

Application of Dynamic Pricing to Retail and Supply Chain Management

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OUTLINE OF PRESENTATION

- **The pricing challenge**
- **The practice of pricing**
- **A pricing model for retail**
- **A pricing model for supply chain management**
- **A fluid delay-based model for pricing and inventory management**
- **Summary**

PRICING IS THE BEST LEVER FOR EARNINGS IMPROVEMENT...

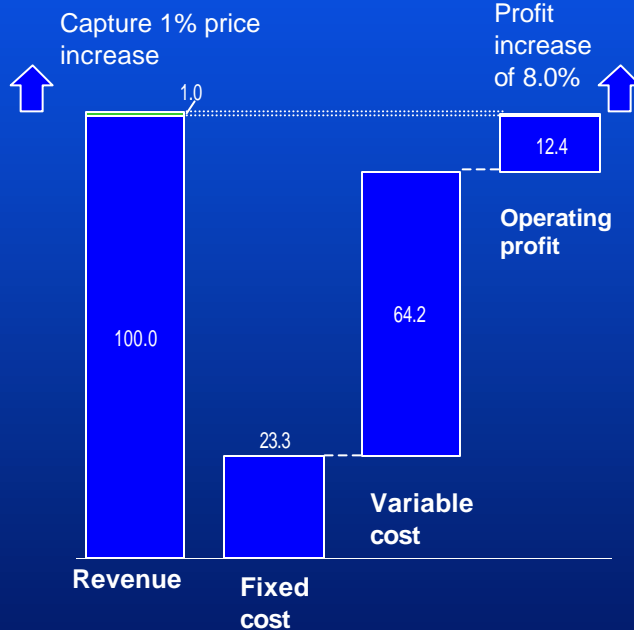
Impact of price increase on operating profit

Improvement in price by 1% increases profitability more than 8.0%

Price is the biggest profit improvement lever

Price decreases are only offset by huge volume uplifts

Percent

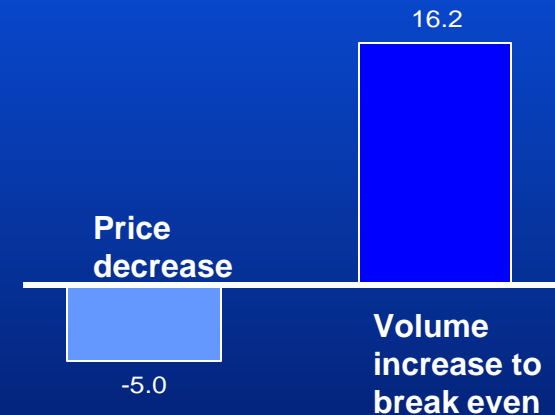
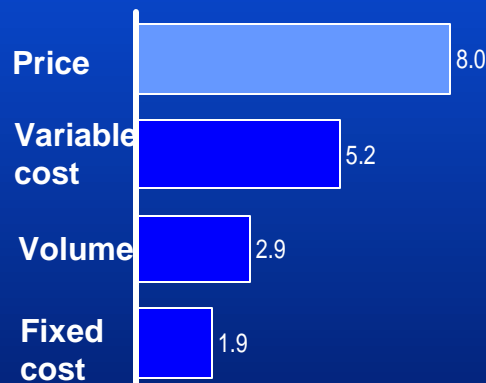


Improving the lever by 1% delivers profit improvement of . . .

Percent

Reducing prices by 5% requires a breakeven volume increase of 16.2%

Percent



Source: Based on 2001 S&P 500 average economics

... AND SHOULD BE ON EVERY CEO AGENDA

Growth drivers



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OVERALL OBJECTIVES OF PRICING IMPROVEMENT PROGRAMS

Achieve significant near-term improvements in profitability through enhanced price performance

Design and institutionalize comprehensive pricing management practices and processes to allow continued improvement into the future

Build systems, skills, incentives, etc. to support, enable, and sustain a high performing price management process

Achieve significant and sustainable gains in profitability through superior pricing management

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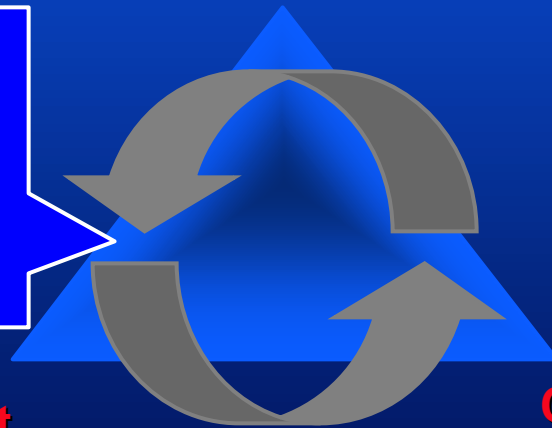
3 Cs OF TACTICAL PRICING

How can companies implement a consistent tactical pricing policy for increasingly dynamic markets?

Who are my customers, and what do they want?

Customer

Tactical pricing
How can I react quickly and correctly?



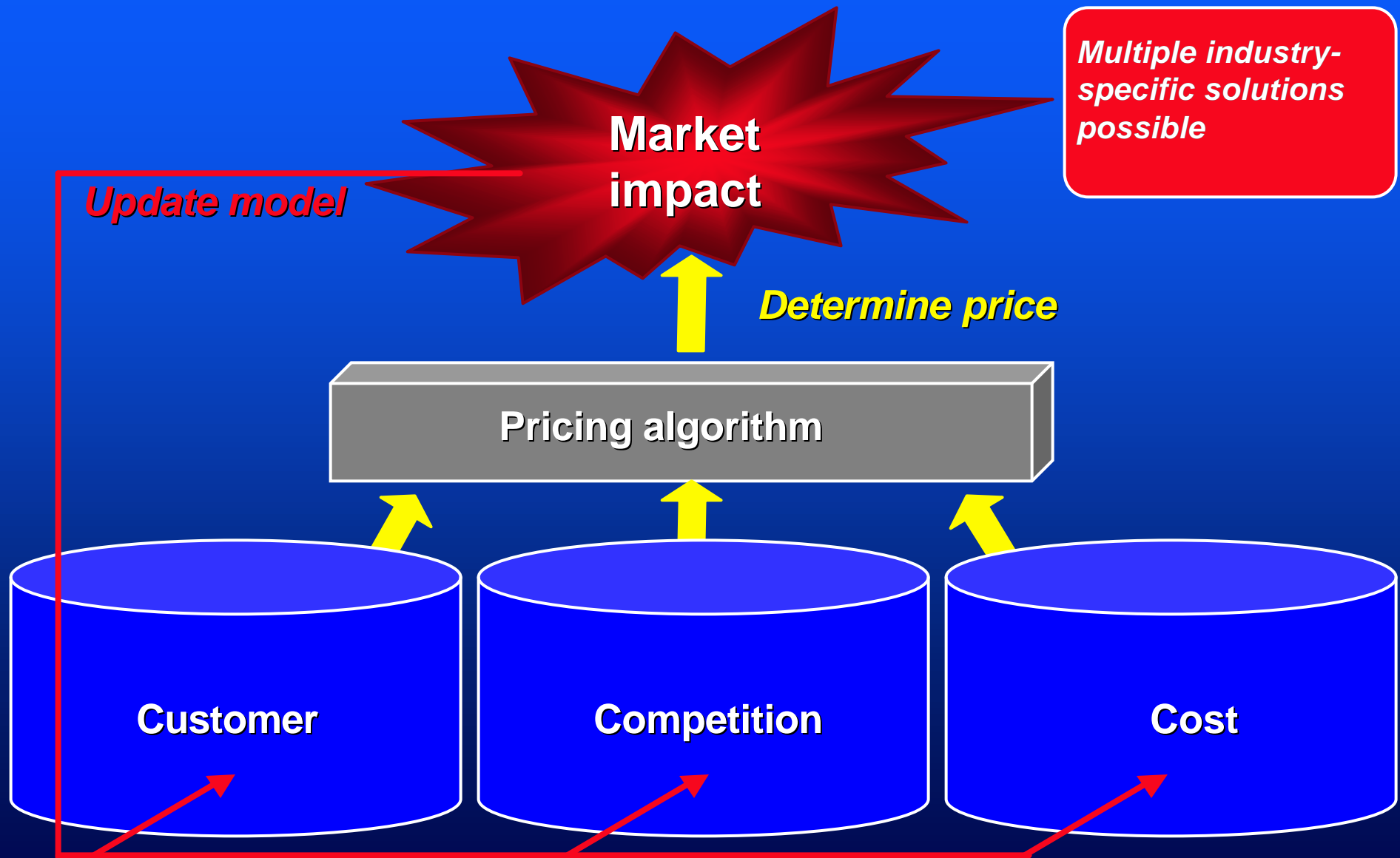
Cost

What are my degrees of freedom to close the sale?

Competition

What competing offers are they looking at?

TACTICAL PRICING FRAMEWORK



CUSTOMER BEHAVIOR

All customers

Loyal customers

Will not switch in linear region

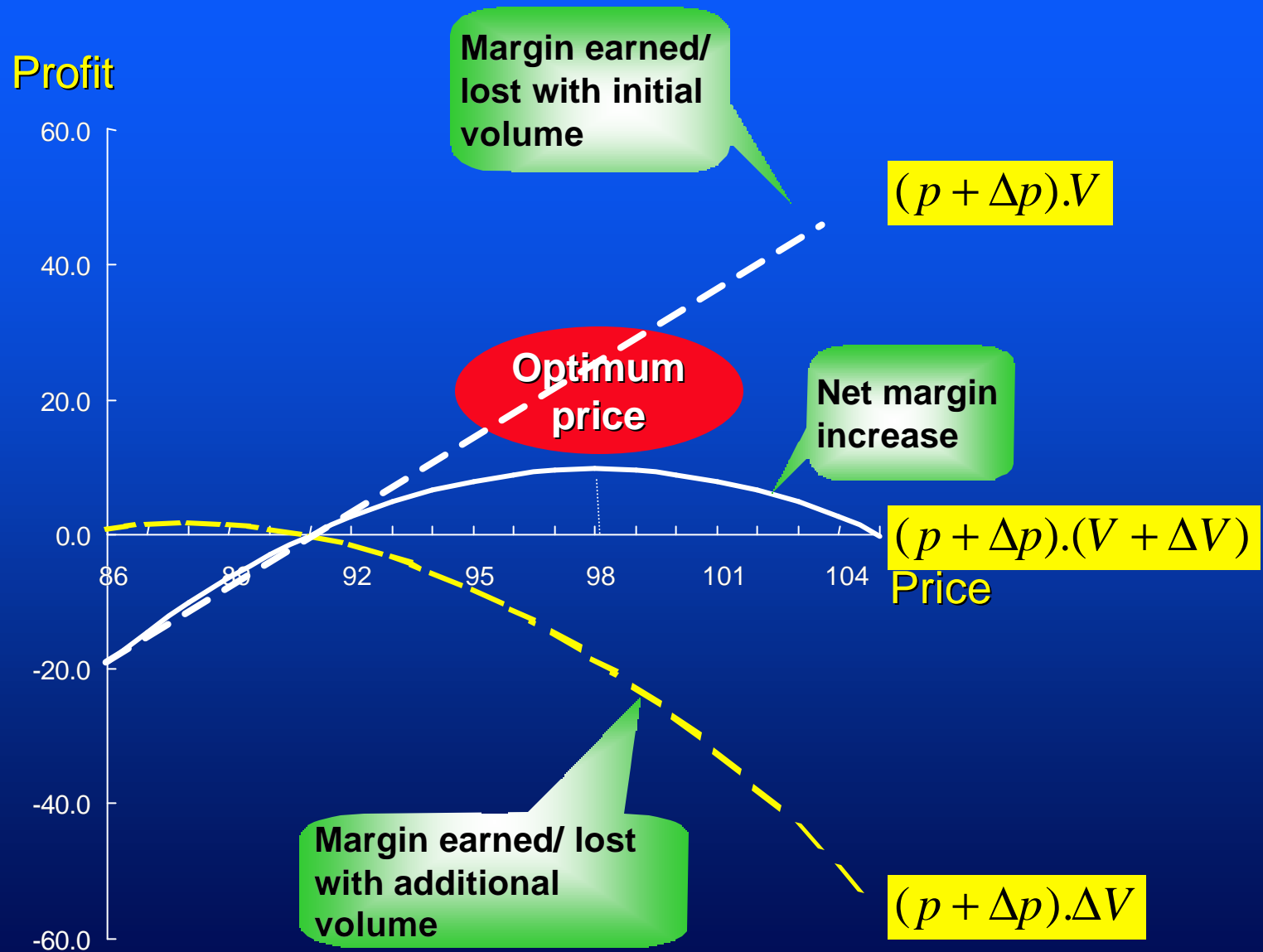
- Switching of loyal customers is highly non-linear
- Switching has hysteresis (i.e., is not immediately and completely reversible)

Shared customers

Will switch over linear region

- Elasticities based on perceived differences in
 - Product
 - Services
 - Channel
 - Promotion
- Switching behavior linearly dependent on cross-elasticity, price differential, and degree of awareness
- Limited to small price band

STATIC NON-LINEAR OPTIMIZATION AT CORE



* Margin = Price- variable cost including channel compensation

PRICING MODEL

- P: price
C: cost
V: volume
b: awareness
CC: shared customers
E: Elasticity

Full
cost

$$(P_i - C_i) \cdot \left(V_{i,0} - V_{R,ind} \cdot E_i(P_{ave} - P_{ave,0}) \pm \sum_j V_{R,comp} \cdot b \cdot CC_{ij} E_{ij}(P_i - P_j) \right)$$

Cost

- Variable cost
- Cash cost
- Full reinvestment cost

Customer

Competition

PRICING MODEL

P: price
C: cost
V: volume
b: awareness
CC: shared customers
E: Elasticity

Industry price

Awareness

Readiness

Attractiveness

$$(P_i - C_i) \cdot \left(V_{i,0} - V_{R,ind} \cdot E_i(P_{ave} - P_{ave,0}) \pm \sum_j V_{R,comp} \cdot b \cdot CC_{ij} E_{ij}(P_i - P_j) \right)$$

Customer

- Industry price
- Awareness
- Readiness
- Attractiveness

Cost

Competition

PRICING MODEL

P: price
C: cost
V: volume
b: awareness
CC: shared customers
E: Elasticity

Industry price

Price differential

$$(P_i - C_i) \cdot \left(V_{i,0} - V_{R,ind} \cdot E_i (P_{ave} - P_{ave,0}) \pm \sum_j V_{R,comp} \cdot b \cdot CC_{ij} E_{ij} (P_i - P_j) \right)$$

Intangibles

Competition

- Industry price
- Intangibles
- Price differential (net of competitive response)

Cost

Customer

UPDATE IS AUTOMATIC

Update model

- Update parameters to reduce difference between predicted and actual
- Several approaches possible (e.g., Kalman filters)



Determine price

Pricing algorithm

Customer

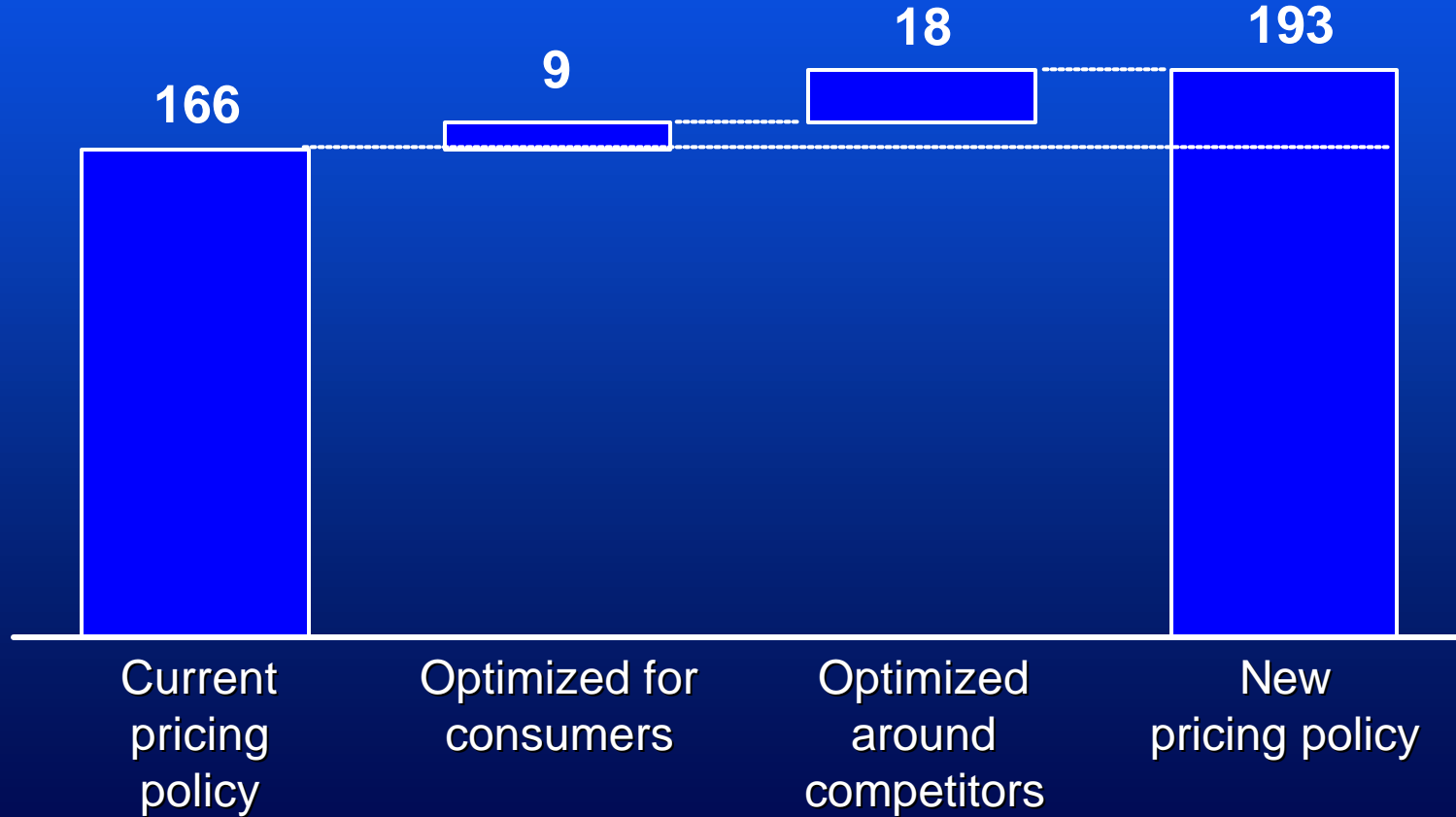
Competition

Cost

IMPACT OF NEW PRICING POLICY

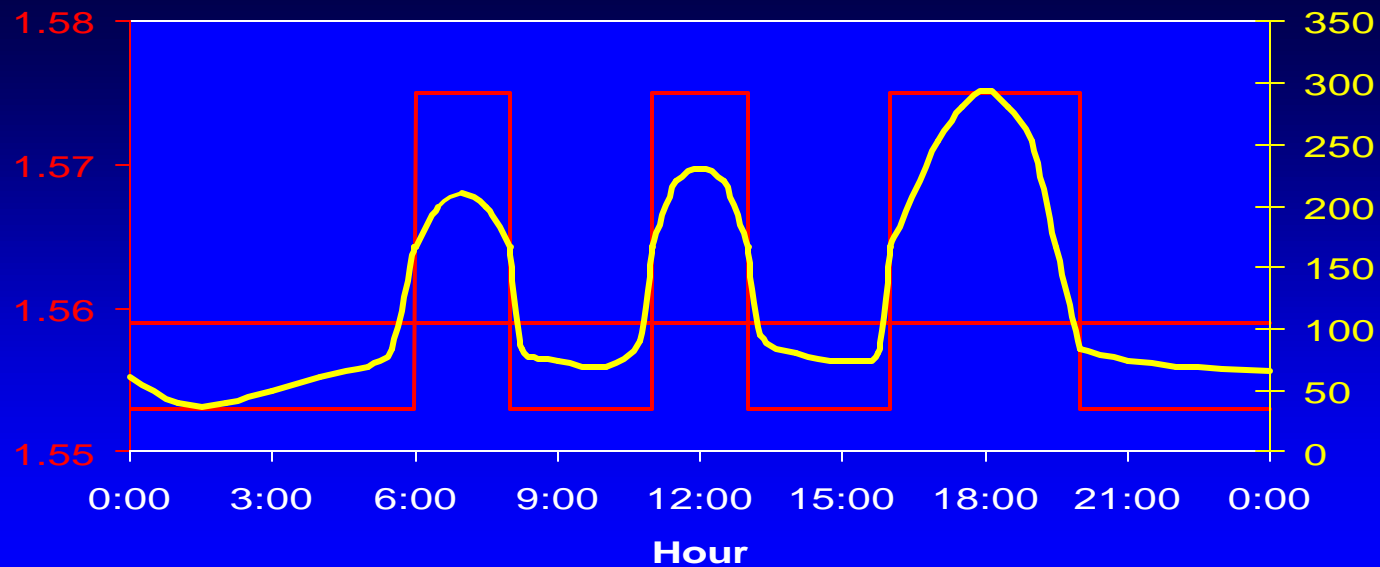
EBITDA impact, \$ million

- 16% gain
- Average 1.5 c/gal increase
- Less than 3% volume loss



INTRA-DAY SEGMENTATION

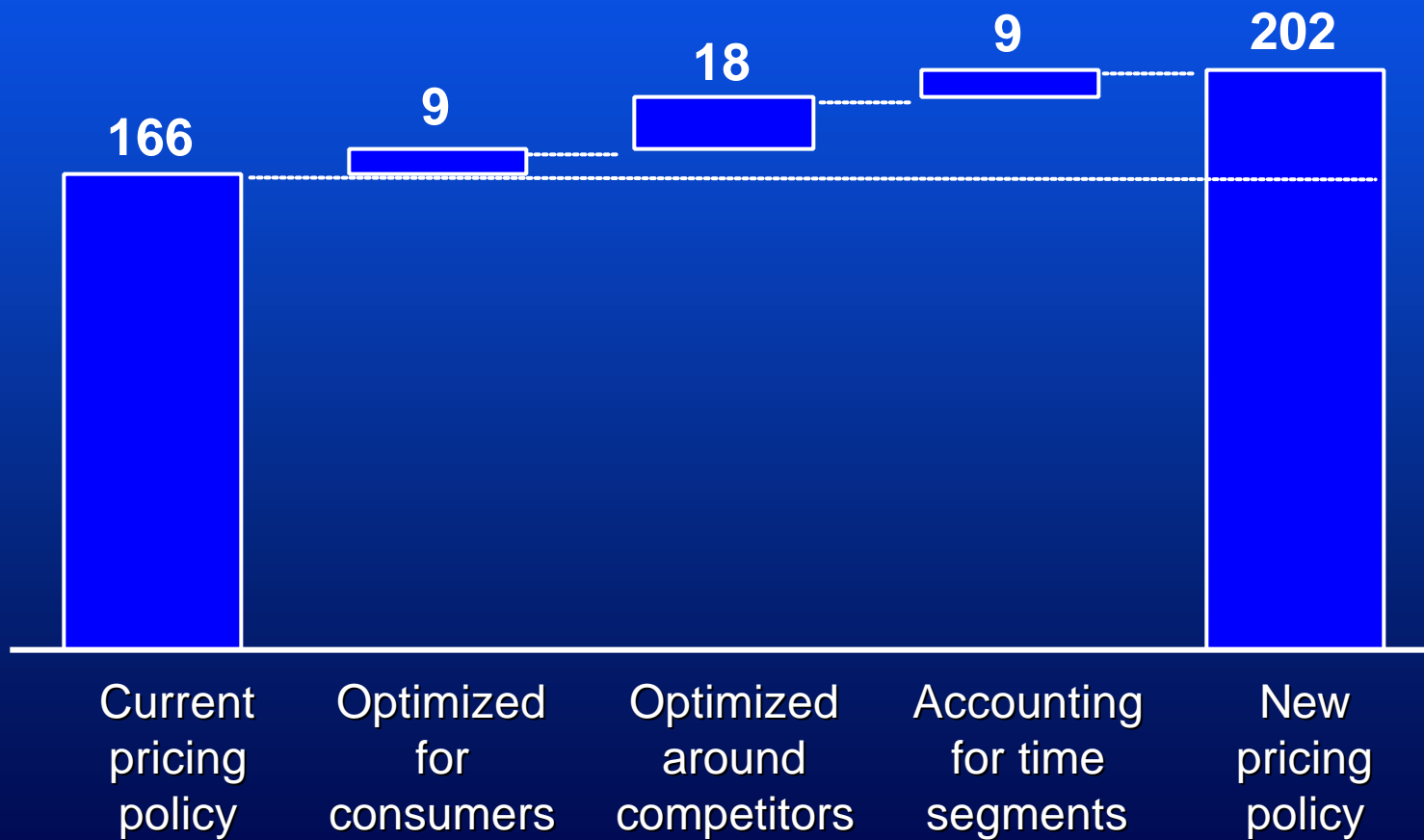
	Customers	Elasticity	
		(light)	(peak)
Loyal	55%	-	-
Shared, Mobil	29%	16%	12%
Shared, Texaco	14%	8%	6%



IMPACT OF IMPROVED NEW PRICING POLICY

EBITDA impact, \$ million

- 22% gain
- Average 2.1 c/gal increase
- Less than 5% volume loss



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BACKGROUND

Scope of the engagement

- Find **improvement opportunities** in key account management for a leading manufacturer (with **75% of market share**) through better price management, without changing the existing mix-structure
- Leverage manufacturer-retailer relationship to develop **win-win situations**

Purpose of the analysis

- Improve manufacturer category profitability by **helping the retailer improve its own category results** through better pricing policies
- Identify **optimal category price structures** for selected categories within the retailer scope of action*

Analyses performed

- **Multiple regression analyses** to determine own-price and cross-price elasticities for each SKU,** using weekly price, volume, and promotional activity data for 2 sample stores
- **Margin optimization** process for each category in both stores, incorporating retailer list prices and manufacturer unit costs per SKU

End-products and impact

- Own-price and cross-price **elasticities** for the top 5 SKUs in each analyzed category
- **Optimal pricing schemes**, resulting in 10% margin improvement for the retailer and 6% for the manufacturer

* This analysis was limited to margin changes only by the retailer. To simulate changes involving the manufacturer price list, competitive reaction must be incorporated

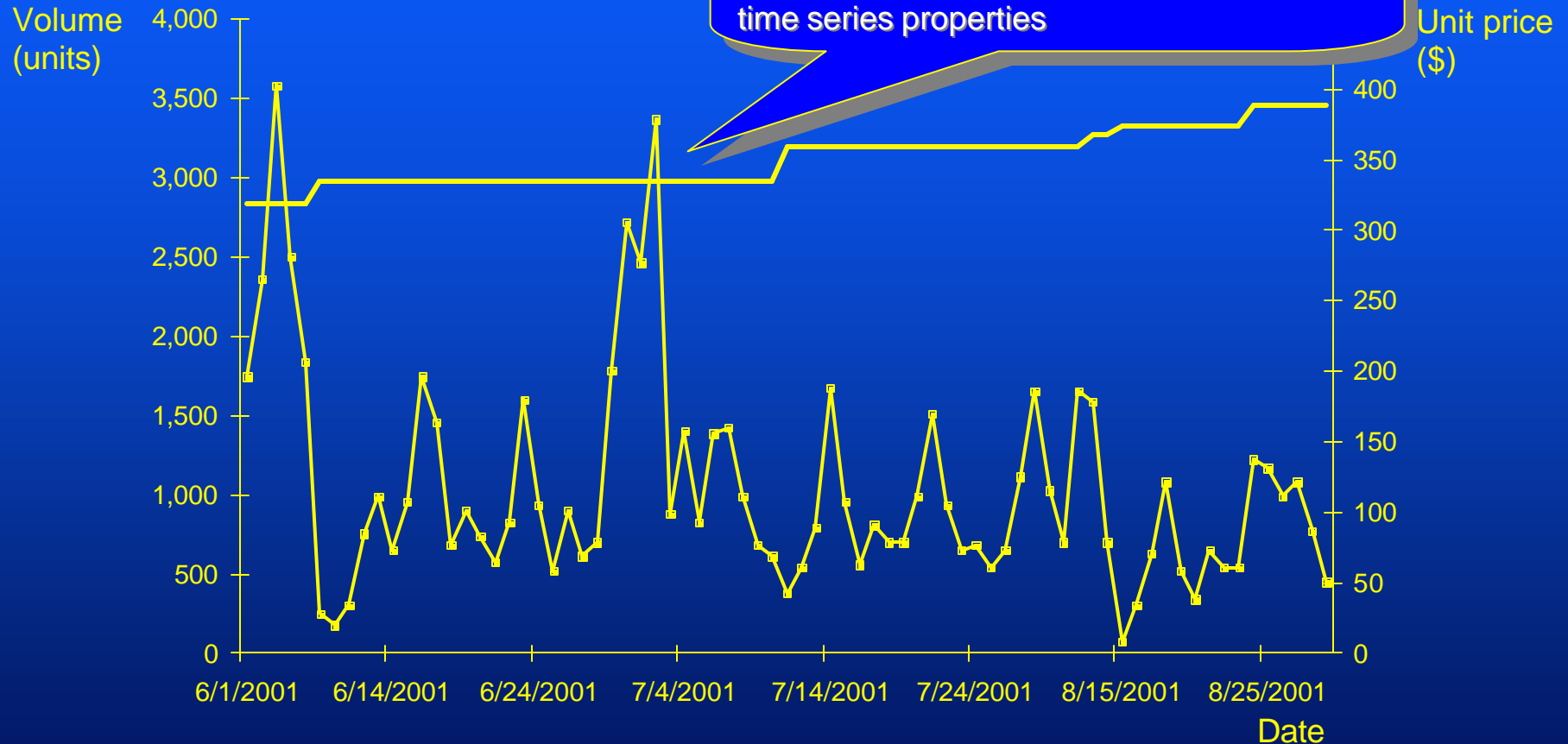
** Stock keeping unit

PRICE CHANGE ANALYSIS



Daily sales for SKU #2201
Store 2

When plotting raw price and volume data, no apparent correlation exists between the 2 variables. Prices and volumes have different time series properties



A simple log price vs. log volume regression in most cases will not be of much use. The complete data set for each regression requires competitor product prices, promotional activity dummy variables, and other qualitative variables, such as seasonality or stock-outs

ESTIMATING ELASTICITY



Analysis

- Econometric analysis has several advantages when it comes to estimating elasticity.* The general form of the log-price equation we used is:

$$\log Q_i = b_0 + b_i \log P_i + \sum_j c_j \log P_j + \sum_k d_k D_k$$

where:

- Q_i is the volume sold and P_i is the price of target SKU i
- P_j is the price of competitor SKU j
- b_i and c_j are own-price and cross-price elasticities
- D_k are dummy variables accounting for promotional activities, store location, seasonality, etc. with their corresponding coefficients d_k

Rationale

- A product's sales volume (Q_i) at a given point in time can be explained in terms of its own price (P_i), other competing product prices (P_j), relevant promotional activities, and other events, such as stock-outs and seasonal patterns (D_k)

Caveats

- Make sure the **correlation** among explanatory ("right-hand") variables is low, especially between continuous (i.e., price) and binary (i.e., "catalog") variables; keep only one of the highly correlated explanatory variables
- Use **alternative model** specifications* (linear demand function, deviation-from-mean model, etc.) to improve model fit
- Given its complexity, it is critical to involve client team members in this process. Client team members should be able to present model assumptions and results to management and thus step away from a conceptual **black-box** perspective

* See "Estimating Price and Promotional Elasticity in Data-Rich Environments," K.K. Davey, PDNet, Nov. 1997

MARGIN OPTIMIZATION MODEL'S PROCESS



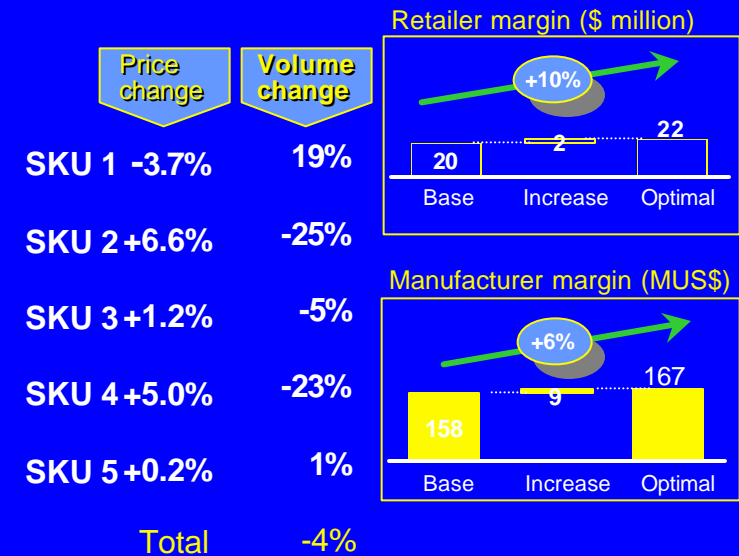
Input

- Daily sales data
 - Prices
 - Volume
 - Cost of goods sold
- Promotional activity log
 - Inclusion in catalogs
 - Temporary exhibit
 - Special event
- Manufacturer margin
- Industry constraints
 - Market share
 - Price/brand positioning
 - Average category pricing

Working steps

- Define the objective function(s)
- Enter demand and cost functions
- Add industry constraints
- Solve the model using a non-linear optimization software
- Stress-test the results
- Discuss recommendations with the retailer and validate results through pilot tests

Output

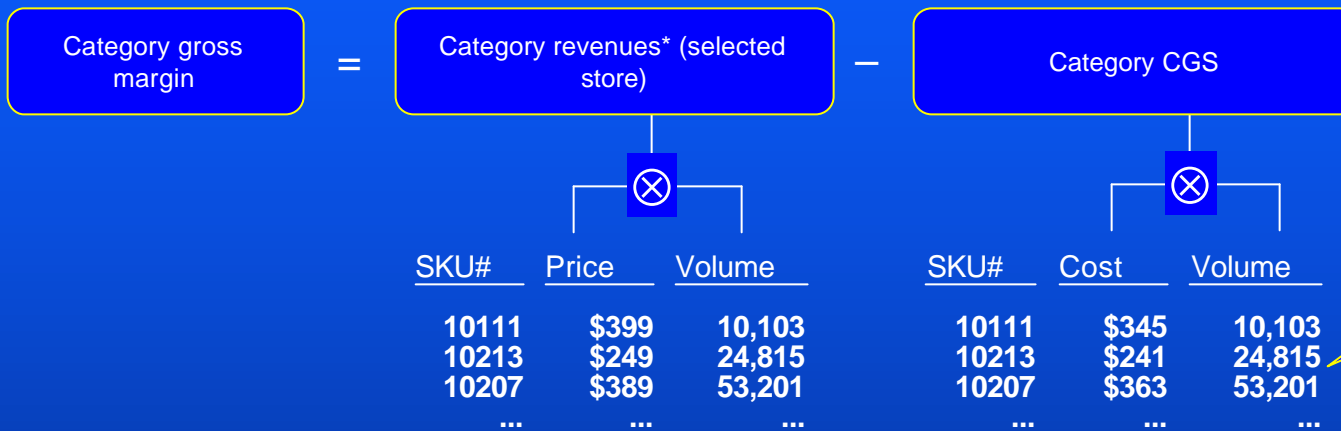


OBJECTIVE FUNCTIONS



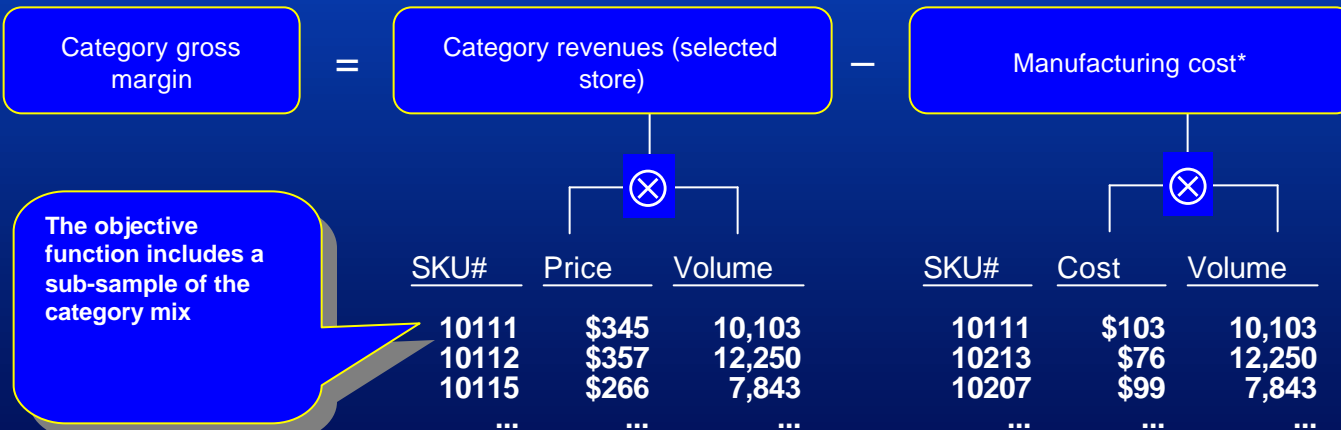
EXAMPLE

Retailer



Volume calculated from the elasticity model assuming a "neutral" setting (i.e., all dummy variables set to zero)

Manufacturer



The objective function includes a sub-sample of the category mix

The retailer focuses on overall category contribution, whereas the manufacturer maximizes its own product mix contribution

* If possible, trade spend and support should be added to the retailer's category revenues and to the manufacturing cost. Trade spend and support includes rappel, volume discount, year-end bonuses, indirect discounts, fixed-trade spend, and cost-to-serve variable expenses.

MATHEMATICAL PROBLEM SETTING



Optimize: $\max \sum_i Q_i (P_i - C_i)$ and/or $\max \sum_i Q_i (C_i - M_i)$

Subject to: $\log Q_i = b_0 + b_i \log P_i + \sum_j c_j P_j \quad \forall i$

SKU demand function as calculated in the elasticity model for each SKU

$$Q_{MAX} \geq \sum_i Q_i \geq Q_{MIN}$$

Total consumer demand for the category needs to stay within acceptable ranges

$$P_1 > \$4.99$$

$$P_2 > P_1 \leq P_4$$

$$\frac{Q_4}{\sum_i Q_i} \geq 30\%$$

Minimum required market share for SKU #4

Where: Q is volume in units sold, P is price per unit, C is retailer cost per unit (and manufacturer list price), M is manufacturer unit cost including trade spend, b s are own-price elasticity estimates, and c s are cross-price elasticity estimates

OUTLINE OF PRESENTATION

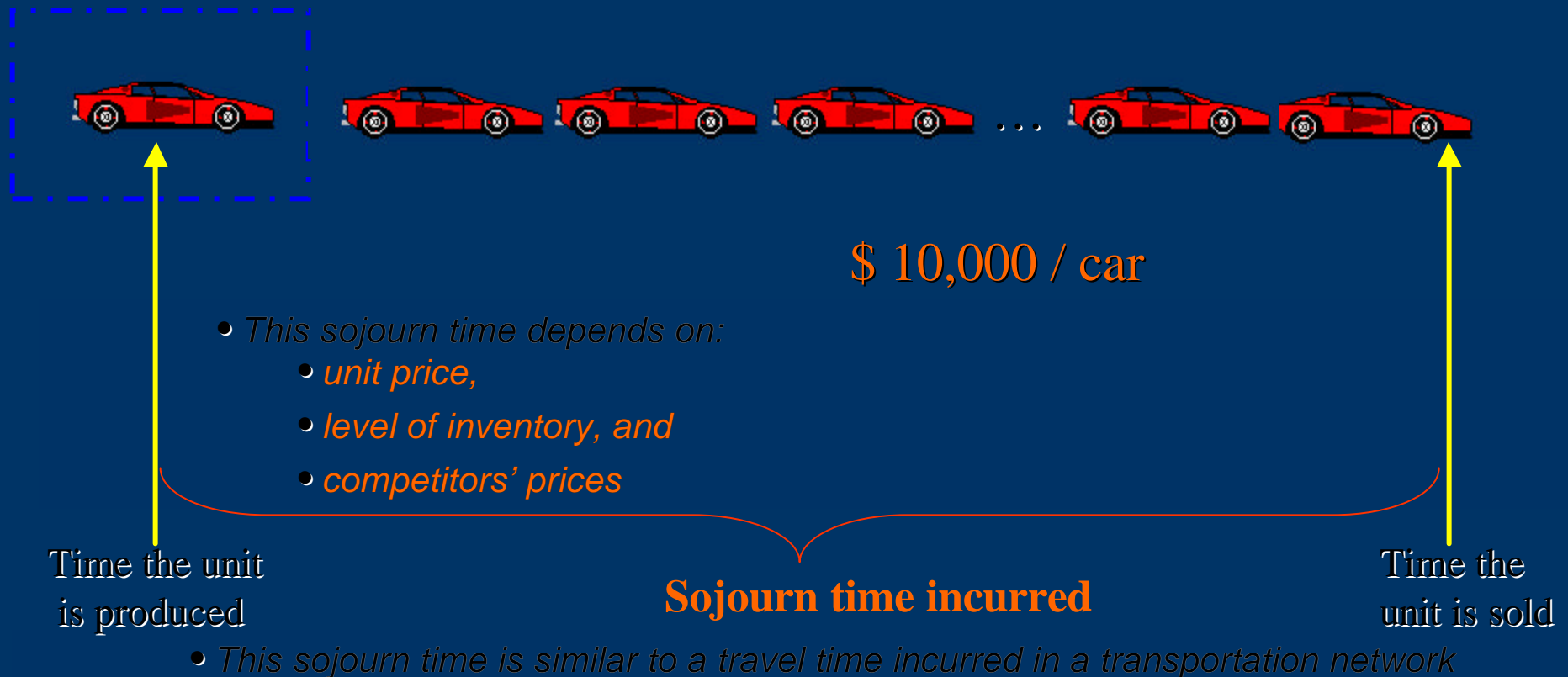
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Introduction

Introduction and Outline

Observation:

- A newly produced unit of good incurs a sojourn time before being sold



Introduction

Introduction and Outline

Observation:

- A newly produced unit of good incurs a sojourn time before being sold
 - *This sojourn time depends on unit price, competitors' prices and level of inventory*
 - *This sojourn is similar to a travel time incurred in a transportation network*

Contribution:

- Propose and study a dynamic pricing model:
 - *Incorporates the delay of price and level of inventory in affecting demand*
 - *Includes pricing, production and inventory decisions in a multi-product environment*

Approach:

- A transportation fluid dynamics model that incorporates:
 - *Price/Inventory level delay function*
 - *Production and sales dynamics*
 - *Production capacity constraints*

Goals:

- Apply analytical methodologies and solution algorithms borrowed from the transportation setting to inventory control and supply chain
- Capture a variety of insightful phenomena that are harder to capture using current models in the literature

Motivation

Literature

Pricing theory has been extensively studied by researchers from a variety of fields:

- *Economics (see for example R. Wilson (1993))*
- *Marketing (see for example G. Lilien et. Al (1992))*
- *Revenue management and supply chain management (see for example G. Bitran and S. Mondschein (1997), LMA. Chan et. al (2000), and, J. McGill and G. Van Ryzin (1999))*
- *Telecommunications (see for example, F. P. Kelly (1994), F. P. Kelly et al. (1998), and, I. Paschalidis and J. Tsitsiklis (1998))*
- *The book by Zipkin (1999), and references therein, provide a thorough review of inventory models.*

Assumptions and Notations

Modeling Assumptions

■ We consider:

- *Stackelberg leader (Monopoly is a special case)*
- *Many products*
- *Common capacity*
- *No substitution between products*
- *Holding costs*
- *No setup costs*
- *Non-perishable products*
- *Unit price is a function of inventory $p_i(I_i)$ (e.g. linear, hyperbolic)*
- *Deterministic model.*

■ Average delay to sell a unit of good

$$A_i(I_i(t)) = T_i(I_i(t), p_i(I_i(t)), p_{i,1}^c(p_i(\cdot)), p_{i,2}^c(p_i(\cdot)), \dots, p_{i,J(i)}^c(p_i(\cdot)))$$

Average time needed to sell, at time t , a unit of product i , given an inventory $I_i(t)$, a unit price $p_i(I_i(t))$, and competitors' prices $p_{i,j}^c(p_i(\cdot))$, $j \in \{1, \dots, J(i)\}$.

- *Provide a methodology to estimate such a function in practice.*
- *Establish connection with the travel functions derived in the transportation context.*

Assumptions and Notations

Notations

- $U_i(t)$: cumulative production flow of product i during interval $[0, t]$;
- $u_i(t)$: production flow rate of product i at time t ;
- $V_i(t)$: cumulative sales flow of product i during interval $[0, t]$;
- $v_i(t)$: sales flow rate of product i at time t ;
- $I_i(t)$: inventory (number of units of product) i at time t ;
- $p_i(I_i(t))$: sales price of one unit of product i given an inventory $I_i(t)$;
- $T_i(I_i(t), p_i(\cdot), (p_{i,j}^c(p_i(\cdot))), j \in \{1, \dots, J(i)\}))$: time needed to sell both a unit of product i produced at time t and all inventory $I_i(t)$, given a unit price $p_i(I_i(t))$ and competitors prices $p_{i,j}^c(p_i(\cdot))$;
- $D_i(\cdot)$: product delay function, a function of the inventory $I_i(t)$, for example, we can choose $D_i(I_i(t)) = T_i(I_i(t), p_i(\cdot), (p_{i,j}^c(p_i(\cdot))), j \in \{1, \dots, J(i)\}))$;
- $s_i(t)$: exit time of a production flow of product type i entering at time t ($s_i(t) = t + D_i(I_i(t))$);
- $c_i(t)$: production cost of product i at time t ;
- $h_i(t)$: inventory cost of product i at time t ;
- $[0, T]$: production period. After time T , the company ceases producing;
- $[0, T_\infty]$: analysis period. It is the interval of time from the instant when the first unit of product is produced to the first instant when all products have been sold.

A Continuous-Time Fluid Dynamics Delay Model for Dynamic Pricing and Inventory Management

Model Formulation

Link dynamics equations

The link dynamics equations express the relationship between the flow variables of a link. They are given by:

$$\frac{dI_i(t)}{dt} = u_i(t) - v_i(t), \quad \forall i \in \{1, \dots, n\}.$$

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Flow propagation equations

Flow propagation equations are used to describe the flow progression over time.

$$V_i(t) = \int_{\omega \in W} u_i(\omega) d\omega, \quad \forall i \in \{1, \dots, n\},$$

where $W = \{\omega : s_i(\omega) \leq t\}$.

If the product exit time functions $s_i(\cdot)$ are continuous, and if the strict First-In-First-Out (FIFO) property is satisfied, then

$$V_i(t) = \int_0^{s_i^{-1}(t)} u_i(\omega) d\omega, \quad \forall i \in \{1, \dots, n\}.$$

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Boundary equations

$$U_i(0) = 0, \quad V_i(0) = 0, \quad I_i(0) = 0, \quad \forall i \in \{1, \dots, n\}.$$

A Continuous-Time Fluid Dynamics Delay Model for Dynamic Pricing and Inventory Management

Model Formulation

Non-negativity and Capacity Constraints

$$u_i(\cdot) \geq 0, \quad i \in \hat{I} = \{1, \dots, n\}, \quad CFR(\cdot) \geq 0.$$
$$\sum_{i=1}^n u_i(t) \leq CFR(t).$$

A Continuous-Time Fluid Dynamics Delay Model for Dynamic Pricing and Inventory Management

Model Formulation

Non-negativity and Capacity Constraints

$$u_i(.) \geq 0, \quad i \in \{1, \dots, n\}, \quad CFR(.) \geq 0.$$
$$\sum_{i=1}^n u_i(t) \leq CFR(t).$$

Objective function

The objective of the company is to maximize its profits. Profits are obtained by subtracting production costs and inventory costs from sales. The objective function can be expressed as:

$$\sum_{i=1}^n \int_0^{T_{\infty}} p_i(I_i(t))v_i(t) - c_i(t)u_i(t) - h_i(t)I_i(t)dt.$$

In general, the DPM is a continuous-time non-linear program. In this system of equations, the known variables are the product delay functions $A_i(.)$ and the shared capacity $CFR(.)$. The unknown variables are $u_i(t)$, $U_i(t)$, $v_i(t)$, $V_i(t)$, $I_i(t)$, $s_i(t)$ and the parameters of $p_i(I_i)$.

Feasibility conditions are similar to the Dynamic Network Loading (DNL) Problem in the dynamic traffic assignment context

Extensive work done on the DNL problem, especially at CRT in Montreal

Discretized DPM Model

Solution Algorithm

Objective Function:

$$Obj = -Min \sum_{i=1}^n (k_i [\sum_{j=0}^{N-1} u_{ij} u_{ij+1} + \sum_{j=0}^N u_{ij}^2] + \sum_{j=0}^N g_{ij} u_{ij})$$

Constraints:

$$\sum_{i=0}^n u_{ij} \leq CFR_j, \quad \forall j \in \{0, 1, \dots, N\}$$
$$u_{ij} \geq 0, \quad \forall i \in \{1, 2, \dots, n\}, \forall j \in \{0, 1, \dots, N\}$$

where $g_{ij} = -\mathbf{d} (p_i^{\max} - c_{ij} - \frac{h_{ij} + h_{ij+1}}{2} \mathbf{d})$,

$$k_i = \frac{\mathbf{e}_i \mathbf{d}^2}{2}, \text{ and } \mathbf{e}_i = \frac{p_i^{\max} - p_i^{\min}}{C_i}$$

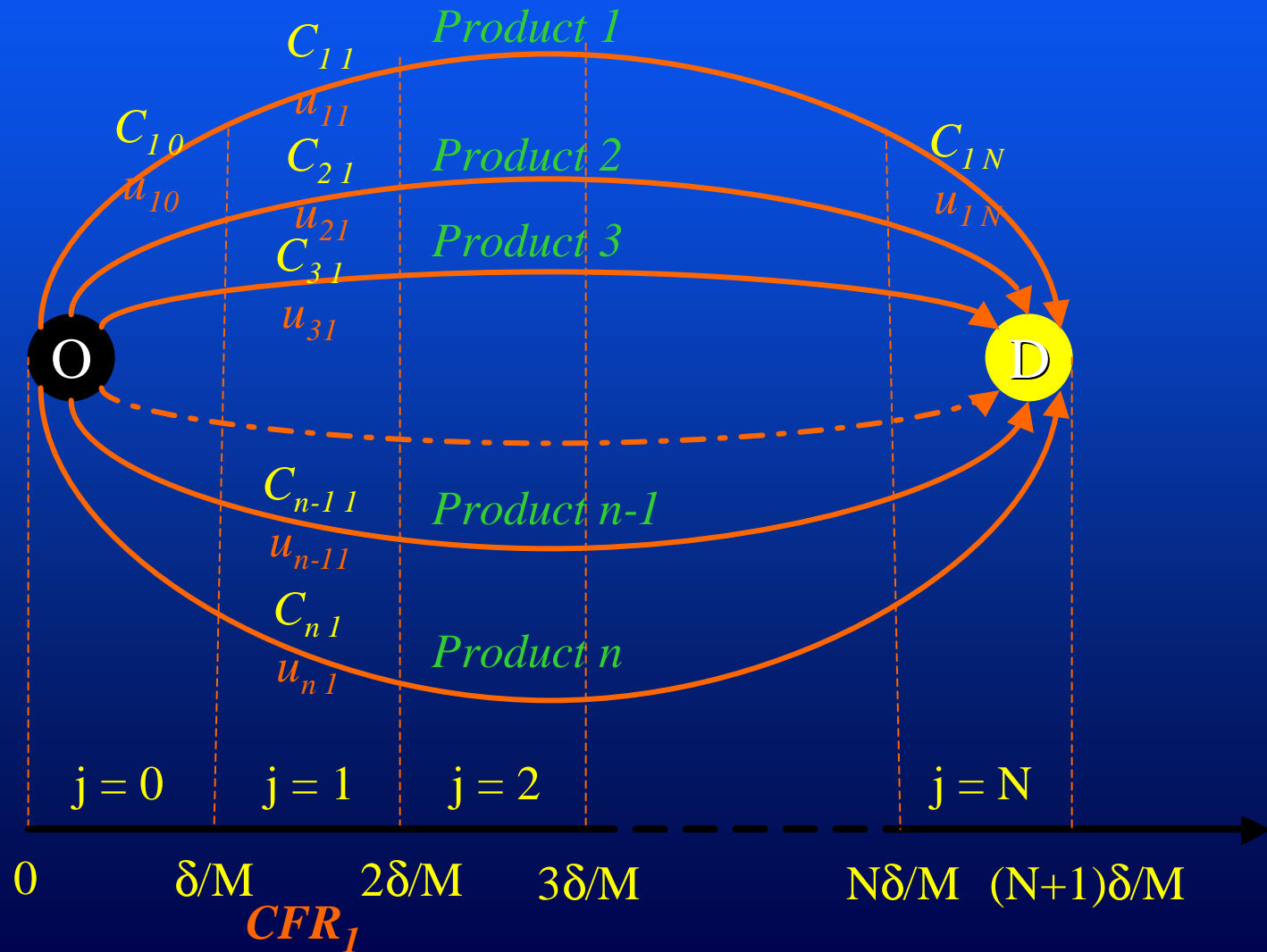
Approach:

$$C_{ij} = -\frac{\partial Obj}{\partial u_{ij}} = 2k_i u_{ij} + k_i (u_{ij+1} + u_{ij-1}) + g_{ij}$$


m_{ij}

Graphical Illustration

$$C_{ij} = 2k_i u_{ij} + k_i (u_{ij+1} + u_{ij-1}) + g_{ij}$$



Discretized DPM Model

Solution Algorithm

Iterative Relaxation Approach

Step 0: ($k=0$) for every $j \in \hat{I} \{0, \dots, N\}$, for every $i \in \hat{I} \{1, \dots, n\}$

$$u_{ij}^0 = \frac{CFR_j}{n}$$

$k = 1$

Step k: for every $j \in \hat{I} \{0, \dots, N\}$:

$$m_{ij}^k = k_i (u_{ij-1}^k + u_{ij+1}^{k-1}) + g_{ij}$$

$$C_{ij}^k = 2k_i u_{ij}^k + m_{ij}^k$$

We order the m_{ij} 's in non-decreasing order

$$m_{\text{order}(1,j)j}^k \leq m_{\text{order}(2,j)j}^k \leq \dots \leq m_{\text{order}(n,j)j}^k$$

Equilibration approach

$$\text{Find } l_j : C_{\text{order}(1,j)j}^k = \dots = C_{\text{order}(l_j,j)j}^k = a_{\text{order}(l_j,j)j}^k \leq C_{\text{order}(l_j+1,j)j}^k, \dots, \leq C_{\text{order}(n,j)j}^k$$

$$u_{\text{order}(1,j)j}^k > 0, \dots, u_{\text{order}(l_j,j)j}^k > 0, \sum_{i=1}^{l_j} u_{\text{order}(i,j)j}^k = CFR_j \quad u_{\text{order}(l_j+1,j)j}^k = \dots = u_{\text{order}(n,j)j}^k = 0$$

Discretized DPM Model

Solution Algorithm

Iterative Relaxation Approach

Step k (continued): for every $j \in \hat{I} \setminus \{0, \dots, N\}$:

$$\text{Let } \mathbf{a}_{\text{order}(i,j)j}^k = \frac{\text{CFR}_j + \sum_{m=1}^i \frac{m_{\text{order}(m,j)j}^k}{2k_{\text{order}(m,j)j}}}{\sum_{m=1}^i \frac{1}{2k_{\text{order}(m,j)j}}}$$

$$\text{Let } l_j = \left\{ \begin{array}{l} \arg \min \{ i \in \{1, \dots, n-1\} : \mathbf{a}_{\text{order}(i,j)j}^k \leq m_{\text{order}(i+1,j)j}^k, \text{ if it exists} \\ n, \text{ otherwise} \end{array} \right\}$$

$$\text{If } i > l_j, \quad u_{\text{order}(i,j)j}^k = 0$$

$$\text{If } i \leq l_j, \quad u_{\text{order}(i,j)j}^k = \frac{\mathbf{a}_{\text{order}(l_j,j)j}^k - m_{\text{order}(i,j)j}^k}{2k_{\text{order}(i,j)j}}.$$

Convergence criterion:

If $u_{ij}^k = 0 \Rightarrow C_{ij}^k \geq \mathbf{a}_{\text{order}(l_j,j)j}^k - \mathbf{e}$, stop. Otherwise $k=k+1$, go to step k.

Main result: The iterative relaxation algorithm converges to the unique optimal solution.

Discretized DPM Model

Small Case Example

Inputs:

5 products, 10 discretization intervals

Price/Inventory Relationship parameters

p_i^{max}	p_i^{min}	
10.1	4.1	<i>Product 1</i>
10.2	4.2	<i>Product 2</i>
10.3	4.3	<i>Product 3</i>
10.4	4.4	<i>Product 4</i>
10.5	4.5	<i>Product 5</i>

Shared Capacity Flow Rate Function

Discretization Interval Index	0	1	2	3	4	5	6	7	8	9
CFR_j	19	21	23	25	27	29	31	33	35	37

Discretized DPM Model

Small Case Example (continued)

Inputs (continued):

	Production Cost									
	0	1	2	3	4	5	6	7	8	9
c_{1j}	8.5938	8.7500	8.8698	8.9709	9.0599	9.1405	9.2144	9.2833	9.3481	9.4093
c_{2j}	5.4000	5.6209	5.7905	5.9333	6.0593	6.1731	6.2778	6.3751	6.4667	6.5533
c_{3j}	4.1698	4.4405	4.6481	4.8231	4.9772	5.1167	5.2449	5.3642	5.4762	5.5823
c_{4j}	4.9209	5.2333	5.4731	5.6751	5.8533	6.0142	6.1622	6.3000	6.4293	6.5517
c_{5j}	7.6599	8.0093	8.2772	8.5033	8.7022	8.8823	9.0478	9.2017	9.3465	9.4833

	Holding Cost									
	0	1	2	3	4	5	6	7	8	9
h_{1j}	1.6037	1.5135	1.4865	1.4236	1.4107	1.3568	1.3504	1.3013	1.2987	1.2528
h_{2j}	1.4138	1.2862	1.2482	1.1591	1.1409	1.0647	1.0556	0.9861	0.9825	0.9176
h_{3j}	1.2331	1.0770	1.0302	0.9213	0.8989	0.8057	0.7944	0.7095	0.7049	0.6254
h_{4j}	1.0574	0.8770	0.8230	0.6972	0.6714	0.5637	0.5507	0.4526	0.4474	0.3556
h_{5j}	0.8846	0.6830	0.6226	0.4820	0.4532	0.3326	0.3182	0.2085	0.2027	0.1000

Discretized DPM Model

Small Case Example (continued)

Intermediary computations:

Modified Margins

	0	1	2	3	4	5	6	7	8	9
m_{1j}	0.0524	0.1500	0.2249	0.2881	0.3437	0.4925	0.4412	0.9005	0.5247	0.8807
m_{2j}	-2.7595	-2.4293	-1.7226	-1.8993	-1.3522	-1.4482	-1.0171	-1.0808	-0.7019	-1.7972
m_{3j}	-4.1725	-2.6537	-2.8482	-2.3648	-2.1954	-2.1620	-1.5787	-2.0477	-0.9823	-3.0201
m_{4j}	-3.7694	-2.5334	-2.5139	-2.1342	-1.9388	-1.8229	-1.4048	-1.6001	-0.8945	-2.4643
m_{5j}	-1.7719	-1.6161	-1.1148	-0.8550	-0.8761	-0.1764	-0.6899	0.4197	-0.5354	-0.0679

Minimum Equilibrium Costs

	0	1	2	3	4	5	6	7	8	9
a_{1j}	-1.5722	-0.8085	-0.4909	-0.1930	0.0922	0.3686	0.6381	0.9023	1.1621	0.4822
a_{2j}	-2.0471	-0.8588	-0.5216	-0.1328	0.3312	0.5089	1.1465	1.0638	1.9404	0.5328
a_{3j}	0.3875	2.3863	2.6718	3.6352	4.2846	4.7980	5.8613	5.8723	7.4177	5.8599
a_{4j}	-1.6909	-0.0736	0.0789	0.7505	1.1729	1.4875	2.2282	2.1361	3.2616	1.6978
a_{5j}	-1.9783	-1.0481	-0.6699	-0.3133	0.0294	0.3376	0.6874	0.9028	1.3215	0.3826

Discretized DPM Model

Small Case Example (continued)

Output:

Optimal Production Flow Rates

	0	1	2	3	4	5	6	7	8	9
u_{1j}	0	0	0	0	0	0	0.8207	0.0074	2.6560	0
u_{2j}	2.9681	5.7549	4.3862	6.6084	5.7568	7.4410	6.8967	8.2630	7.7667	9.0827
u_{3j}	8.8559	6.6900	9.0762	8.5477	9.2700	10.4151	9.2370	12.2918	8.9352	14.1780
u_{4j}	7.1760	6.1885	7.6836	7.5871	8.2005	9.0021	8.5122	10.4268	8.5692	11.8622
u_{5j}	0	2.3666	1.8540	2.2568	3.7727	2.1418	5.5334	2.0110	7.0729	1.8772

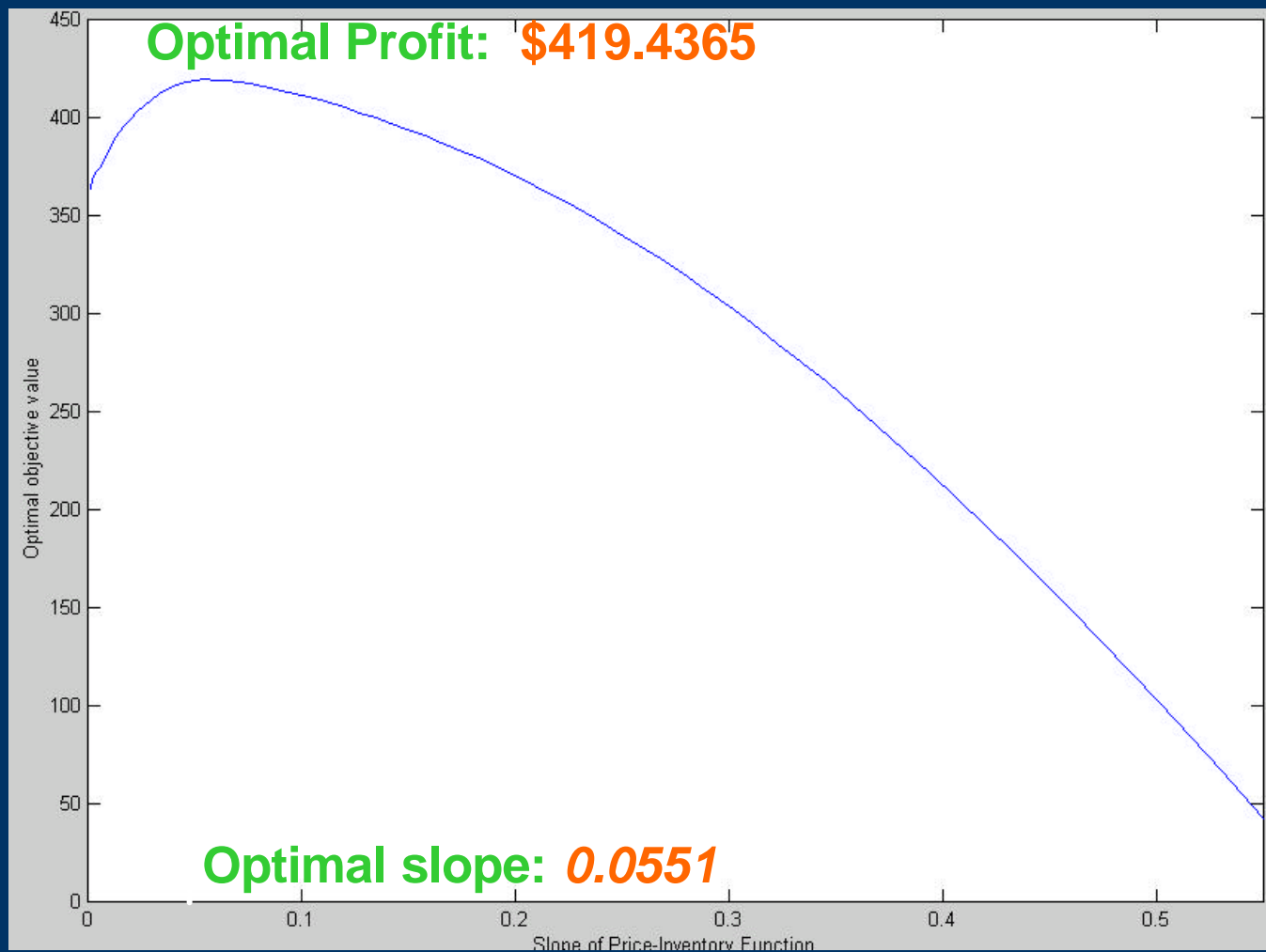
Optimal Profit: **\$405.0681**

Discretized DPM Model

Small Case Example (continued)

Joint pricing and inventory management:

Optimal Profit as function of the slope (i.e. $(s+s^2)/C$)



Competitive Setting

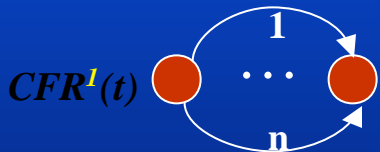
$l = 1, \dots, K$ competing retailers
 n products



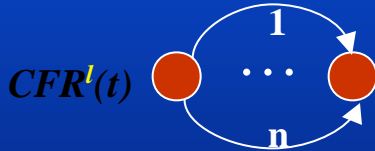
In what follows, we express p_i^l as $p_i^l(I_i(t), I_{i^{-l}}(t))$. This assumes:

- Knowledge of inventories of all players
- $\{p_i^l(I_i(t), p_{i^{-l}}), l=1, \dots, K\}$ is “invertible”

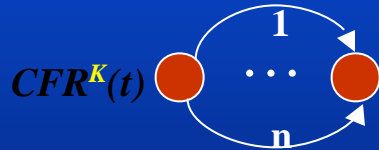
Best Response Problem for Retailer l



...



...



Retailer 1

Retailer l

Retailer K

Best Response Problem Retailer l

$$\text{Max} \sum_{i=1}^n \int_0^{T_\infty} p_i^l(I_i^l(t), I_i^{-l}(t)) v_i^l(t) - c_i^l(t) u_i^l(t) - h_i^l(t) I_i^l(t) dt$$

$$I_i^l(t) = U_i^l(t) - V_i^l(t) \quad \forall i \in \{1, \dots, n\}$$

such that

$$\sum_{i=1}^n u_i^l(t) \leq \text{CFR}^l(t)$$

$$u_i^l(\cdot) \geq 0 \quad \forall i \in \{1, \dots, n\}$$

$$I_i^l(0) = U_i^l(0) = V_i^l(0) = 0 \quad \forall i \in \{1, \dots, n\}$$

If the product exit time functions $s_i^l(\cdot)$ continuous, if **strict FIFO** holds, then

$$s_i^l(w) = w + D_i^l(I_i^l(w), I_i^{-l}(w))$$

$$V_i^l(t) = \int_0^{s_i^{l-1}(t)} u_i^l(w) dw \quad \forall i \in \{1, \dots, n\}$$

Discretized DPM Model

Pricing function:

$$p_i^l(I_i(t)) = p_i^l \max - e_i^l I_i^l(t) + \sum_{k \text{ not } l} f_i^k I_i^k(t)$$
$$e_i^l > 0$$

Discretization: intervals of length d/M where M : discretization accuracy

For every discretization interval $j \in \hat{I} = \{0, 1, \dots, (N+1)M - 1\}$, and for every $t \in \hat{I} = [j d/M, (j+1) d/M)$:

$$CFR(t) = CFR_j, \quad u_i(t) = u_{ij}, \quad c_i(t) = c_{ij}, \quad \text{and} \quad h_i(t) = h_{ij}$$

Piecewise constant

Decision variables:

Production levels: u_{ij} for every product i and for every discretization interval index j

For simplicity of the presentation, we consider $M=1$

Best Response – Retailer l

$$Obj^l = -Min \sum_{i=1}^n (k_i^l [\sum_{j=0}^{N-1} u_{ij}^l u_{ij+1}^l + \sum_{j=0}^N u_{ij}^{l^2}] + \sum_{j=0}^N g_{ij}^l u_{ij}^l - \sum_{k \neq l} l_i^k [\sum_{j=0}^{N-1} u_{ij}^l u_{ij+1}^k + \sum_{j=0}^N u_{ij}^l u_{kj}^k])$$

such that

$$\sum_{i=0}^n u_{ij}^l \leq CFR_j^l, \quad \forall j \in \{0, 1, \dots, N\}$$

$$u_{ij}^l \geq 0, \quad \forall i \in \{1, 2, \dots, n\}, \forall j \in \{0, 1, \dots, N\}$$

$$p_i^l(I_i(t)) = p_i^{l \max} - e_i^l I_i^l(t) + \sum_{k \text{ not } l} f_i^k I_i^k(t)$$

$$k_i^l = \frac{e_i^l d^2}{2} \quad l_i^k = \frac{f_i^k d^2}{2}$$

$$g_{ij}^l = -d \left(p_i^{l \max} - c_{ij}^l - \frac{h_{ij}^l + h_{ij+1}^l}{2} d \right)$$

Best Response Model and Nash Equilibrium

Best response model:

- The best response problem is a strictly convex quadratic problem
- There exists a solution to the best response problem, and this solution is unique

Nash equilibrium:

- If $e_i^l > S_{knot l} |f_i^k|$, there exists a Nash Equilibrium, and this equilibrium is unique

Solution Algorithm

Main ideas behind the solution algorithm

➤ **Non-separability by retailer is overcome using an iterative learning algorithm: **outer loop****

- **We start with initial production policies for every retailer**
- **At each iteration, retailers solve the QP using information from past iteration about other retailers: **inner loop****
 - ✓ **In the inner loop, non-separability by time period and shared capacity constraint among products are overcome using an iterative relaxation algorithm**

Solution Algorithm - Inner Loop: For each retailer

Objective Function:

$$Obj = -Min \sum_{i=1}^n (k_i [\sum_{j=0}^{N-1} u_{ij} u_{ij+1} + \sum_{j=0}^N u_{ij}^2] + \sum_{j=0}^N g_{ij} u_{ij})$$

Constraints:

$$\sum_{i=0}^n u_{ij} \leq CFR_j, \quad \forall j \in \{0, 1, \dots, N\}$$

$$u_{ij} \geq 0, \quad \forall i \in \{1, 2, \dots, n\}, \forall j \in \{0, 1, \dots, N\}$$

where $g_{ij} = -\mathbf{d} (p_i^{\max} - c_{ij} - \frac{h_{ij} + h_{ij+1}}{2} \mathbf{d}) - \sum_{k \text{ competitors}} l_i^k (u_{ij}^k + u_{ij+1}^k)$
 k competitors
 past iteration of outerloop

$$k_i = \frac{e_i \mathbf{d}^2}{2}, \text{ and } e_i = \frac{p_i^{\max} - p_i^{\min}}{C_i}$$

Approach: $C_{ij} = -\frac{\partial Obj}{\partial u_{ij}} = 2k_i u_{ij} + k_i (u_{ij+1} + u_{ij-1}) + g_{ij}$

m_{ij}

Solution Algorithm - Inner Loop: For each retailer

Step 0: ($k=0$) for every $j \in \hat{I} \{0, \dots, N\}$, for every $i \in \hat{I} \{1, \dots, n\}$

$$u_{ij}^0 = \frac{CFR_j}{n}$$

$k = 1$

Step k: for every $j \in \hat{I} \{0, \dots, N\}$:

$$m_{ij}^k = k_i(u_{ij-1}^k + u_{ij+1}^{k-1}) + g_{ij}$$

$$C_{ij}^k = 2k_i u_{ij}^k + m_{ij}^k$$

We order the m_{ij} 's in non-decreasing order

$$m_{\text{order}(1,j)j}^k \leq m_{\text{order}(2,j)j}^k \leq \dots \leq m_{\text{order}(n,j)j}^k$$

Equilibration approach

$$\text{Find } l_j : C_{\text{order}(1,j)j}^k = \dots = C_{\text{order}(l_j,j)j}^k = a_{\text{order}(l_j,j)j}^k \leq C_{\text{order}(l_j+1,j)j}^k, \dots, \leq C_{\text{order}(n,j)j}^k$$

$$u_{\text{order}(1,j)j}^k > 0, \dots, u_{\text{order}(l_j,j)j}^k > 0, \sum_{i=1}^{l_j} u_{\text{order}(i,j)j}^k = CFR_j \quad u_{\text{order}(l_j+1,j)j}^k = \dots = u_{\text{order}(n,j)j}^k = 0$$

Solution Algorithm - Inner Loop: For each retailer

Step k (continued): for every $j \in \hat{I} \setminus \{0, \dots, N\}$:

$$\text{Let } \mathbf{a}_{order(i,j)j}^k = \frac{CFR_j + \sum_{m=1}^i \frac{m_{order(m,j)j}^k}{2k_{order(m,j)}}}{\sum_{m=1}^i \frac{1}{2k_{order(m,j)}}}$$

$$\text{Let } l_j = \begin{cases} \operatorname{argmin}\{i \in \{1, \dots, n-1\} : \mathbf{a}_{order(i,j)j}^k \leq m_{order(i+1,j)j}^k, \text{ if it exists} \\ n, \text{ otherwise} \end{cases}$$

$$\text{If } i > l_j, \quad u_{order(i,j)j}^k = 0$$

$$\text{If } i \leq l_j, \quad u_{order(i,j)j}^k = \frac{\mathbf{a}_{order(l_j,j)j}^k - m_{order(i,j)j}^k}{2k_{order(i,j)j}}$$

Convergence criterion:

$$\text{If } u_{ij}^k = 0 \Rightarrow C_{ij}^k \geq \mathbf{a}_{order(l_j,j)j}^k - \mathbf{e}, \text{ stop. Otherwise } k=k+1, \text{ go to step } k.$$

Main results:

- The Iterative Relaxation Algorithm converges to the unique optimal solution of the inner-loop problem
- The Iterative Learning&Relaxation Algorithm converges to the the unique Nash Equilibrium

OUTLINE OF PRESENTATION

- The pricing challenge
- The practice of pricing
- A pricing model for retail
- A pricing model for supply chain management
- A fluid delay-based model for pricing and inventory management
- **Summary**

Questions?

Thank You

The text 'Thank You' is rendered in a 3D, blocky font. Each letter is filled with a different color from a rainbow spectrum: 'T' is pink, 'h' is red, 'a' is orange, 'n' is yellow, 'k' is light green, 'Y' is blue, and 'o' is purple. The letters are positioned on a dark blue surface, and each letter casts a white shadow to its left and slightly forward, giving the text a three-dimensional appearance.