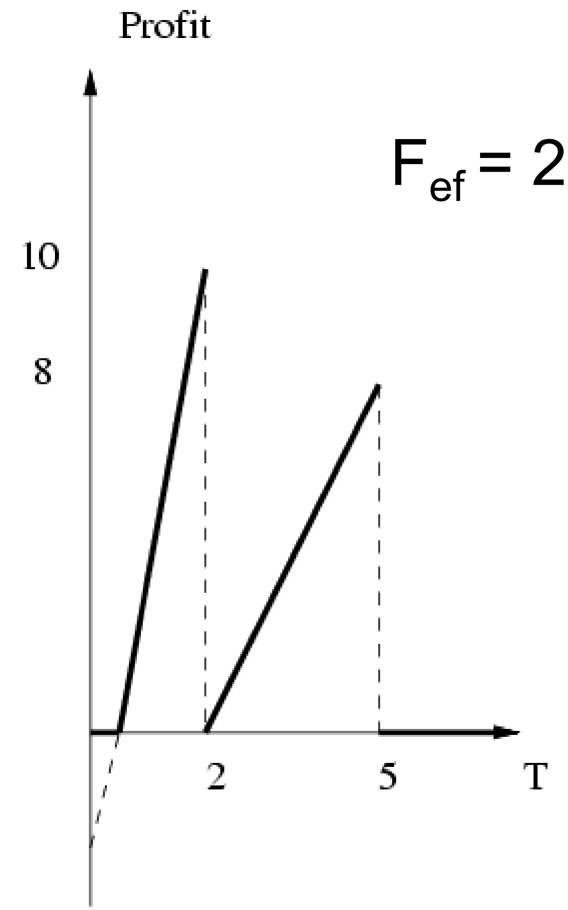
Profit Function



$2: 1 \rightarrow 2$
$4: 3 \rightarrow 4$



$F_{ef} = 2$



Properties

1. One commodity
   $\Rightarrow$ JDP is a toll setting problem

2. Upper bound on the profit

3. Moving constraints from the lower level
   to the upper level

Moving constraints from second to first level

$$\begin{aligned}
\max_{T,v} \quad & \sum_{k\in\mathcal{K}} \sum_{a\in\mathcal{A}_1} n^k T_a x_a^k - \sum_{a\in\mathcal{A}_1} f_a v_a - \sum_{k\in\mathcal{K}} \sum_{a\in\mathcal{A}_1} n^k c_a x_a^k \\
\text{s.t.} \quad & v_a \in \{0,1\} \qquad \forall a \in \mathcal{A}_1, \\
\min_{x,y} \quad & \sum_{k\in\mathcal{K}} n^k \Big(\sum_{a\in\mathcal{A}_1} T_a x_a^k + \sum_{a\in\mathcal{A}_2} d_a y_a^k \Big) \\
& A_1 x^k + A_2 y^k = e^k \qquad \forall k \in \mathcal{K}, \\
& \boxed{x_a^k \le v_a \qquad \forall k \in \mathcal{K}\ \forall a \in \mathcal{A}_1} \\
& x^k, y^k \ge 0 \qquad \forall k \in \mathcal{K}.
\end{aligned}$$

Solution Procedure for JDP

Lagrangian Relaxation

$$x_a^k \leq v_a \qquad \forall k \in \mathcal{K},\ \forall a \in \mathcal{A}_1$$

$$u = (u_a^k) \geq 0,$$

$$\begin{aligned}
(LSP(u)) \quad ZL(u) = \max_{T,v,x,y,\lambda} \quad & \sum_{a\in\mathcal{A}_1}\sum_{k\in\mathcal{K}} n^k T_a x_a^k - \sum_{a\in\mathcal{A}_1} f_a v_a \\
& - \sum_{a\in\mathcal{A}_1}\sum_{k\in\mathcal{K}} n^k c_a x_a^k \boxed{+ \sum_{a\in\mathcal{A}_1}\sum_{k\in\mathcal{K}} u_a^k (v_a - x_a^k)} \\
\text{s.t.} \quad & v_a \in \{0,1\} \qquad \forall a \in \mathcal{A}_1, \\
\min_{x,y} \quad & \sum_{k\in\mathcal{K}} n^k \Big(\sum_{a\in\mathcal{A}_1} T_a x_a^k + \sum_{a\in\mathcal{A}_2} d_a y_a^k \Big) \\
\text{s.t} \quad & Ax^k + By^k = e^k \qquad \forall k \in \mathcal{K}, \\
& x^k, y^k \geq 0 \qquad \forall k \in \mathcal{K},
\end{aligned}$$

Dual Lagrangian Problem

$$\min_{u\geq 0} \ z(u)$$

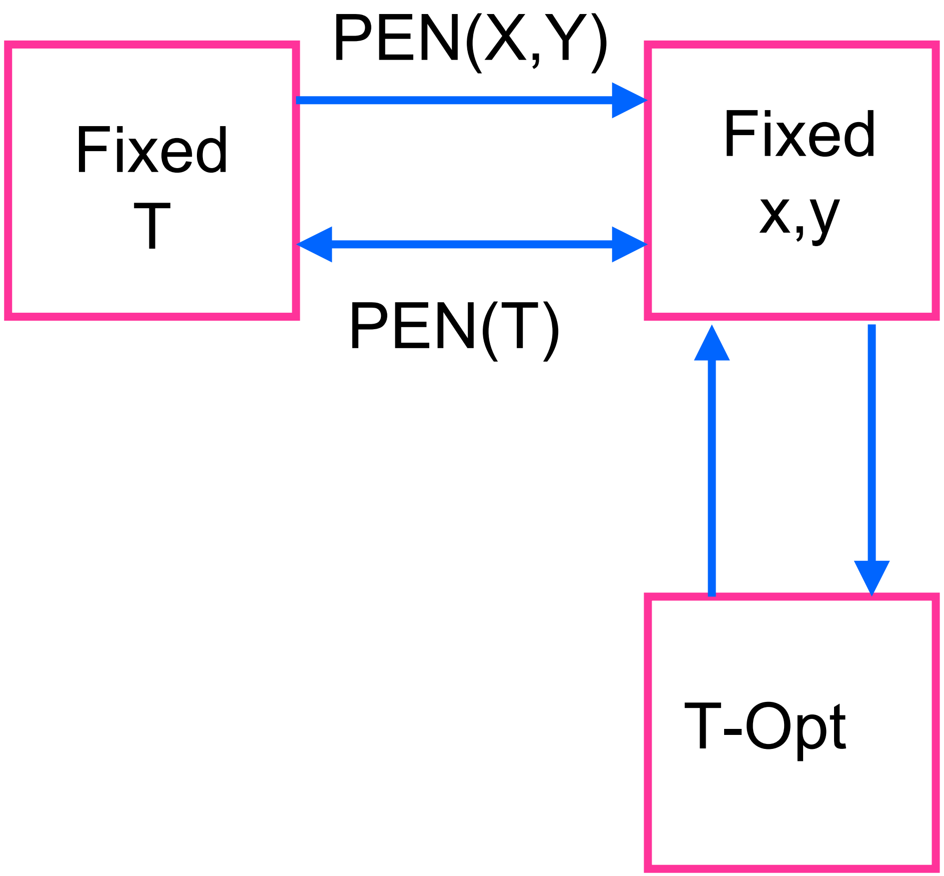
Primal Dual Algorithm for Lagrangian Relaxation

$$
\begin{aligned}
(PEN)\quad \max_{T,v,x,y,\lambda}\quad & \sum_{k\in\mathcal{K}}\sum_{a\in\mathcal{A}_1} n^k T_a x_a^k - \underline{\sum_{a\in\mathcal{A}_1} f_a v_a} - \sum_{k\in\mathcal{K}}\sum_{a\in\mathcal{A}_1} n^k c_a x_a^k \\
& + \underline{\sum_{k\in\mathcal{K}}\sum_{a\in\mathcal{A}_1} u_a^k}(v_a - x_a^k) \boxed{- M_1 \sum_{k\in\mathcal{K}} n^k (Tx^k + dy^k - \lambda^k e^k)} \\
\text{s.t.}\quad & A_1 x^k + A_2 y^k = e^k \qquad \forall k\in\mathcal{K}, \\
& \lambda^k A_1 \le T \qquad \forall k\in\mathcal{K}, \\
& \lambda^k A_2 \le d \qquad \forall k\in\mathcal{K}, \\
& v_a \in \{0,1\} \qquad \forall a\in\mathcal{A}_1, \\
& x_a^k \ge 0 \qquad \forall k\in\mathcal{K},\ \forall a\in\mathcal{A}_1, \\
& y_a^k \ge 0 \qquad \forall k\in\mathcal{K},\ \forall a\in\mathcal{A}_2.
\end{aligned}
$$

Binaries Variables : v

$$
\begin{aligned}
(PEN1(v)) \quad \max_{v} \quad & \sum_{a \in \mathcal{A}_1} \sum_{k \in \mathcal{K}} (u_a^k - f_a) v_a \\
\text{s.t.} \quad & v_a \in \{0,1\} \qquad \forall a \in \mathcal{A}_1.
\end{aligned}
$$

Gauss-Seidel method for PEN



PEN(X,Y)
Fixed
T
Fixed
x,y
PEN(T)
T-Opt

Inverse Optimization Program

$$
\begin{aligned}
(T-OPT)\quad \max_{T,\lambda}\quad & \sum_{a\in\mathcal{A}_1}\sum_{k\in\mathcal{K}} n^k T_a x_a^k - \sum_{a\in\mathcal{A}_1}\sum_{k\in\mathcal{K}} n^k c_a x_a^k \\
\text{s.t.}\quad & \lambda^k A \leq T \qquad \forall k\in\mathcal{K}, \\
& \lambda^k B \leq d \qquad \forall k\in\mathcal{K}, \\
& n^k(Tx^k + dy^k - \lambda^k e^k) = 0 \qquad \forall k\in\mathcal{K}.
\end{aligned}
$$

Numerical Results

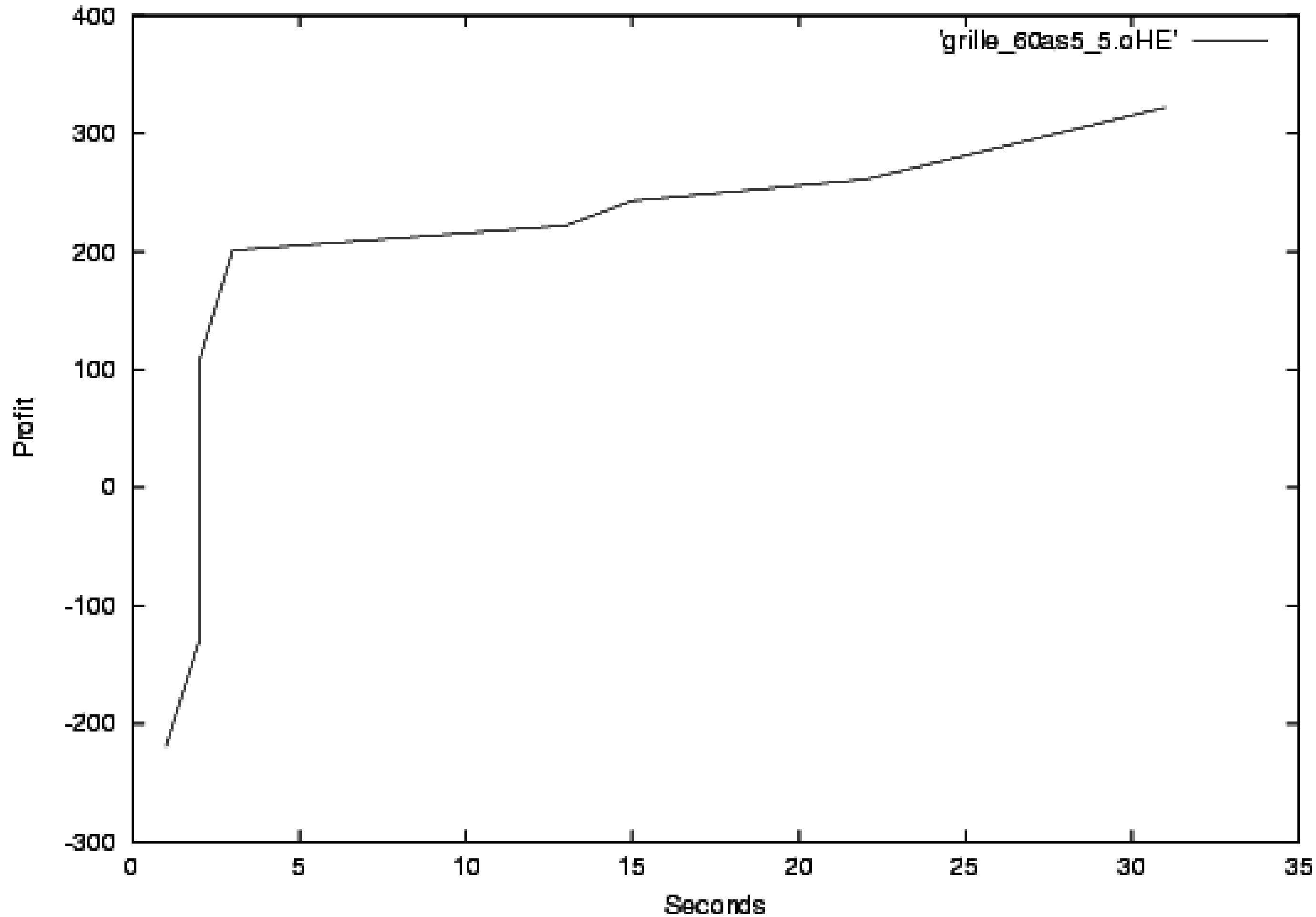
- Grid Networks : 60 nodes- 208 arcs
- Commodities: 10 – 20
- 5% to 20% of tariff arcs
- Heuristic / MIP formulation solved with Cplex

Heuristic					Cplex	
%TA	#TA	#BA	%OPT	CPU	GAP	CPU
5	4.8	159.6	0.98	62.0	73.62	433.0
10	5.4	187.4	0.97	84.8	58.20	781.0
15	9.8	196.0	1.08	118.4	46.95	10371.4
20	12.6	348.2	1.22	268.6	51.28	9223.8

Grid networks (60,208) - 20 Commodities

		Heuristic			Cplex	
f_a	%TA	NOPT	%OPT	CPU	GAP	CPU
0	5	5	0.99	28.4	30.29	1546.
	15	0	1.09	53.4	28.61	4958.
30	5	5	0.98	6.02	73.62	433.0
	15	4	1.08	118.4	46.95	10371
60	5	5	1.00	67.8	111.81	35.0
	15	5	1.00	140.8	44.92	2333.

Grid networks (60,208) - 20 Commodities
Fixed cost Sensitivity



Grid networks (60,208) - 20 Commodities
Upper level envelopes on profit function

9. Conclusion

Algorithm based on a novel application of Lagrangian relaxation within a bilevel programming framework

Extensions :

capacities on the link of the network

Yield management (determine the good price and the good services availability at the right time)