Real Options in Dynamic Pricing and Revenue Management

Chris Anderson
Ivey School of Business, Univ. of Western Ontario
London, Ontario, Canada
canderson@ivey.ca

Agenda

- Financial Options
- Real Options
- Services & Dynamic Pricing/ RM
- RO & RM
  - Motivation (car rentals)
  - Pricing and demand models
- RO and low price guarantees

Options

- Derivatives
- A financial instrument that gives you the right to buy or sell a share at a specified price
- Call option - the right to buy at a specific price (the exercise price)
- Put option - the right to sell at a specific price (the exercise price)

Basic Types of Options

- European
  - call - gives the owner the right to buy on a specific date
  - put - gives the owner the right to sell on a specific date
- American
  - gives the right to buy or sell at any time prior to a specific date

Other Terminology

- A long position - you actually own the security
  - e.g. the way most of us invest in stocks or mutual funds
- A short position - you have sold a security that you don't own
- Note: if you sell or write an option contract, you have a short position on that contract

Why Buy Options

1. Cheaper than underlying securities - can get a huge position on a security for a low price
2. Risk management - option pays if stock rises or falls by a large amount - can protect your portfolio from a very volatile market
3. Regulatory reasons (e.g. some places don't allow short selling)

Value of a European Call

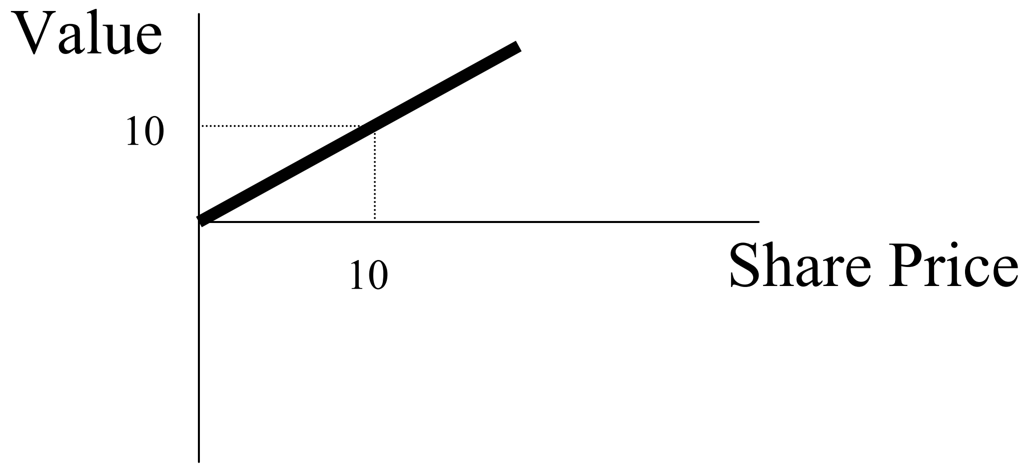
- Gives you the right to buy a stock for \$K at some future date, T
- When would you use it?
- If the stock price, S_T, is bigger than K, then you exercise your option and buy a share for \$K, then immediately sell it for \$$S_T$, and make a profit of S_T - K
- If the future price is less than \$K, you do nothing
- Thus, the value at time T is max(S_T-K,0)

Value of a European Put

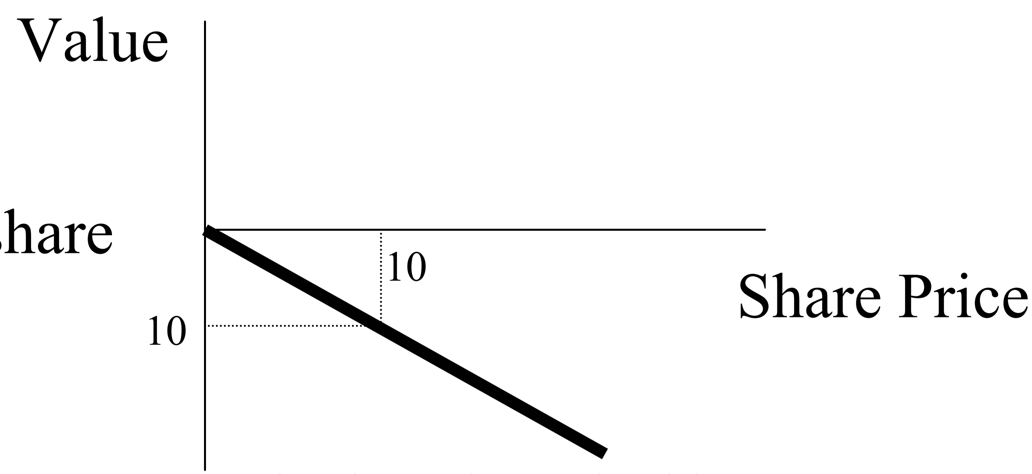
- Gives you the right to sell a stock for \$K at some future date, T
- When would you use it?
- If the stock price, S_T, is smaller than K, then you buy a share on the market for \$$S_T$, then exercise your option and sell it for \$K, and make a profit of $K-S_T$
- If the future price is more than \$K, you do nothing
- Thus, the value at time T is $\max(K-S_T,0)$

Payoff Diagrams

Buy a share

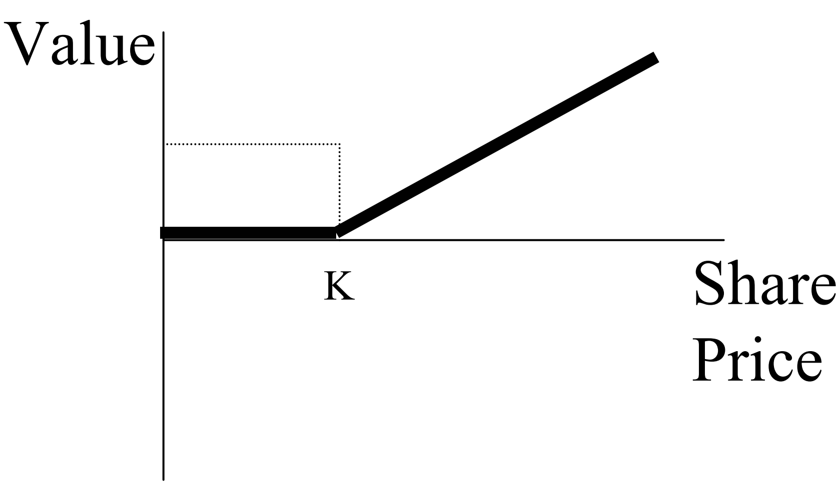


Sell (short) a share

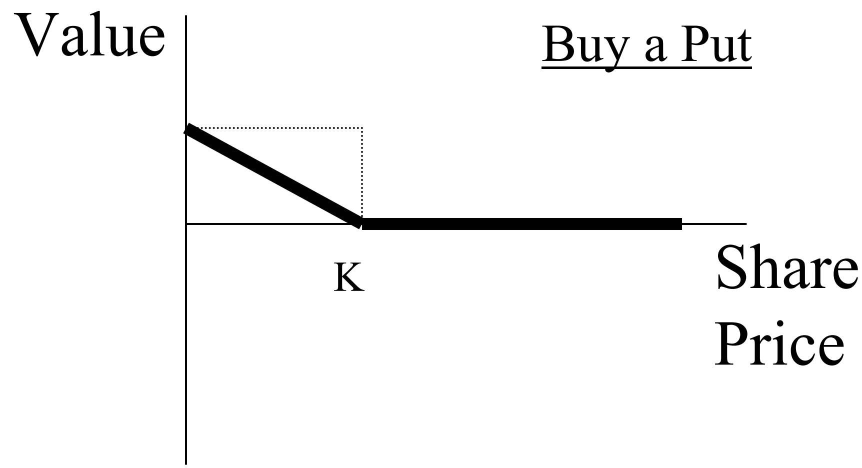


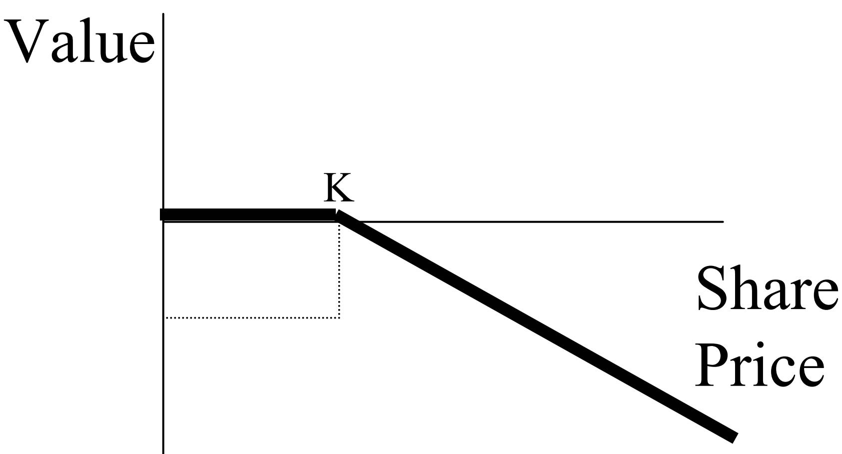
Payoff Diagrams

Buy a Call

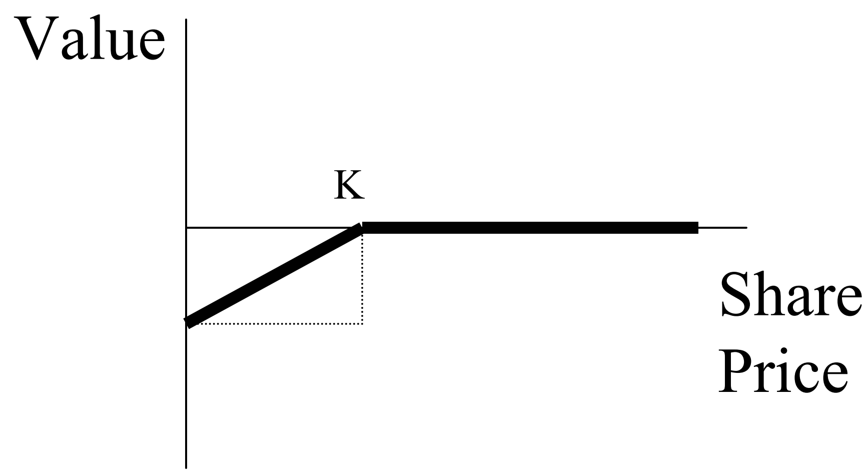


Buy a Put





Sell a Call



Sell a Put

Methods for Pricing

Valuing Options by Arbitrage Methods

If an investment has no risk, it should yield the risk-free rate of return (T-Bills), if not we can create wealth – money making machine

Example

- Stock trades at $40
- European call has exercise price of $40
- Risk free rate is 1/9% (per period)
- A very simplified world...
- In one period, one of two things will happen:
  - stock trades at $32
  - stock trades at $50
- Form a Portfolio
  - x shares of stock, sell 1 call

Arbitrage Pricing

Stock Price	Portfolio
50	50x-10
32	32x-0

W/ no risk, portfolio must be equal under both stock price scenarios

50x-10=32x   18x=10 or x=5/9

A portfolio of 5/9 shares, short a call – no risk, gen. Risk free return

Value in 1 period * 1/(1+r) = initial value

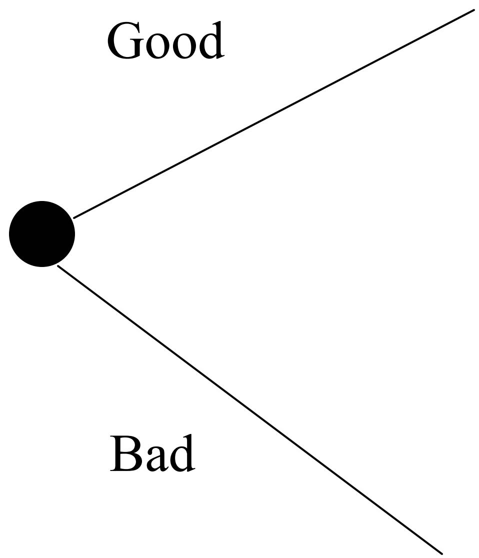
5/9*40 –c =1/(1+1/9)*32*5/9   c=56/9

2^{nd} Example

- Stock trades at \$20
- European call has exercise price of \$21
- Risk free rate is 12%
- A very simplified world...
- In 3 months, one of two things will happen:
  - stock trades at \$18
  - stock trades at \$22
- Form a Portfolio
  - 0.25 shares of stock, sell 1 call

Example

This construction gives us a risk-free portfolio whose value is the same no matter what happens in the future!



Value of stocks = .25×22 = \$5.50
Value of options = -1×(22-21) = -\$1.00
Value of portfolio = \$5.50-\$1.00 = \$4.50

Value of stocks = .25×18 = \$4.50
Value of options = -1×(0) = 0
Value of portfolio = \$4.50

Risk Neutral Pricing

- If this portfolio is not subject to risk, then investors must be indifferent between this portfolio and a risk free bond with the same payoff ($4.50) in 3 months

- Why? If they weren't, you could buy one and sell the other to create a risk free "money pump"

Value of portfolio in three months = \$4.50

Value of portfolio now = $4.50 \times e^{-.12 \times .25}$ = \$4.37

$4.37 = .25 \times 20 - 1 \times$ (price of call today)

price of call today = 5.00 - 4.37 = .63

And if it wasn't \$0.63, we would have an arbitrage opportunity.

- In a risk-neutral world, investors do not demand any premium to take on extra risk
- (In the real world, risky investments have a higher average growth rate than safe ones - a risk-return tradeoff.)

- Thus, in a risk-neutral world, all assets grow at the risk free rate.
- Why? If asset A grew faster than asset B, all investors would prefer A since they are neutral to risk.

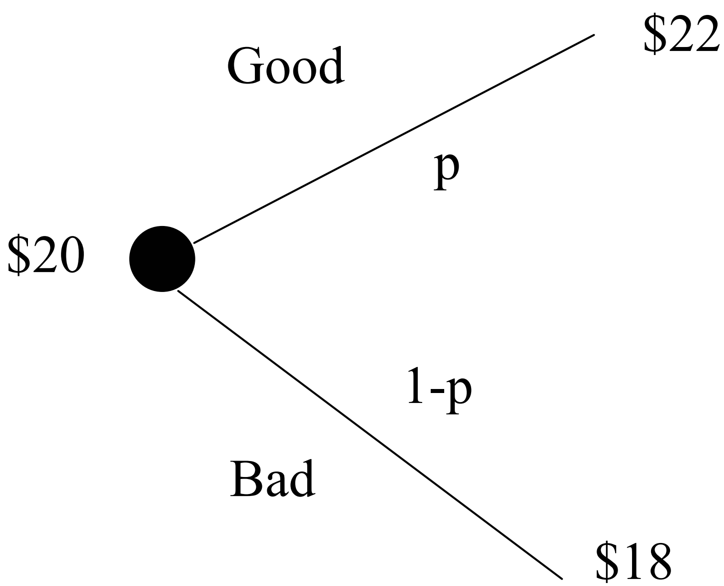
- We use this observation to determine the probability that the stock price rises or falls in a risk-neutral world.

Since investors are risk neutral, the stock grows on average at the risk-free rate.

Price in 3 months = $20 \times e^{.12 \times .25} = \$20.61$

Then the expected stock price after 3 months must equal $20.61.

Let p = probability that the stock goes up in a risk-neutral world.



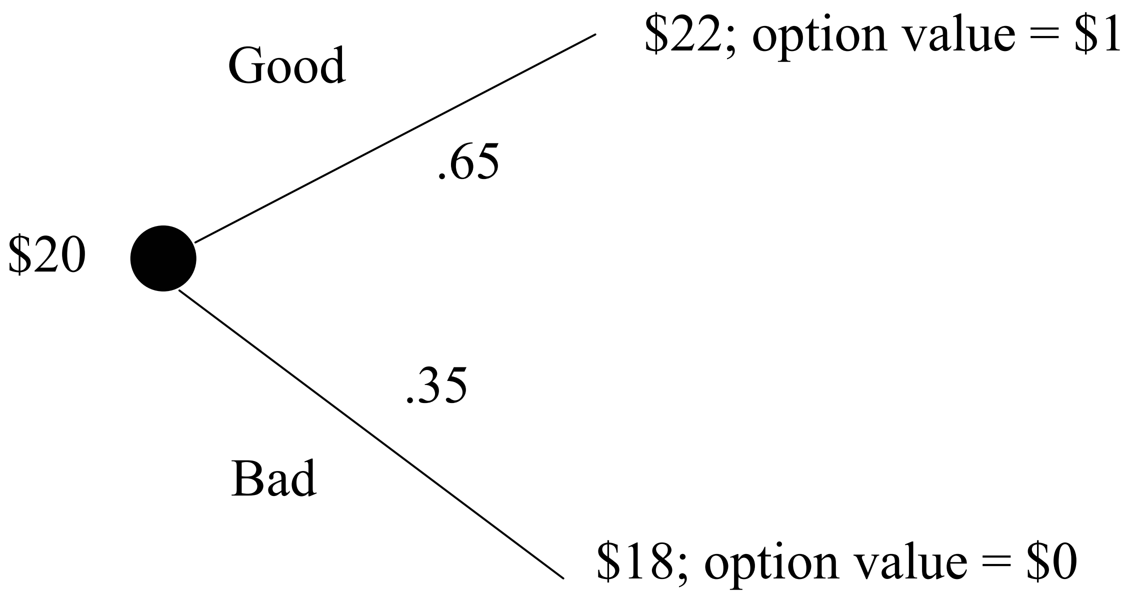
EV stock price

$$= p \times 22 + (1-p) \times 18$$
$$= 18 + 4p$$

Thus, $20.61 = 18+4p$

$p = .65$

Determining the Option’s Value



At 3 months:

EV option = $.65\times1 + .35\times0 = .65$

EV now = $.65\times e^{-.12\times.25} = \$0.63$

Basic Approaches to Pricing

- For “vanilla” European options (puts and calls), a formula exists
- For exotic European options, can simulate
- For American-style options, need to use a decision tree approach

The Black-Scholes Formula

- Scholes, Merton received Nobel Prize in Economics in 1997
- Based on dynamic application of risk neutral pricing

The Black-Scholes Formula

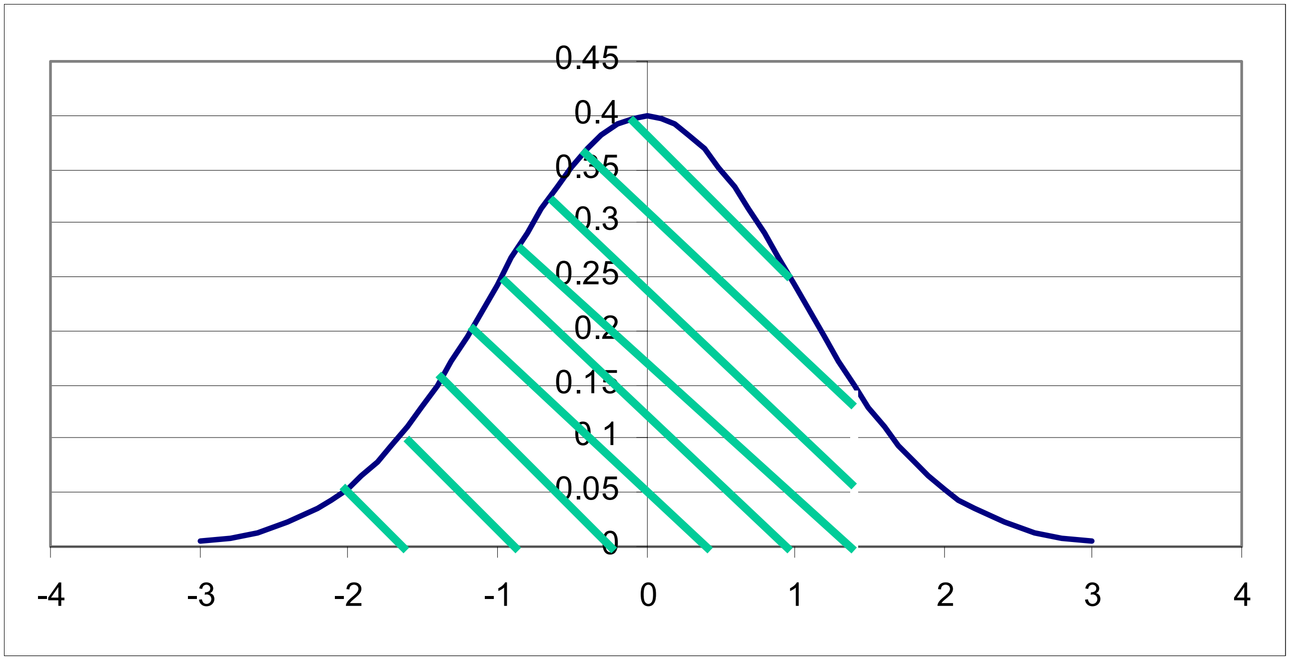
$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r_f + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \qquad d_2 = d_1 - \sigma\sqrt{T}$$

Value of a call: $$C = S_0 N(d_1) - Ke^{-r_f T} N(d_2)$$

Value of a put: $$P = Ke^{-r_f T} N(-d_2) - S_0 N(-d_1)$$

What is $N(d_1)$?

$N(d_1)$



Note: $N(-d_1) = 1-N(d_1)$

in Excel, $N(d_1)$ = normsdist(d_1) or normdist(d_1,0,1,1)

So, $N(d_1)$ = the probability that the return is less than a certain amount

Pricing via Simulation

- Basic premise of finance – an asset's value is derived from its future discounted (expected) cash flow

- Simulate the underlying value driver or asset (stock)
- Calculate payoffs
- Replicate
- Average payouts, discount

Lognormal model of stock prices

- Over time stock goes up
- More uncertainty farther out try to estimate
- Positive values

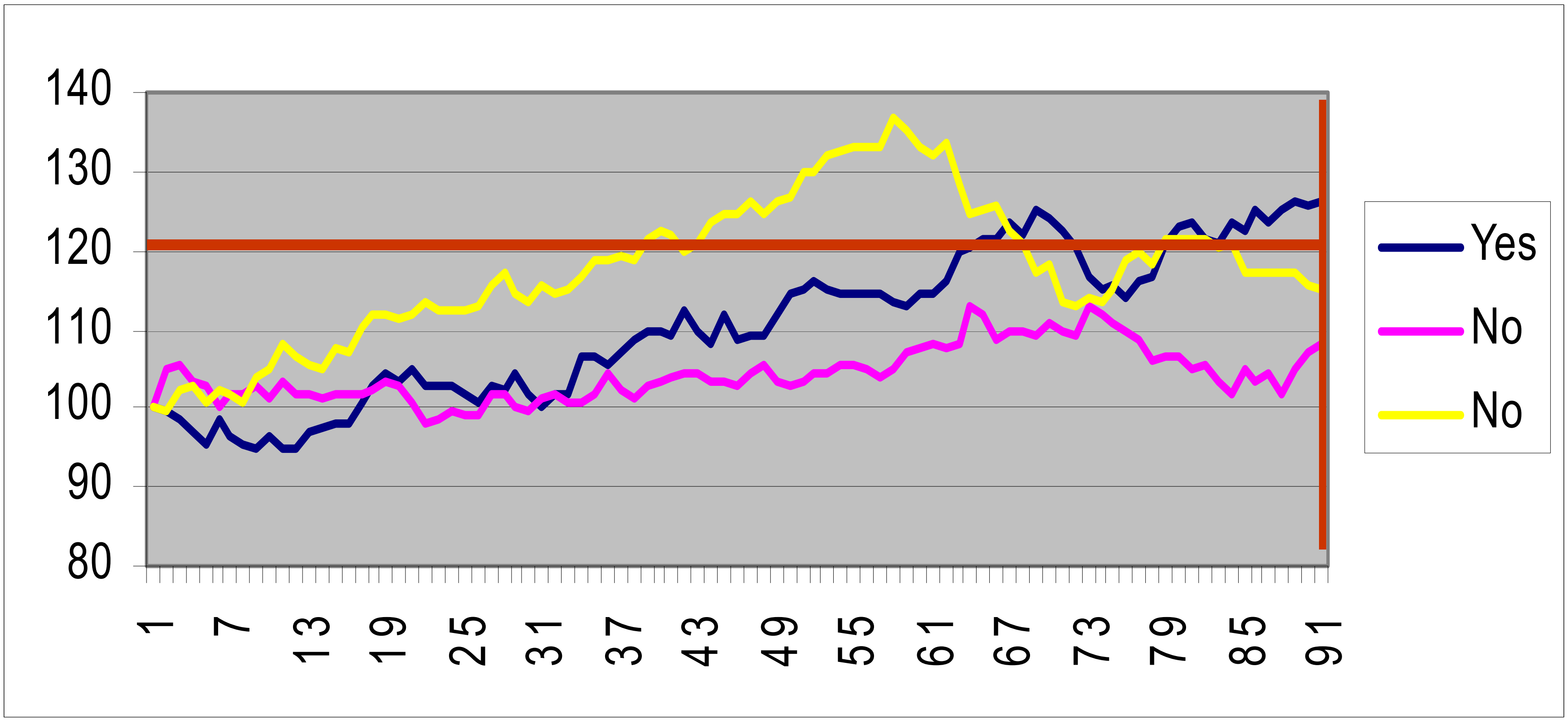
$$\frac{\delta S}{S} \sim \phi(u\,\delta t, \sigma\sqrt{\delta t})$$

$$\ln S_T \sim \phi(\ln S_0 + (u - \frac{\sigma^2}{2})\mathrm{T}, \sigma\sqrt{T})$$

$$S_T = S_0 \exp[\phi((u - \frac{\sigma^2}{2})\mathrm{T}, \sigma\sqrt{T})]$$

Pricing Options

Consider a (European) call option with an exercise price of $120.
In which cases does it have value at the end of 3 months?



Real Options

Many investments, not just those involving stocks, may be viewed as combinations of puts and calls – if we know the value of the puts/calls we can value many real investment opportunities

Managerial View of Real Options

- RO is a modern methodology for economic evaluation of projects and investment decisions <u>under uncertainty</u>
  - RO approach *complements* the corporate tools
- RO considers the uncertainties and the options (managerial flexibilities), giving two answers:
  - The *value of the investment opportunity (value of the option)*; and
  - The *optimal decision rule* (*threshold*)
- RO can be viewed as an <u>optimization problem</u>:
  - Maximize the NPV (typical objective function) *subject to*:
  - (a) Market uncertainties (price);
  - (b) Technical uncertainties (volume) and
  - (c) Relevant Options (managerial flexibilities)

Value in the Real Option

Real options increase in value as greater the uncertainties and the flexibility to respond

Low — Likelihood of receiving new information — High

Uncertainty

High — Ability to respond — Low

Room for Managerial Flexibility

Moderate Flexibility Value	**High Flexibility Value**
Low Flexibility Value	**Moderate Flexibility Value**

•**Option to purchase** an airplane 3 years from now for \$20 million, P=value(t=3), P uncertain (economic cycle etc..), cash flow max(P-20,0) – call option, if you can value the call, you can value the option to purchase

•**Abandonment Option** a R&D project, in 5 years can sell devl't for \$80 million, P= value(t=5), value of option max(80-P,0) – put

•**Expansion** – option at *t* to double investment

•**Contraction** – option at *t* to cut scale

•**Postponement** – option to delay launch till time *t*

•**Pioneer** – option to enter new markets at time *t,* buy –ve NPV firm

•**Flexibility** – build expensive plant that can build three types (cars) versus one

•**Licensing** – license a drug, such that if sales > \$50 million get 20% gross sales (as developer)

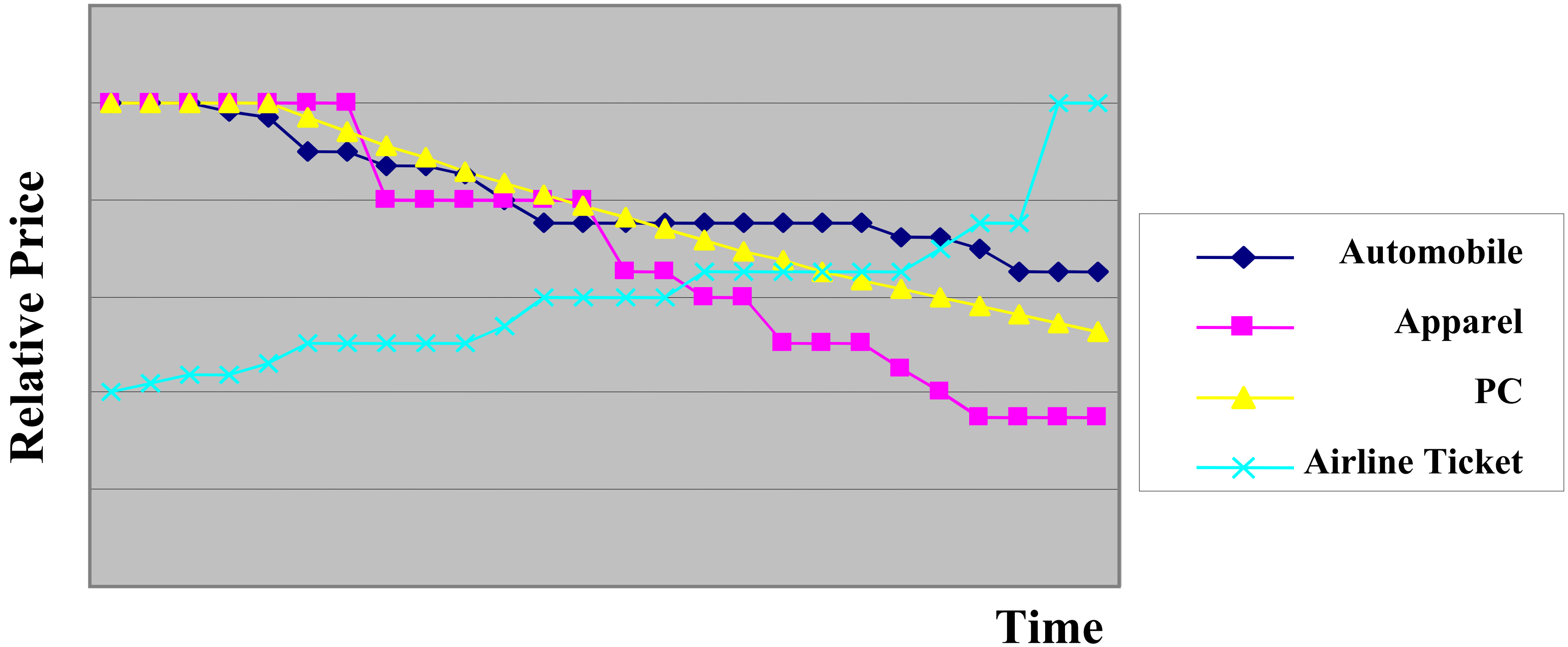
Complexities of Service

- In manufacturing, we assume that capacity can be adjusted over time to match supply w/ demand
- In services
  - Capacity is often fixed
  - Outputs rarely storable
  - Sales opportunity lost if not met
  - Demand often temporal

Coping Strategies for time-varying demand.

- Inventories, overtime, backlogging, and many of the other strategies we use for production planning aren't available to us in service businesses.
- How do service operations managers address the problem of matching capacity to time varying demand?
- With pricing tactics

DP in practice



DP&RM - the basics

- Setting and updating prices with a wide variety of customers, products, or channels.
- Aligning prices with market conditions
  - Customer sensitivity
  - Competition's pricing
  - Corporate objectives
- Airlines, hotels, rental cars, fashion goods, more each day

Current Approaches

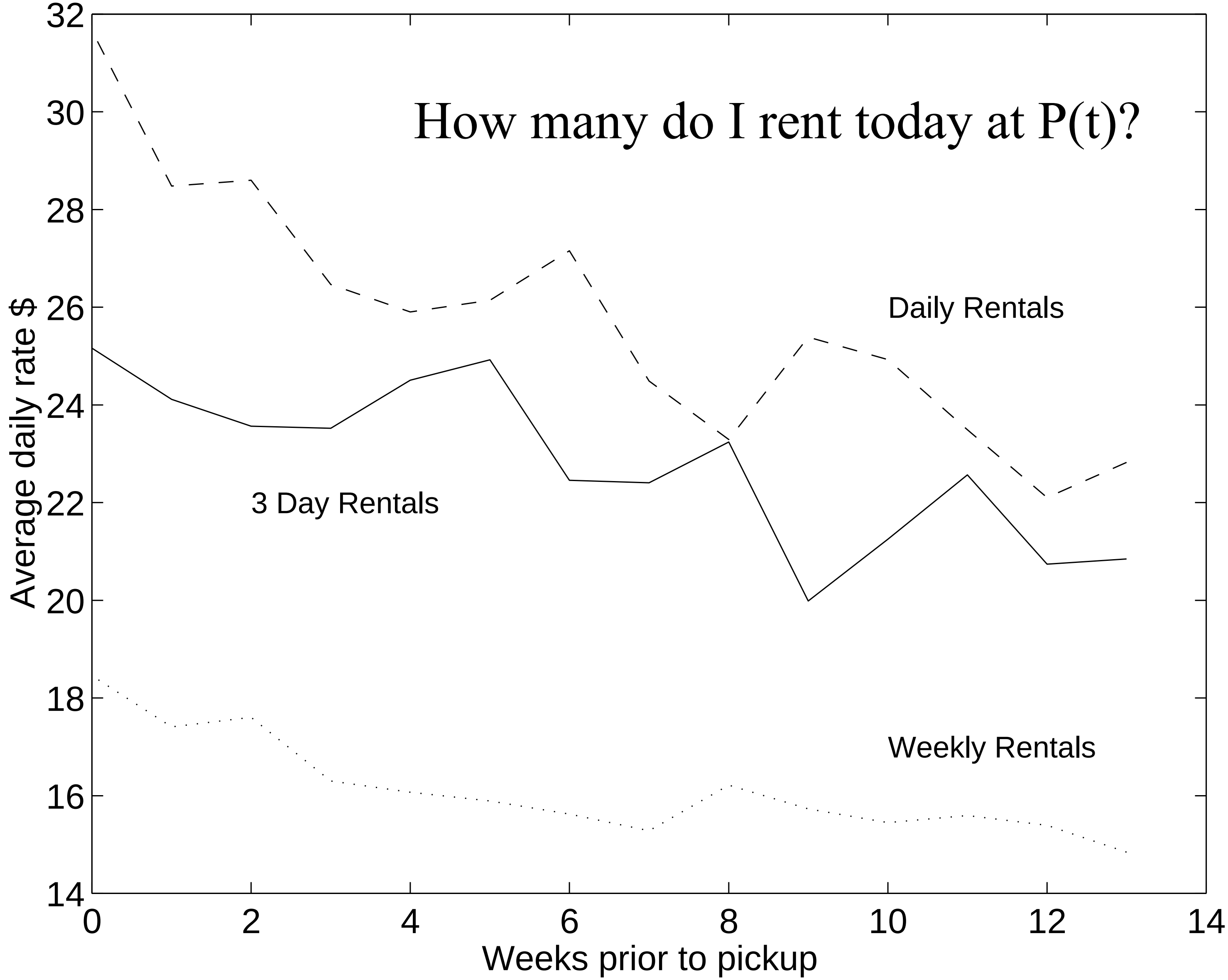
- Marginal analysis (inc. gain vs. inc. loss)
- Math Programming (usually deterministic)
- Threshold curves (comparison to historical perf.)
- "Managerial experience"

Focus of Car Rentals

- 90 day planning horizon, relatively fixed capacity (sunk costs), very low variable costs
- Decisions
  - Price to post, accept or deny a request for rental
  - LOR, upgrades, overbooking

Optionality

- View reservation as an option, exercised if booking allowed, held if capacity reserved
- Tradeoff between rate today & potential higher rate (uncertain demand) later
- Call option w/limited demand


How many do I rent today at P(t)?
Daily Rentals
3 Day Rentals
Weekly Rentals
Average daily rate $
Weeks prior to pickup
32
30
28
26
24
22
20
18
16
14
0
2
4
6
8
10
12
14

A Price and Demand Model

Market price is key driver!

$$dP = u(P,t)dt + b(P,t)dX$$

$$u(P,t) = \alpha(\overline{P}(t) - P)$$

$$b(P,t) = \sigma P$$

$$D = \beta_0 + \beta_1 P$$

$$D = \beta_0 P^{\beta_1}$$

Payouts

M cars to rent over time T subdivided into periods (daily)

$V_m^T = 0$ no revenue from unrented vehicles

$$\theta = V_{m+1}^{j+1} + P_j$$

$$\theta = V_{m+k}^{j+1} + kP_j$$

$$\theta = \max(V_m^{j+1}, V_{m+1}^{j+1} + P_j, V_{m+2}^{j+1} + 2P_j, \ldots, V_{m+k}^{j+1} + kP_j)$$

$$m + k \le M, k \le F(P_t)$$

Valuing the option

$$\prod = V - \Delta S$$

portfolio of option and fraction of stock

$$\prod = V_1 - \Delta V_2$$

portfolio of one type of car and another

$$d\prod = r\prod dt$$

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 P^2 \frac{\partial^2 V}{\partial P^2} + (\mu - \lambda\sigma P)\frac{\partial V}{\partial P} - rV = 0$$

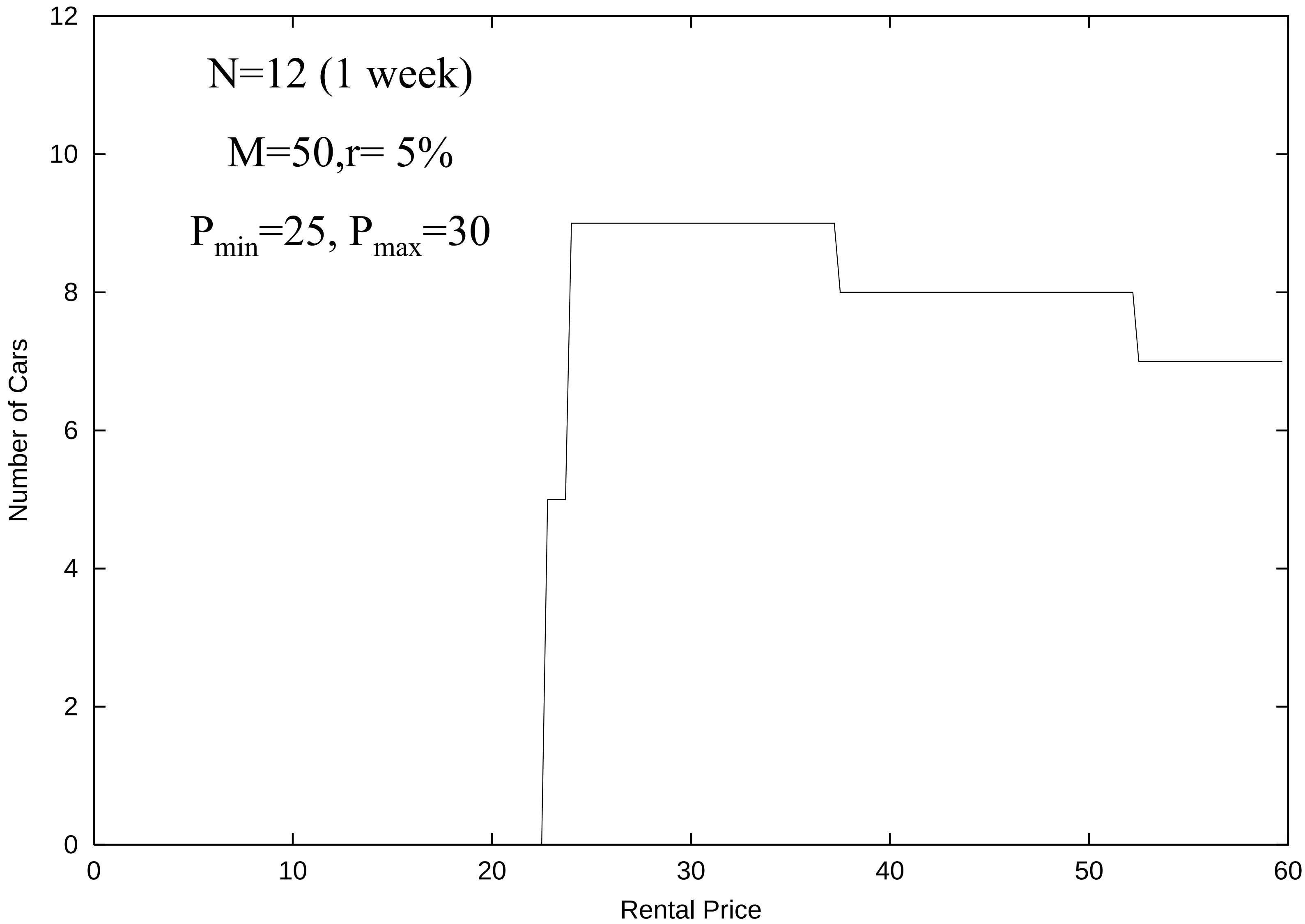
Solution

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 P^2\frac{\partial^2 V}{\partial P^2}+(\mu-\lambda\sigma P)\frac{\partial V}{\partial P}-rV=0$$

Requires numerical solution.

Analytical under conditions of excess capacity and large prices.

Also under low volatility in the price process.


N=12 (1 week)
M=50,r= 5%
P_{min}=25, P_{max}=30
Number of Cars
Rental Price
0
2
4
6
8
10
12
0
10
20
30
40
50
60

More general approaches

$$dP = u(P,t)dt + b(P,t)dX$$

$$u(P,t) = \alpha_p(\overline{P}(t) - P)$$

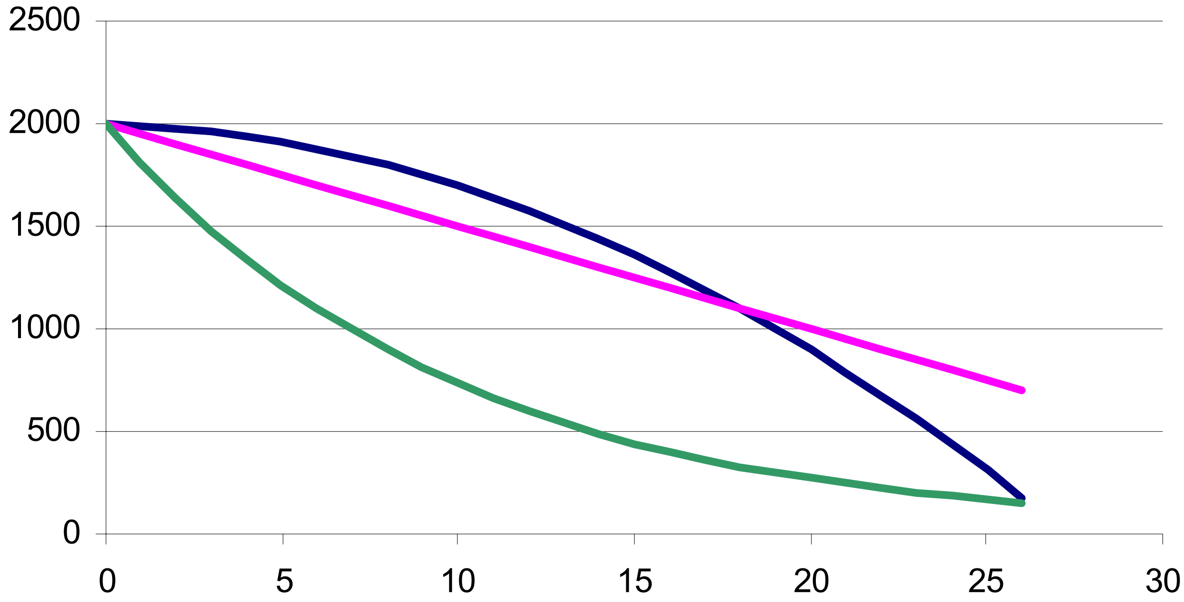
$$b(P,t) = \sigma P$$

$$\frac{dD}{dP} = \alpha_d(\hat{P}(t) - P)$$

$$dP = u_d(P,t)dt + b_d(D,t)dX$$

$$u_d(P,t) = \alpha_p(\overline{P}(t) - P)\alpha_d(\hat{P}(t) - P)$$

Decaying Price (Perishables)



Decaying Price

Fashion goods, electronics

$$dD = u(P,t)dt + b(D,t)dX$$

$$u(P,t) = \alpha_1 \left(\frac{\hat{P}(t) - P}{P}\right)^n$$

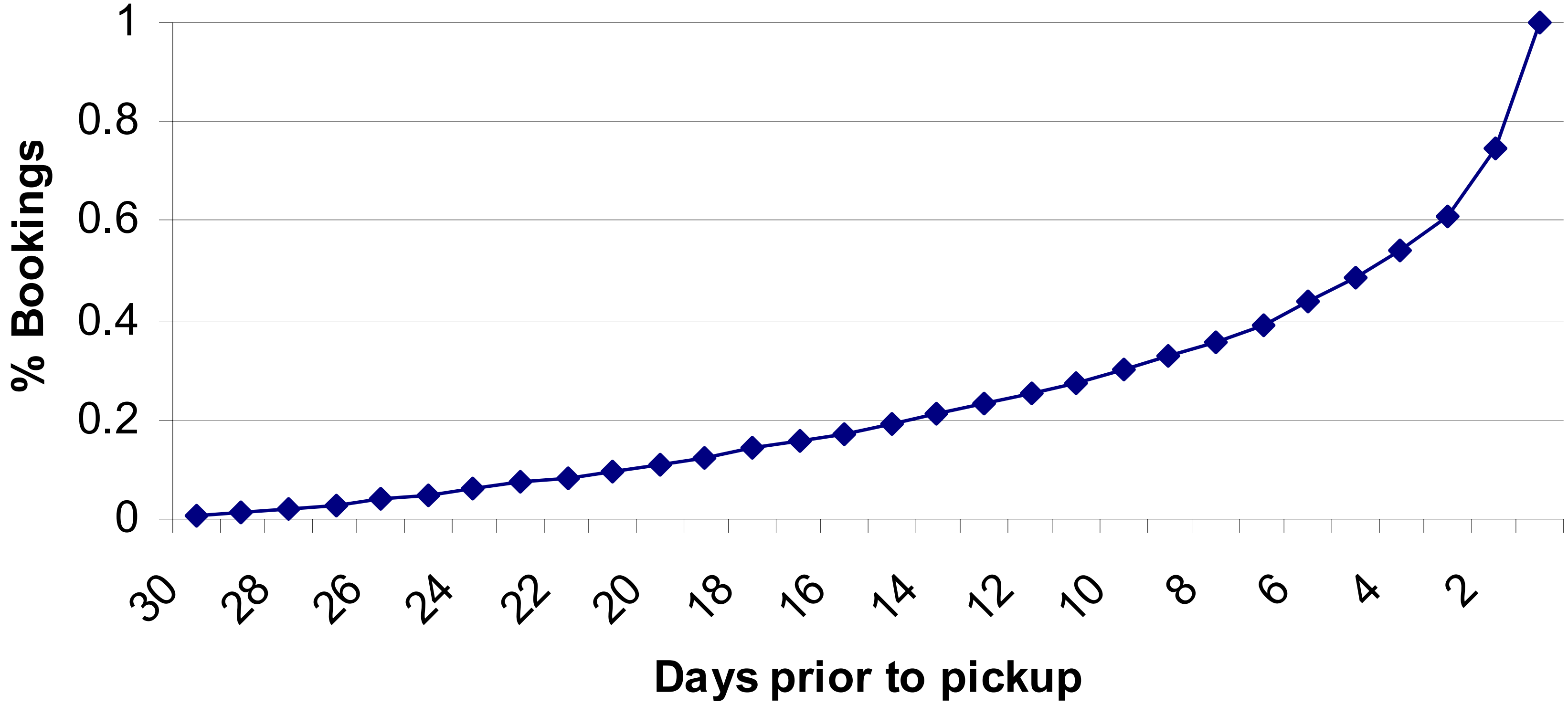
- substitution and own price effects

if u is a function of D then have exponential demand

if not, linear demand.

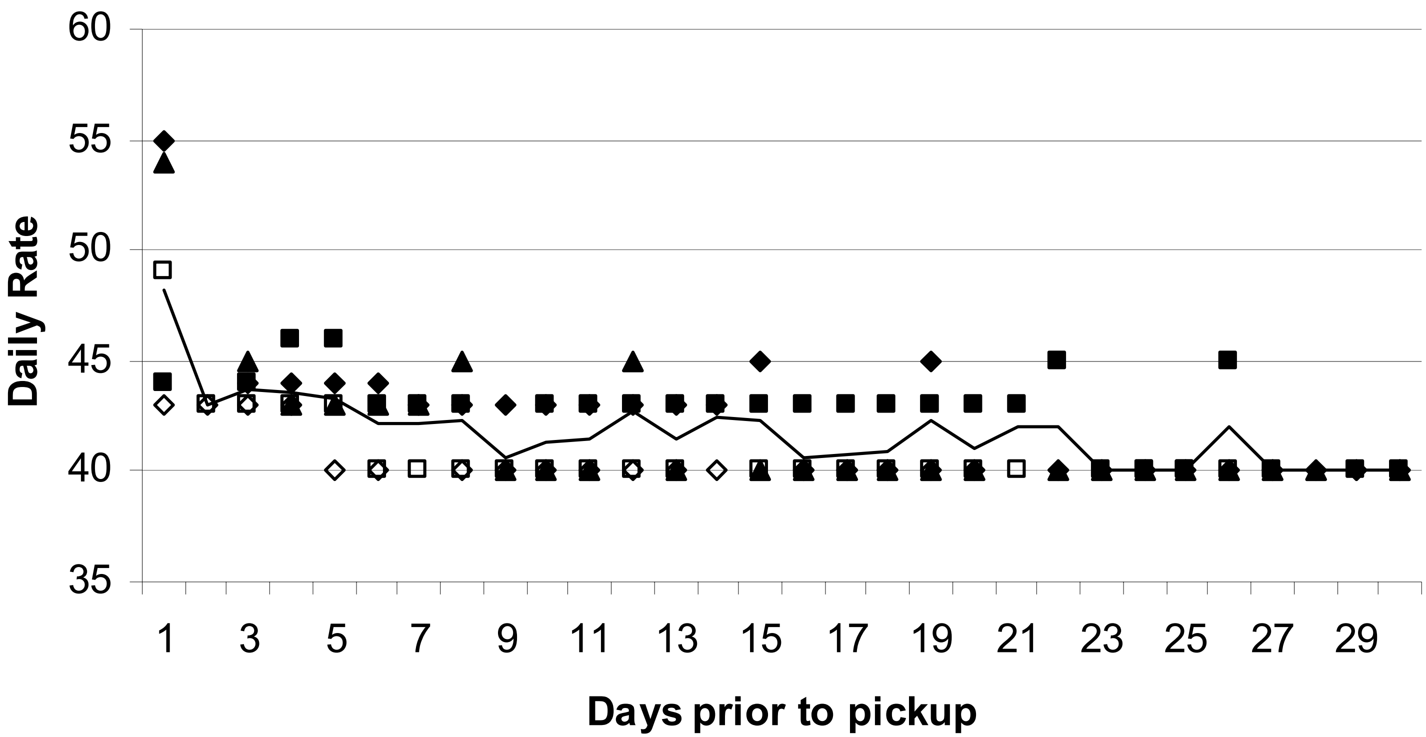
Low Price Guarantees

- Motivation
  - DTAG pays 15% commissions on bookings through 3rd party websites (Expedia, Travelocity, Orbitz, etc…) versus allocated costs of $0.75/rental for bookings on Dollar.com and Thrifty.com
- Book now, if rates drop will rebate!
  - Common in cruise industry
- Most Favoured Customer & Meet Or Release
  - Retailers, big box stores

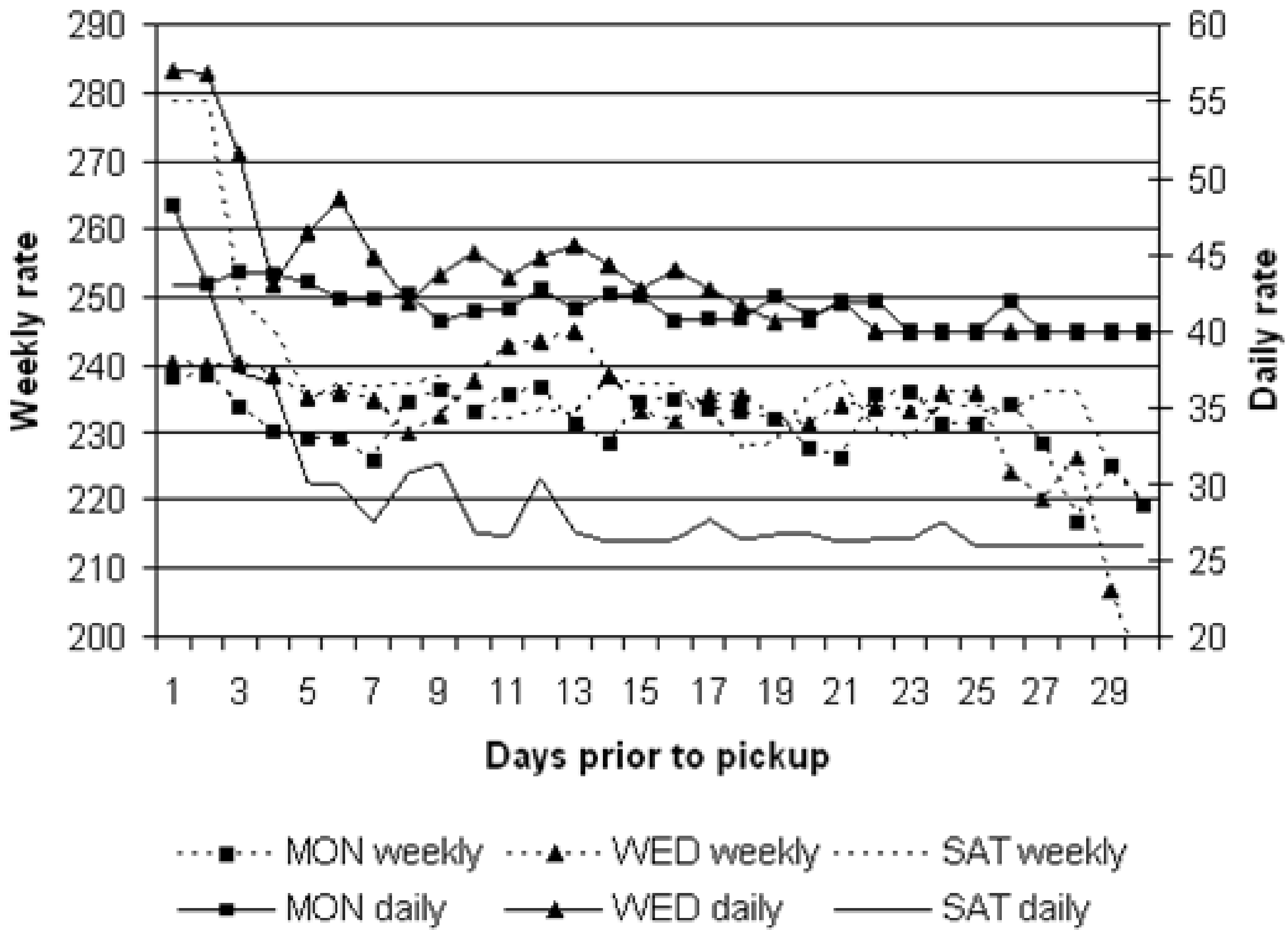

1
0.8
0.6
0.4
0.2
0
% Bookings
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
Days prior to pickup

Channel	**% of total**	**Cost to DTAG (per rental)**
WALKUPS	14.3%	5%
800 NUMBER	16.3%	$6.00
Dollar.com, Thrifty.com	26.5%	$0.75
Travel agent bookings	12.9%	15%
Internet Site A	14.5%	15%
Internet Site B	7.5%	15%
Internet Site C	5.6%	15%
Other Internet sites	2.2%	15%

29.8%



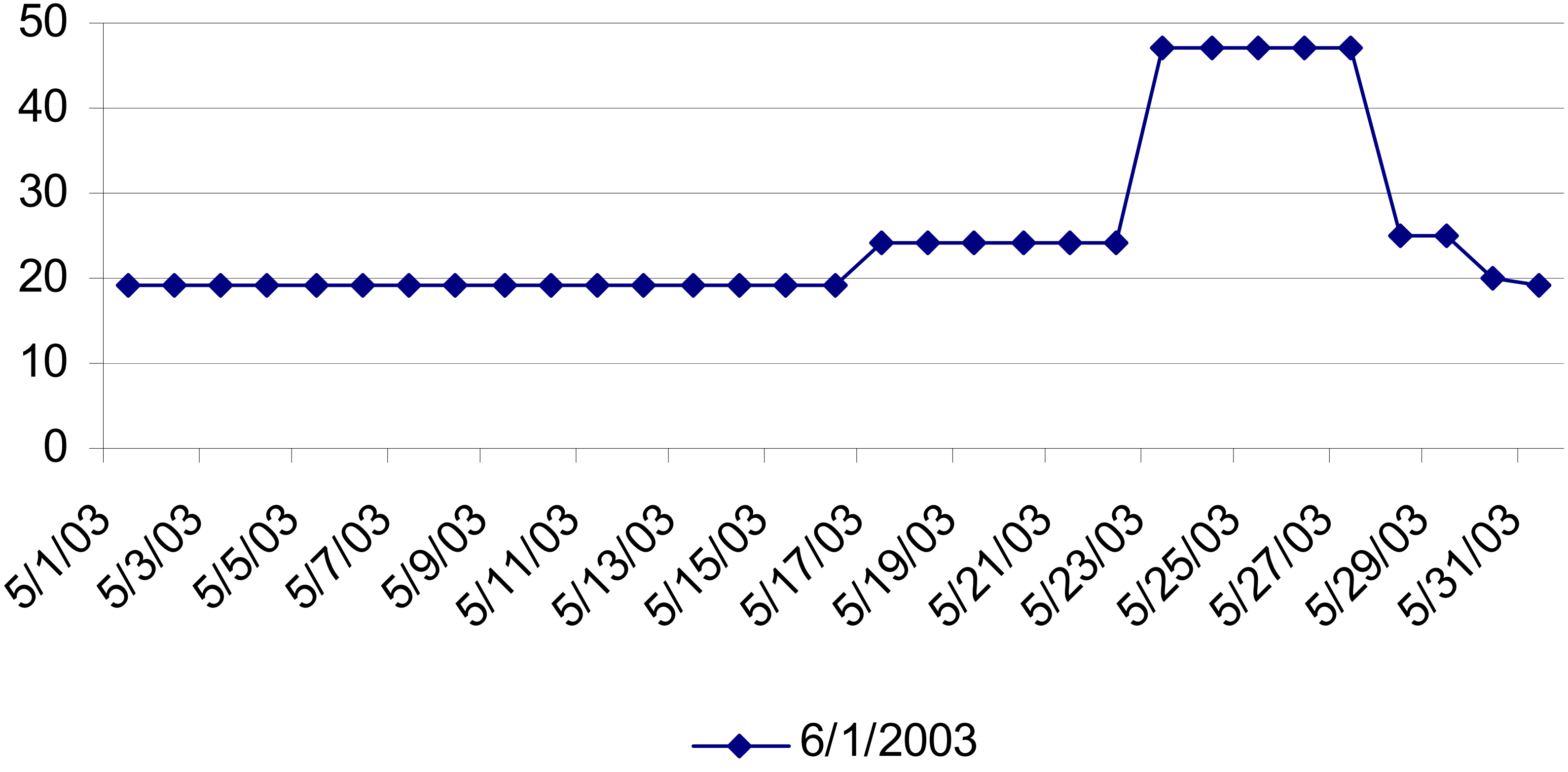
60
55
50
45
40
35
Daily Rate
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
Days prior to pickup



Weekly rate
Daily rate
Days prior to pickup
MON weekly
WED weekly
SAT weekly
MON daily
WED daily
SAT daily

Goal – reduce distribution costs

- Move traffic from 3rd party sites to DTAG sites
- Two elements in promo
  1. DTAG sites have lowest DTAG rates
  2. DTAG rebates consumer if rates drop after they have booked
- 1 is simply spin as all channels use the same rate engine
- 2 might be very costly
  - Cost?
  - Break-even?
- 2 actually already exists!



50
40
30
20
10
0
5/1/03
5/3/03
5/5/03
5/7/03
5/9/03
5/11/03
5/13/03
5/15/03
5/17/03
5/19/03
5/21/03
5/23/03
5/25/03
5/27/03
5/29/03
5/31/03
6/1/2003

LPG Option

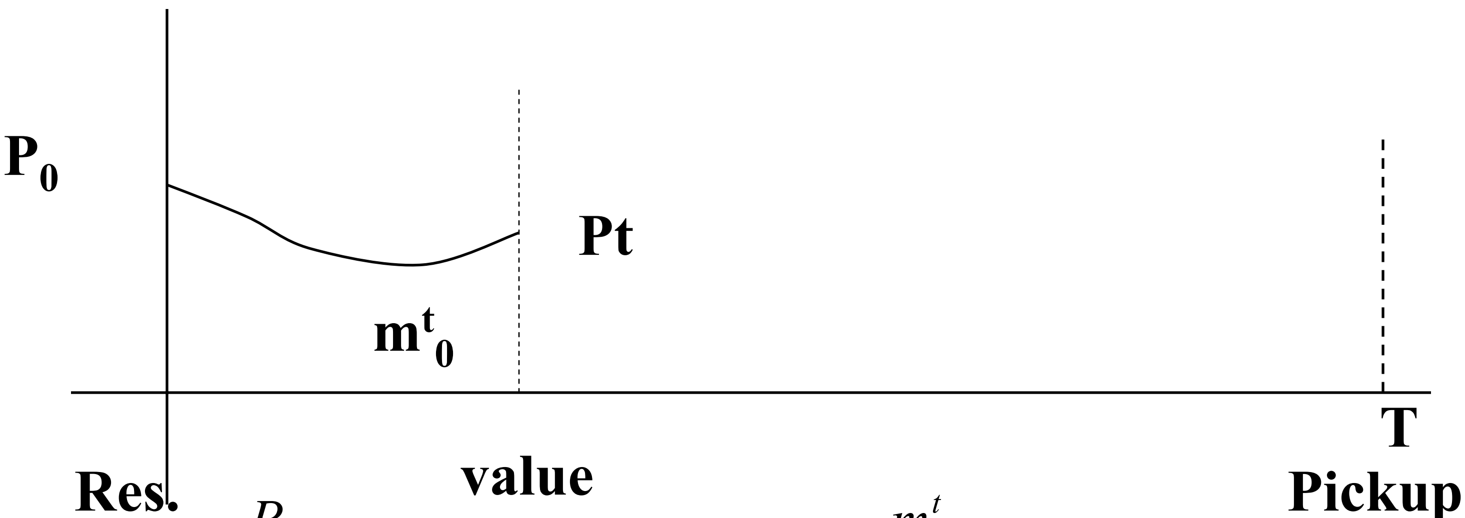
- Consumer gets a free option
- Payout

$$\max[0, \text{Reserved price - lowest from time of reservation to pickup}]$$

$$\max[0, P_t - m_t^T]$$

$$e^{-r(T-t)} \max[0, P_t - E[m_t^T \mid P_t]]$$

LPG Option – assume LogN



$$N\left(\frac{\ln\frac{P_t}{m_0^T}+\mu(T-t)}{\sigma\sqrt{T-t}}\right)-\left(\frac{P_t}{m_t^T}\right)^{\frac{2\mu}{\sigma^2}}N\left(\frac{\ln\frac{m_0^t}{P_t}+\mu(T-t)}{\sigma\sqrt{T-t}}\right)$$

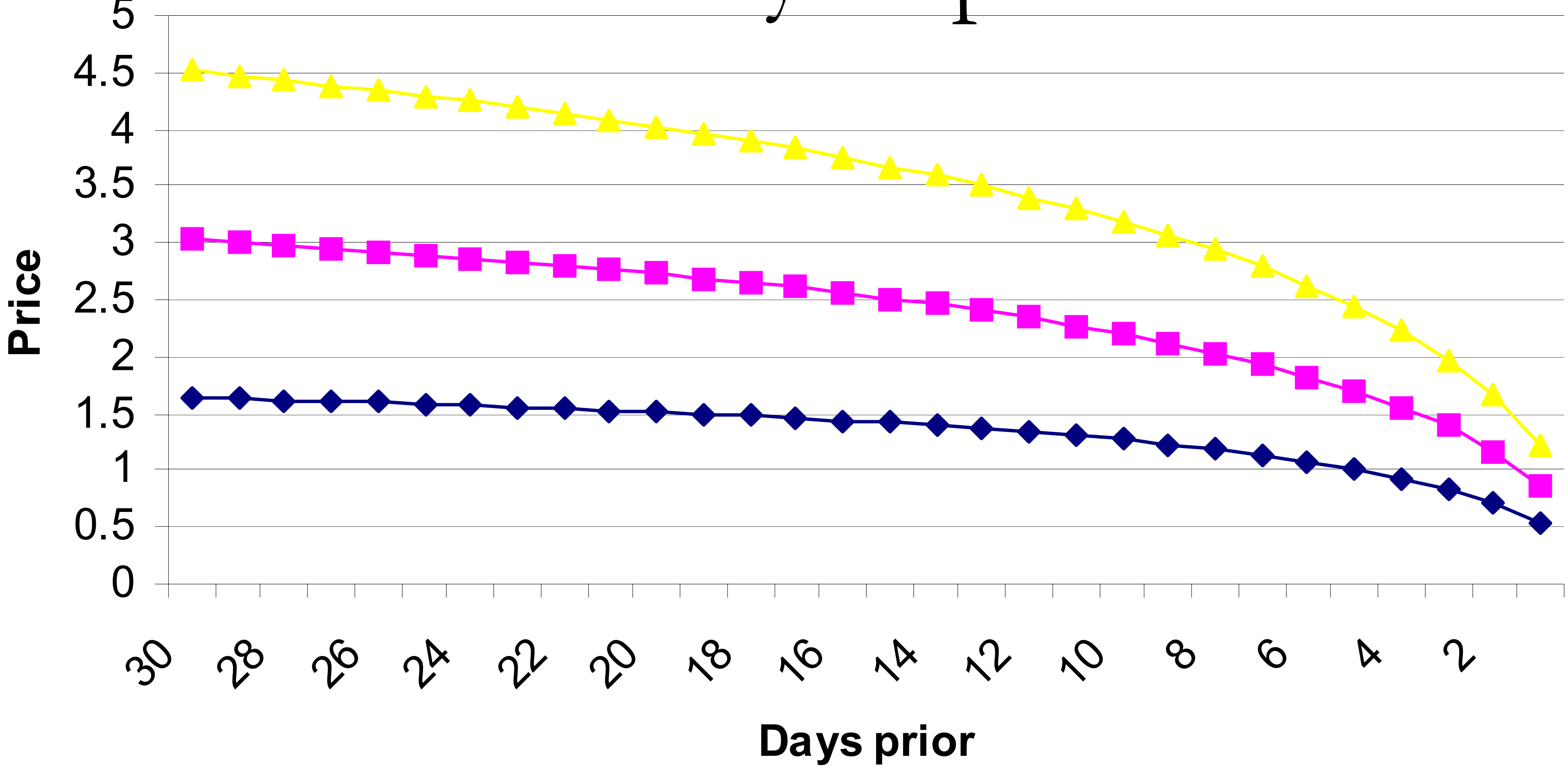
$$\sim P[r>\ln\frac{P_t}{m_0^T}]$$

LPG Option price

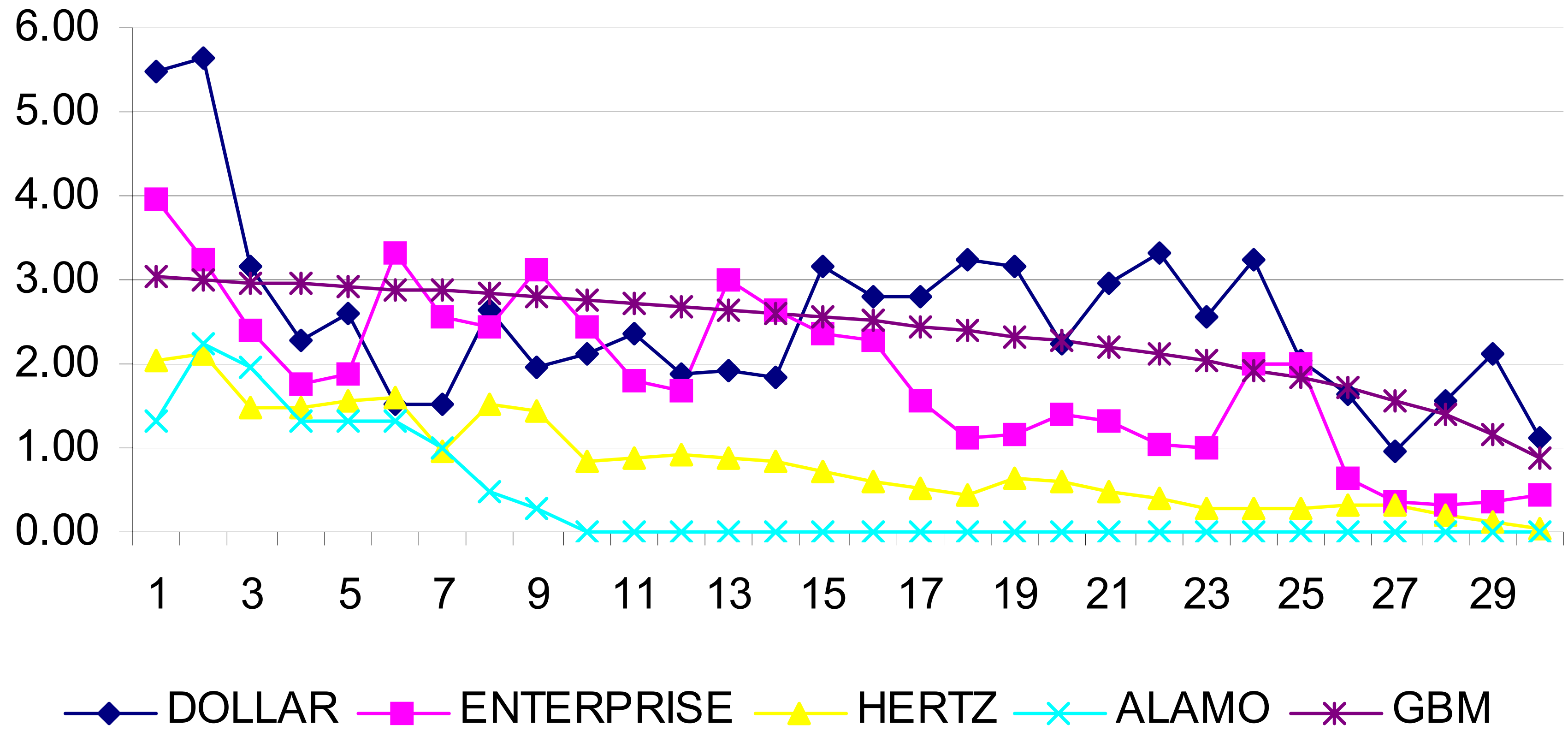
$$p(P_t, m_0^t, T) = -P_t e^{(\frac{\sigma^2}{2} - \mu - r)\tau} N(-d\, \frac{2(\sigma^2 + \mu)\sqrt{\tau}}{\sigma}) +$$

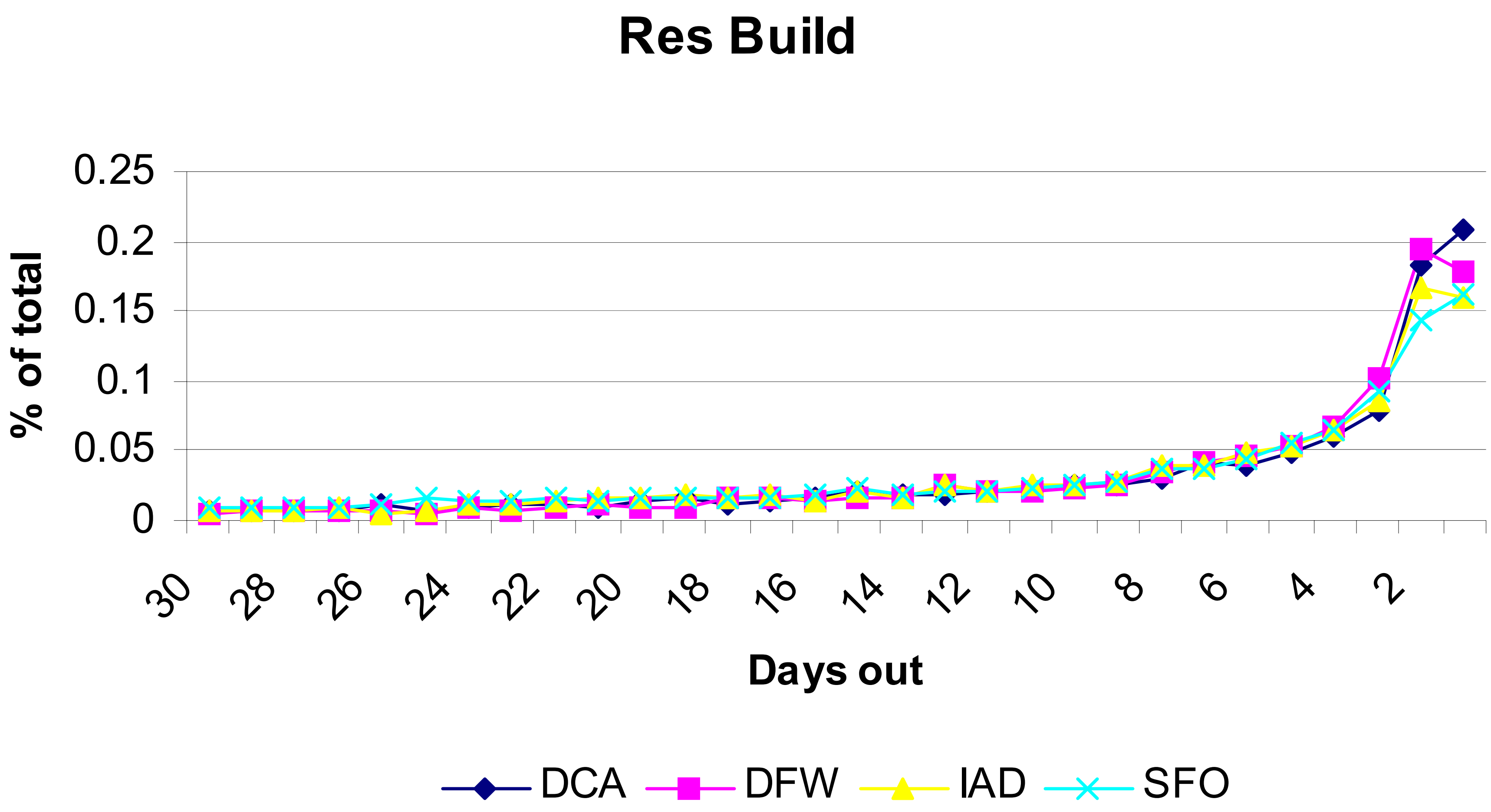
$$\frac{\sigma^2 e^{-r\tau}}{\sigma^2 + 2\mu} P_t \left\{ \left(\frac{P_o}{P_t} \right)^{\frac{\sigma^2 + 2\mu}{\sigma^2}} N(-d + \frac{2(\sigma^2 + \mu\sqrt{\tau})}{\sigma}) - e^{(\frac{\sigma^2}{2} + \mu)\tau} N(-d) \right\}$$

Volatility Impacts



Intermediate cars, overnight DFW





Res Build
% of total
0.25
0.2
0.15
0.1
0.05
0
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
Days out
DCA
DFW
IAD
SFO

Impact

- Sum_t (Value * % booked)
- ~$1.80

- Implications
    - Need to move a lot of traffic
    - Market share
- Cancellation fee?

Other Models

- Mean-reversion
- Exponential curve
    - Exponential smoothing
    - Moving average

Published rental rates - Microsoft Internet Explorer

File Edit View Favorites Tools Help

Back Forward Stop Refresh Home Search Favorites History Mail Print Edit Discuss SideStep

Address http://www.expedia.com/pub/agent.dll?qscr=carw&itid=&subm=1 Go Links

Can I use a credit card with a billing address outside the U.S.?

Is it safe to buy online?

Need help with this page?

Other FAQs

Verify rate and continue

$31.99 /Day

Air Conditioning, Automatic Transmission, In the Airport Terminal

Midsize Car : Show description

DOLLAR

Verify rate and continue

$38.99 /Day

Air Conditioning, Automatic Transmission, In the Airport Terminal

Full Size Car : Show description

Thrifty

Verify rate and continue

$38.99 /Day

Air Conditioning, Automatic Transmission, In the Airport Terminal

Standard Car : Show description

Thrifty

Verify rate and continue

$39.99 /Day

Air Conditioning, Automatic Transmission, In the Airport Terminal

Standard Car : Show description

DOLLAR

Verify rate and continue

$39.99 /Day

Air Conditioning, Automatic Transmission, In the Airport Terminal

Full Size Car : Show description

DOLLAR

Verify rate and continue

$47.99 /Day

Air Conditioning, Automatic Transmission, In the Airport Terminal

Economy Car : Show description

AVIS

Verify rate and continue

$47.99 /Day

Air Conditioning, Automatic Transmission, In the Airport Terminal

Economy Car : Show description

Hertz

Verify rate and continue

Internet

Orbitz: Car Search Results - Microsoft Internet Explorer

File Edit View Favorites Tools Help

Back Forward Stop Refresh Home Search Favorites History Mail Print Edit Discuss SideStep

Address http://www.orbitz.com/App/ProcessCarSearch?cache=1047073116382&requestId=223&lastPage=interstitial

San Francisco International, CA
at or near airport

change search

drop-off March 13, 2003 9:00 AM
same as pickup

your preferred company No Preference

Daily Rates*	Payless Car Rental	Fox	Budget	Alamo	Enterprise	Thrifty Car Rental	Dollar Rent A Car	National Car Rental	Hertz	Avis
Economy	$15.19+	$16.90+	$20.99+	$20.95+	$22.47+	$27.99+	$28.99+	$49.99+	$47.99+	$47.99+
Compact	$16.09+	$17.90+	$24.99+	$20.95+	$23.37+	$28.99+	$29.99+	$51.95+	$53.99+	$53.99+
Midsize	$16.59+	$18.50+	$30.99+	$21.95+	$25.17+	$30.99+	$31.99+	$52.95+	$59.99+	$59.99+
Standard	$23.29+	$25.90+	$38.99+	$29.95+	$30.57+	$38.99+	$39.99+	$46.99+	$67.99+	$67.99+
Full Size	$24.19+	$26.90+	$39.99+	$29.95+	$30.57+	$38.99+	$39.99+	$46.99+	$67.99+	$67.99+
Premium			$47.99+	$58.99+	$50.30+	$55.91+	$55.99+	$60.99+	$84.99+	$84.99+
Luxury			$53.99+	$68.95+	$62.95+	$63.91+	$63.99+	$78.99+	$94.99+	$94.99+
Specialty									$99.99+	$67.99+
Convertible			$31.99+	$55.99+		$79.91+	$79.99+		$72.99+	
Van	$48.89+	$50.90+	$37.99+	$64.95+	$53.97+	$63.91+	$63.99+	$59.95+	$72.99+	$72.99+
SUV	$500.88+		$30.99+	$61.99+	$50.37+	$57.91+	$57.99+	$62.95+	$64.99+	$67.99+
Pickup			$19.99+						$82.99+	
All Terrain									$149.99+	
Other	$28.94+	$16.01+							$90.99+	

*Rates listed here only reflect standard rate plan periods (daily, weekly, or monthly) and do not include additional charges, taxes, or fees. Additional days beyond this period may be charged at a different rate or with additional week, day, or hourly charges. Weekend Daily rate applies to Fridays, Saturdays, Sundays, and often Mondays. Any additional days will be charged at the Extra day rate. All rates are shown in US Dollars. Rates in local currencies may be displayed by selecting a rate from this page and then viewing the details of that rate.

the Orbot

*required

*city name, or airport code

Done Internet

Questions

- canderson@ivey.ca