



DIRO
IFT 6150

TRAITEMENT D'IMAGES

BINARISATION ET MORPHOLOGIE

MATHÉMATIQUE

Max Mignotte

Département d'Informatique et de Recherche Opérationnelle.

Http : [//www.iro.umontreal.ca/~mignotte/ift6150](http://www.iro.umontreal.ca/~mignotte/ift6150)

E-mail : mignotte@iro.umontreal.ca

BINARISATION ET MORPHOLOGIE MATHÉMATIQUE

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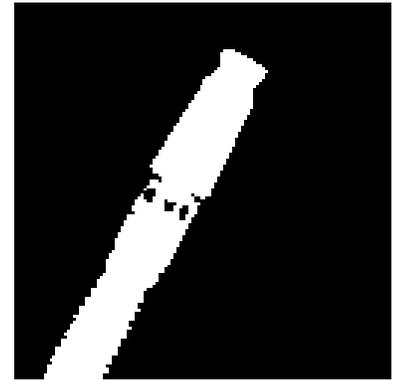
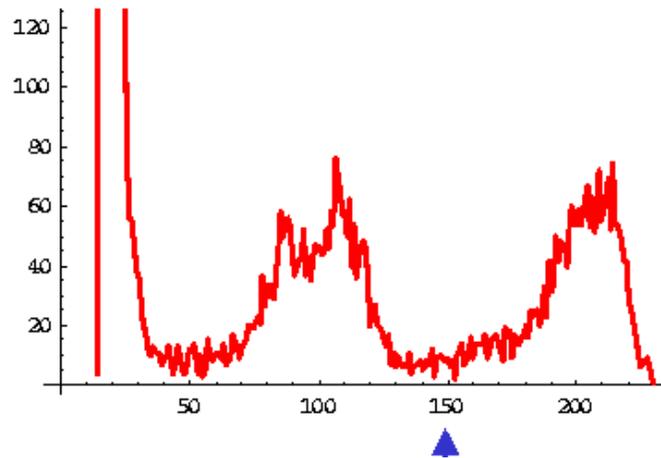
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BINARISATION

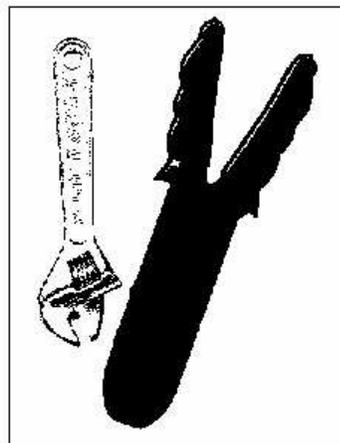
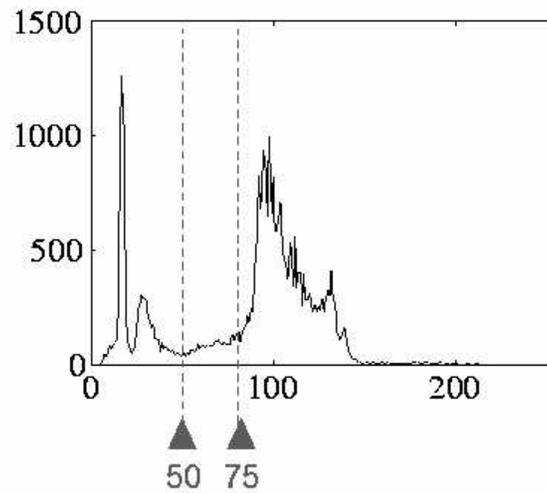
SEUILLAGE PAR INSPECTION (1)

Seuillage par inspection de l'histogramme

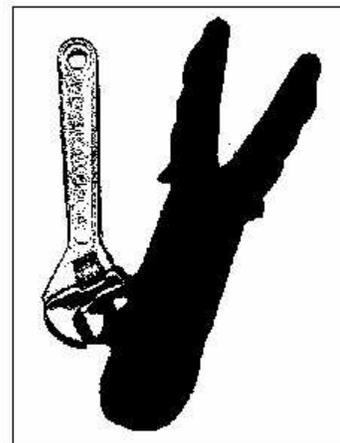


Original

Histogram



Threshold = 50



Threshold = 75

BINARISATION

SEUILLAGE PAR INSPECTION (2)

Seuillage par inspection de l'histogramme

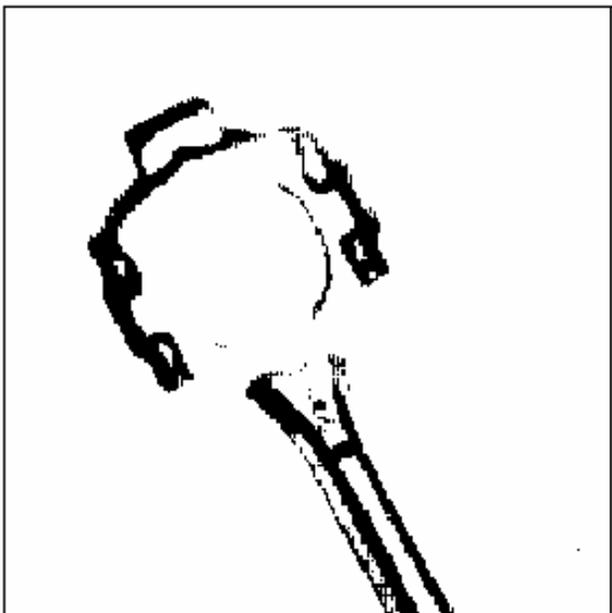
Original Image



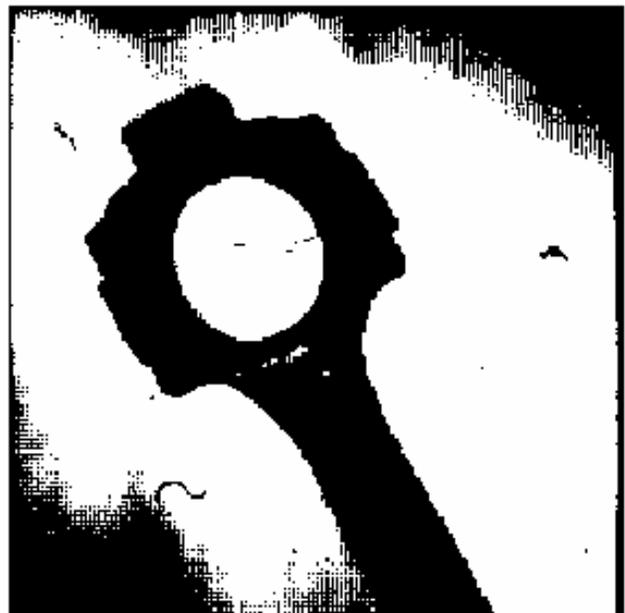
Binary Image



Threshold too low



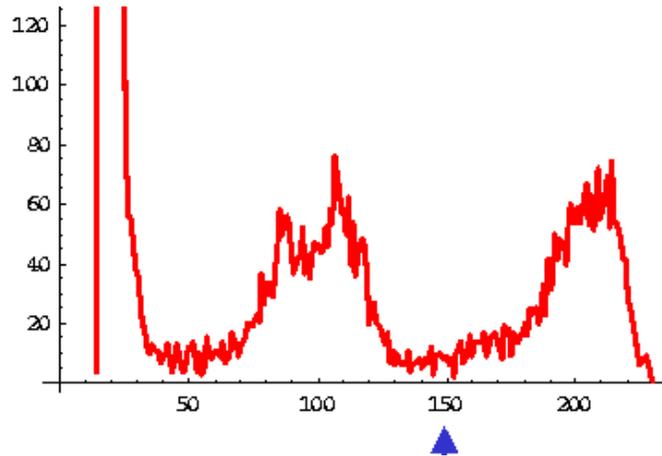
Threshold too high



BINARISATION

SEUILLAGE OPTIMAL (1)

Seuillage optimal



Soit deux régions (arrière-plan et objet)
présent dans une image

La probabilité $P(z)$ d'avoir une valeur de niveau de gris z ($z \in [0, 255]$ par ex.) dans l'image est donnée par

$$p(z) = P_1 p_1(z) + P_2 p_2(z)$$

P_i : prop. de pixels appartenant à la région i ($P_1 + P_2 = 1$)

$p(z)$: Histogramme de l'image

$p_i(z)$: proba. d'un pixel \in région i d'avoir un ng = z

Si nous supposons que $p_1(z)$ et $p_2(z)$ sont distribués selon une loi normale, nous obtenons

$$p_1(z) = \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{(z - \mu_1)^2}{2\sigma_1^2}\right)$$

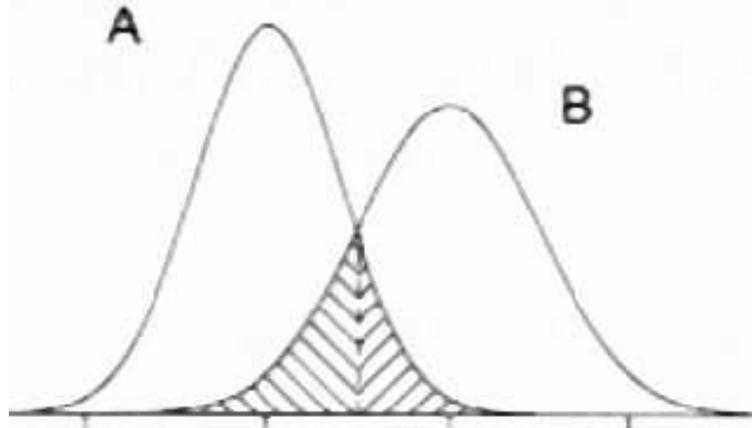
$$p_2(z) = \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left(-\frac{(z - \mu_2)^2}{2\sigma_2^2}\right)$$

BINARISATION

SEUILLAGE OPTIMAL (2)

$p(z)$ devient alors

$$p(z) = P_1 \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{(z - \mu_1)^2}{2\sigma_1^2}\right) + P_2 \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left(-\frac{(z - \mu_2)^2}{2\sigma_2^2}\right)$$



Soit E_1 , la probabilité de classer un pixel dans la classe 1 lorsqu'il appartient à la classe 2 et E_2 , la proba. de classer un pixel dans la classe 2 lorsque celui-ci appartient à la classe 1.

Les probabilités d'erreur sont données par

$$E_1(T) = \int_{-\infty}^T p_2(z) dz \quad \text{et} \quad E_2(T) = \int_T^{\infty} p_1(z) dz$$

La probabilité d'erreur totale est alors donnée par

$$E(T) = P_2 E_1(T) + P_1 E_2(T)$$

Cherchons une valeur de T qui minimise $E(T)$

$$\begin{aligned} E'(T) &= -P_1 p_1(T) + P_2 p_2(T) = 0 \\ P_1 p_1(T) &= P_2 p_2(T) \end{aligned}$$

BINARISATION

SEUILLAGE OPTIMAL (3)

Après simplification, nous obtenons une expression de la forme

$$AT^2 + BT + C = 0$$

Avec

$$\begin{aligned} A &= \sigma_1^2 - \sigma_2^2 \\ B &= 2(\mu_1 \sigma_2^2 - \mu_2 \sigma_1^2) \\ C &= \mu_2^2 \sigma_1^2 - \mu_1^2 \sigma_2^2 + 2\sigma_1^2 \sigma_2^2 \ln\left(\frac{\sigma_2 P_1}{\sigma_1 P_2}\right) \end{aligned}$$

- Lorsque les variances sont égales

$$\begin{aligned} \sigma_1^2 &= \sigma_2^2 = \sigma^2 \\ T &= \frac{\mu_1 + \mu_2}{2} + \frac{\sigma^2}{\mu_1 - \mu_2} \ln\left(\frac{P_2}{P_1}\right) \end{aligned}$$

- Lorsque les variances et les proportions sont égales

$$\begin{aligned} P_1 &= P_2 \\ T &= \frac{\mu_1 + \mu_2}{2} \end{aligned}$$

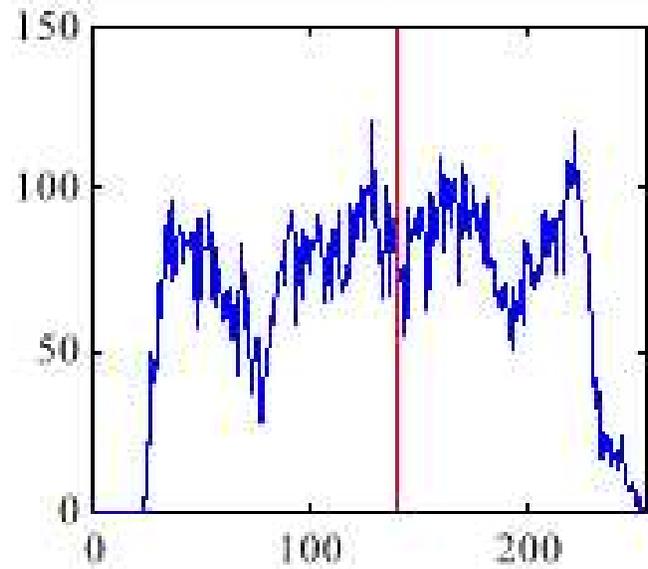
BINARISATION

SEUILLAGE LOCALE (1)

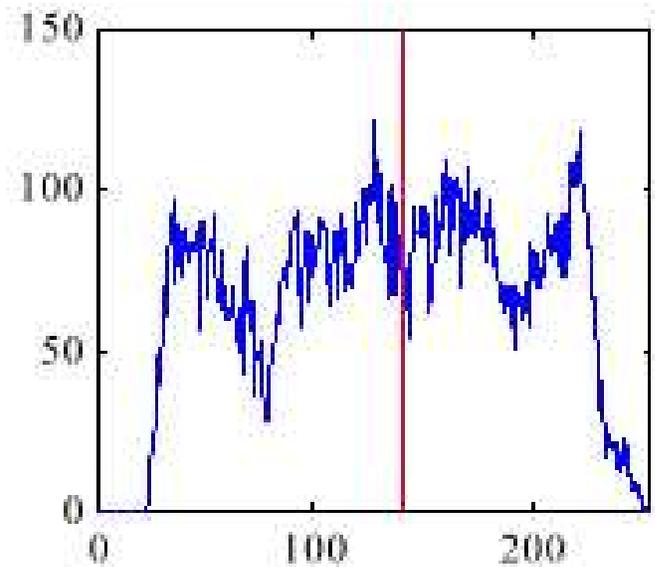
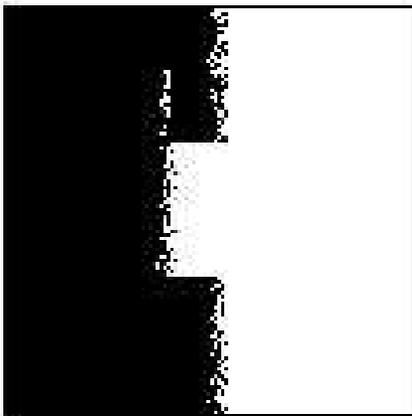
Original



Histogram



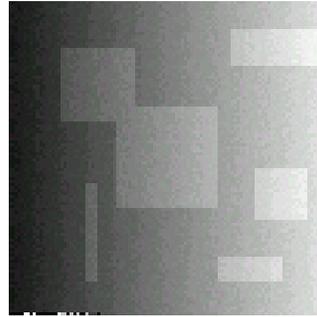
Single Global Threshold



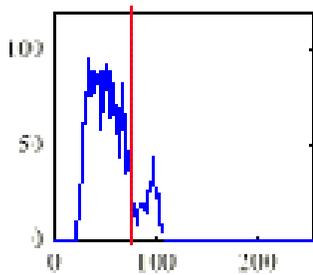
T = 128

BINARISATION

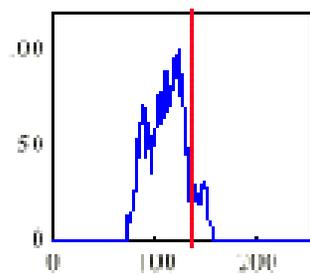
SEUILLAGE LOCALE (2)



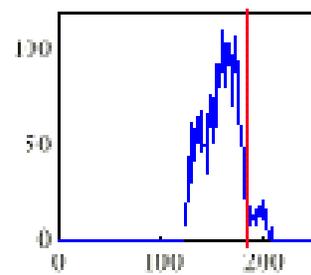
Seuillage Local



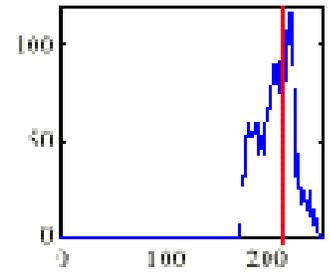
$T = 80$



$T = 128$



$T = 188$

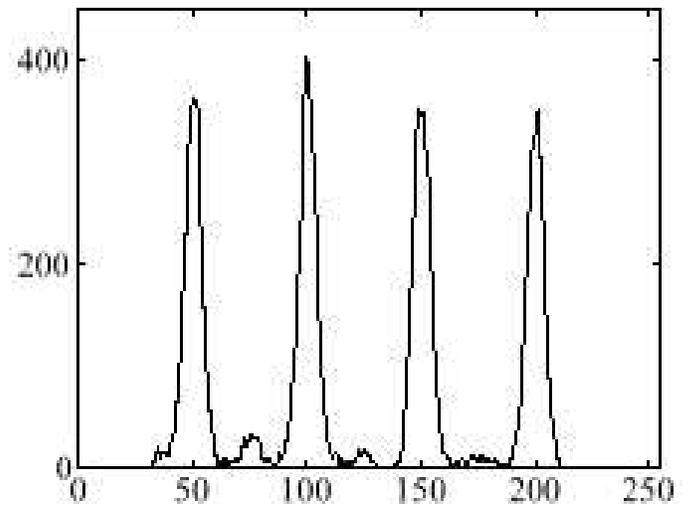
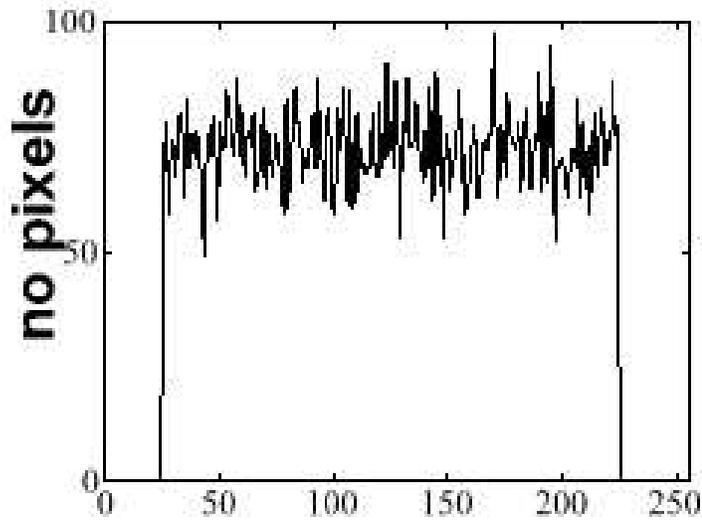
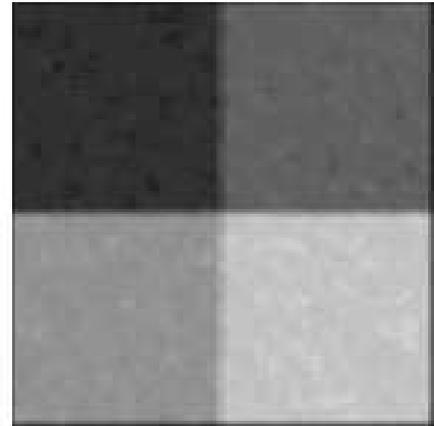
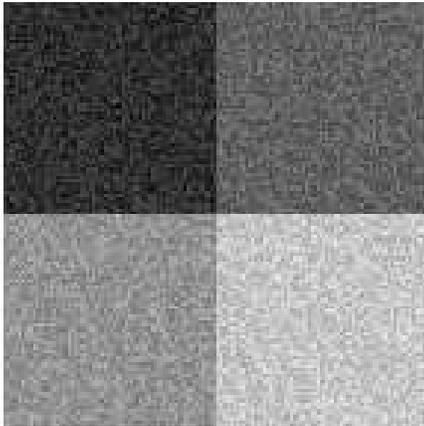


$T = 226$



BINARISATION

SEUILLAGE & FILTRAGE



Le bruit complique la sélection d'un seuil
► Filtrage passe-bas

MORPHOLOGIE MATHÉMATIQUE

THÉORIE DES ENSEMBLES

► Traitement d'image basé sur la théorie des ensembles



Quelques images binaires

$A, B =$ Ens. des pixels = Ens. des coordonnées

$A \cup B = \{ x \mid x \in A \text{ ou } x \in B \}$ Union

$A \cap B = \{ x \mid x \in A \text{ et } x \in B \}$ Intersection

$A^C = \{ x \mid x \notin A \}$ Complément

$A - B = \{ x \mid x \in A, x \notin B \}$ Différence

$(A)_C = \{ x \mid x = a + c, a \in A \}$ Translation

$\hat{A} = \{ x \mid -x \in A \}$ Inversion

MORPHOLOGIE MATHÉMATIQUE

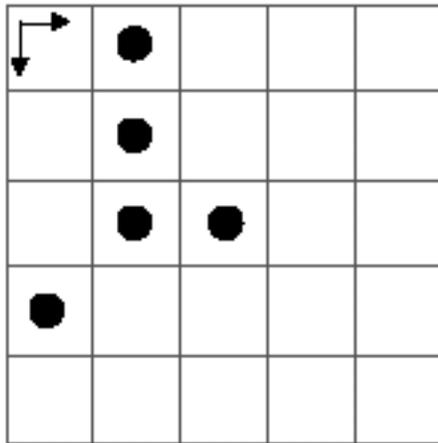
TRANSLATION

Translation

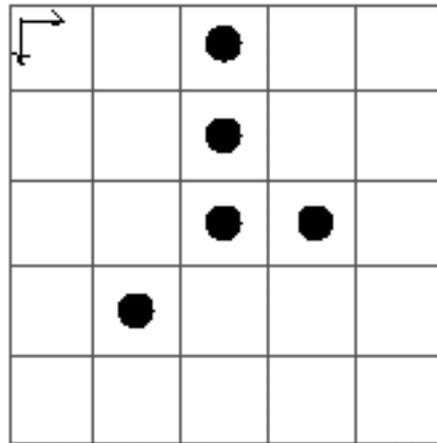
$$A = \{(1,0), (1,1), (1,2), (2,2), (0,3)\}$$

$$x = (1,0)$$

$$(A)_x = \{(2,0), (2,1), (2,2), (3,2), (1,3)\}$$



A



$(A)_x$

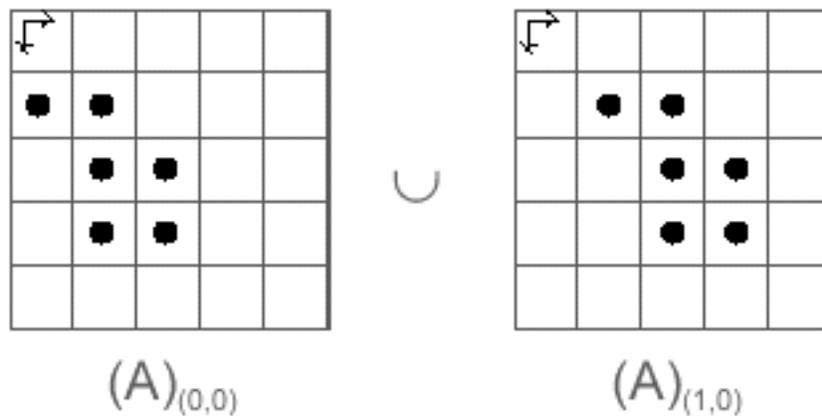
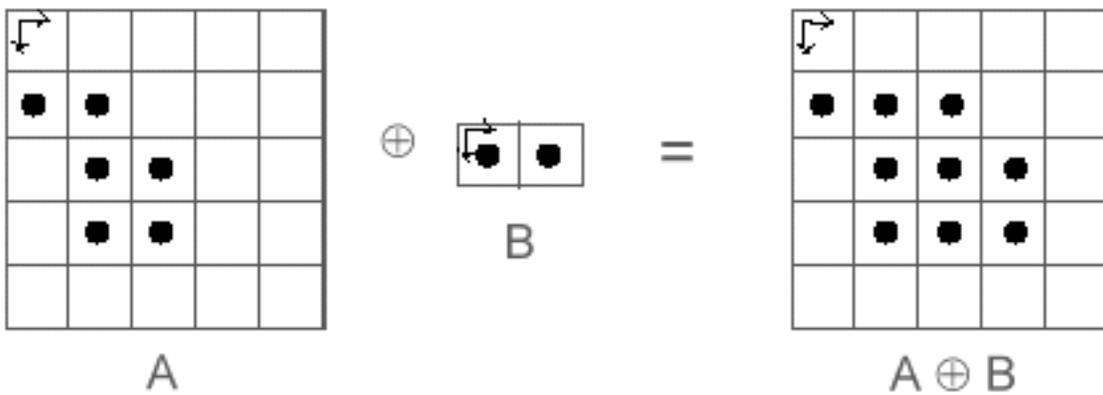
MORPHOLOGIE MATHÉMATIQUE

DILATATION (1)

Dilatation

$$A \oplus B = \{ x \mid x=a+b, a \in A, b \in B \}$$

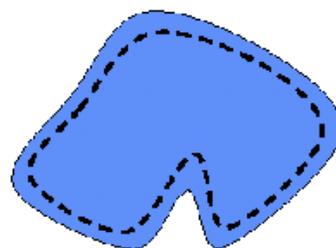
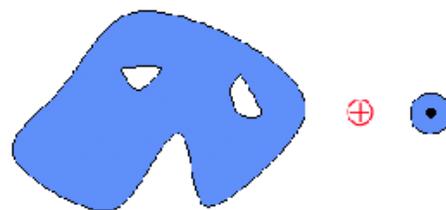
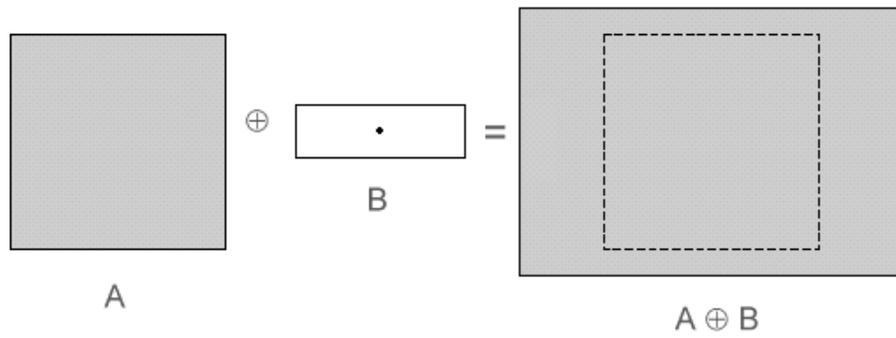
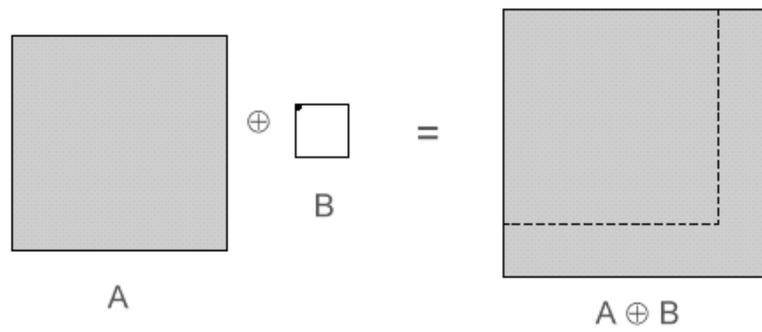
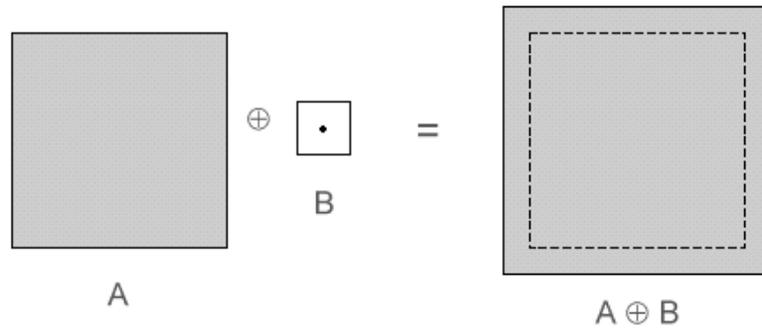
$$A \oplus B = \bigcup_{b \in B} (A)_b$$



B : Élément structurant

MORPHOLOGIE MATHÉMATIQUE

DILATATION (2)



MORPHOLOGIE MATHÉMATIQUE

DILATATION (3)

Propriétés de la dilatation

$$A \oplus B = B \oplus A$$

Commutative

$$A \oplus (B \oplus C) = (A \oplus B) \oplus C$$

Associative

$$A \subseteq B \Rightarrow C \oplus A \subseteq C \oplus B$$

Monotone

$$(A \cup B) \oplus C = (A \oplus C) \cup (B \oplus C)$$

Distributive

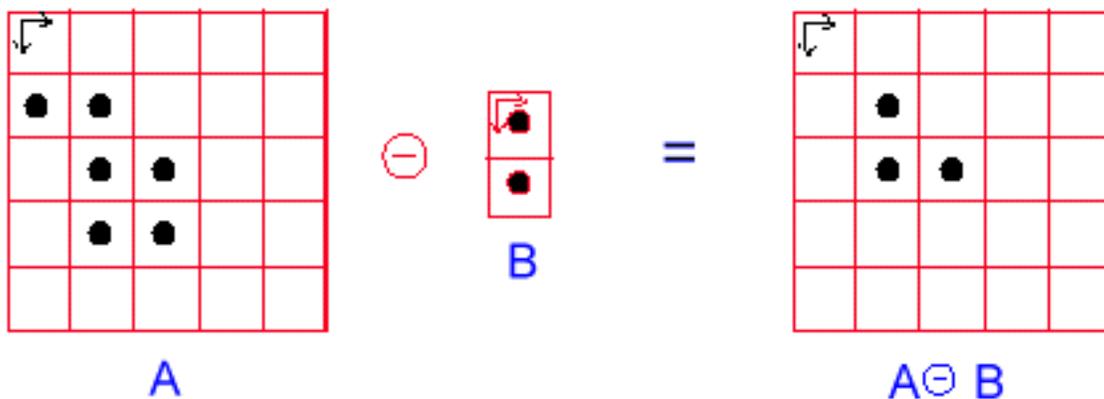
$$(A \cap B) \oplus C \subseteq (A \oplus C) \cap (B \oplus C)$$

Érosion

$$A \ominus B = \{x \mid x+b \in A, \forall b \in B\}$$

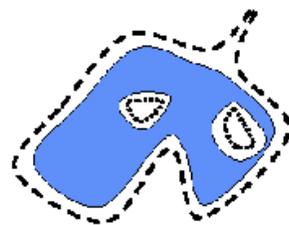
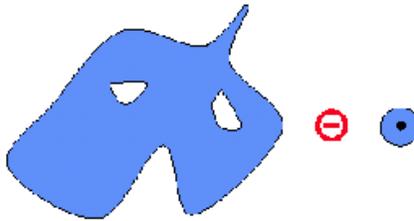
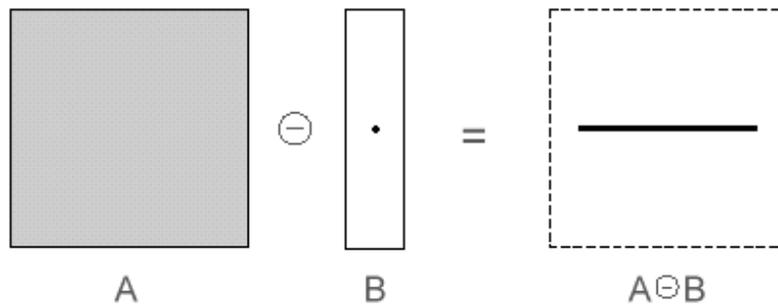
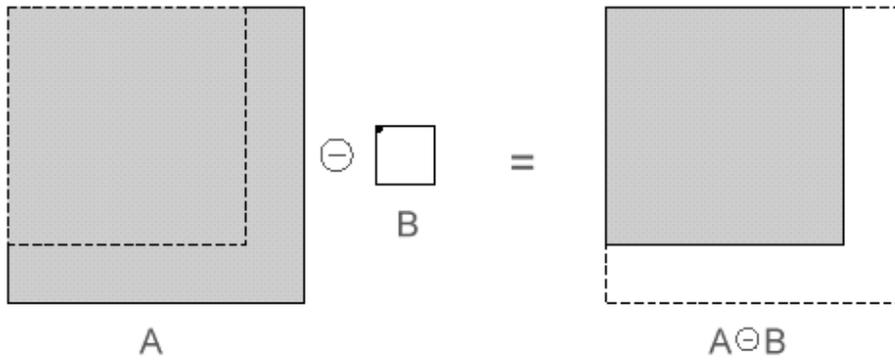
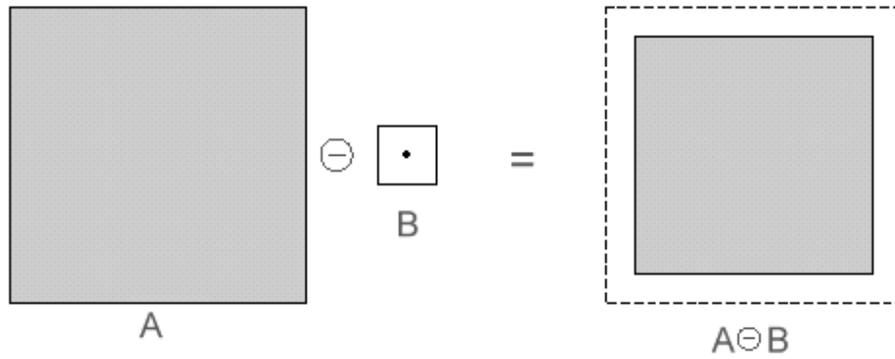
$$A \ominus B = \{x \mid (B)_x \subseteq A\}$$

$$A \ominus B = \bigcap_{b \in B} (A)_{-b}$$



MORPHOLOGIE MATHÉMATIQUE

ÉROSION (2)



MORPHOLOGIE MATHÉMATIQUE

ÉROSION (3)

Propriétés de l'érosion

$$A \subseteq B \Rightarrow A \ominus C \subseteq B \ominus C$$

Monotone

$$A \supseteq B \Rightarrow C \ominus A \subseteq C \ominus B$$

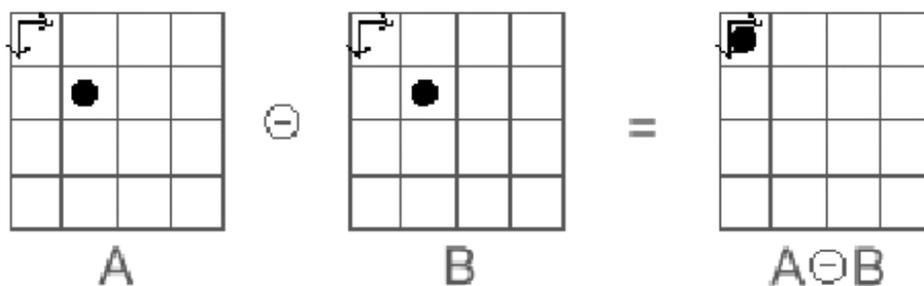
$$(A \ominus B)^c = A^c \oplus \hat{B}$$

Erosion vs Dilation

Note:

$$(A \ominus B) \subseteq A \quad ?$$

Non! Exemple:



MORPHOLOGIE MATHÉMATIQUE

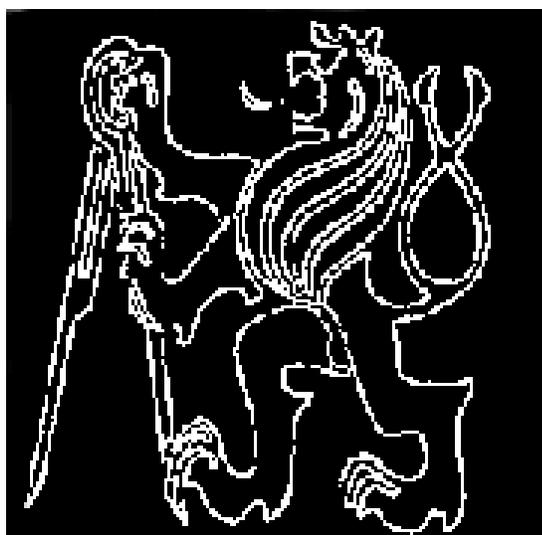
DÉTECTION DE CONTOURS



Original



Eroded



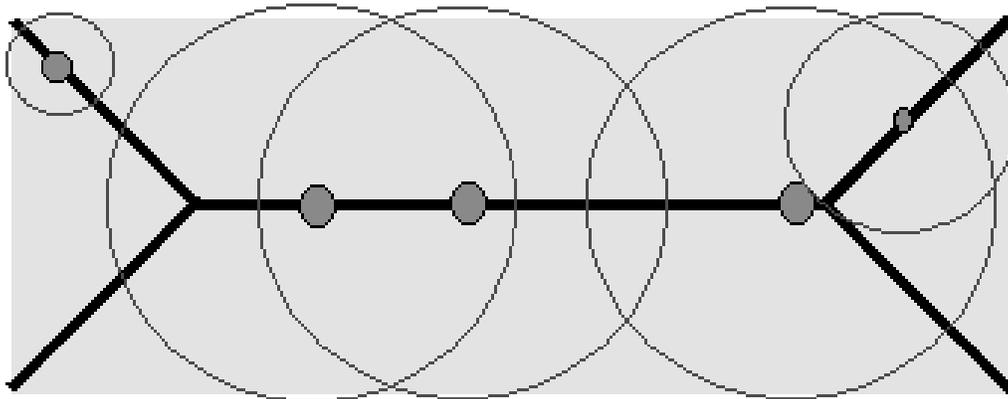
Edges = Original - Eroded

MORPHOLOGIE MATHÉMATIQUE

ESTIMATION DE SQUELETTE

Squelette d'un objet

- Estimation de l'axe intermédiaire d'un objet
(Méthode des feux de prairies ou des disques circulaires)



Original



Erosion

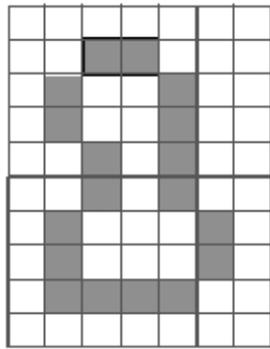


Multiple Erosion =
Skeleton

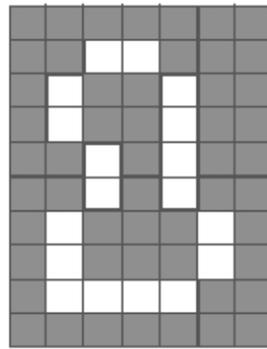
Si on conserve les derniers résultats juste avant
la disparition du point ou du segment

MORPHOLOGIE MATHÉMATIQUE

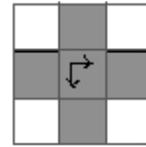
REPLISSAGE



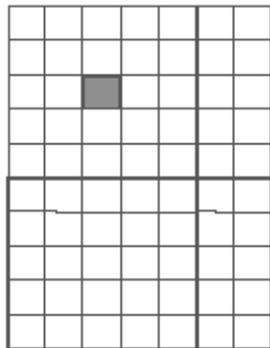
A



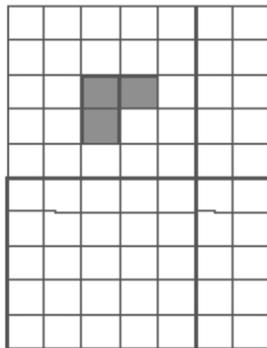
A^c



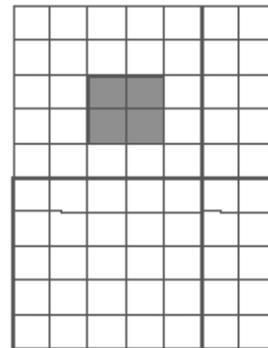
B



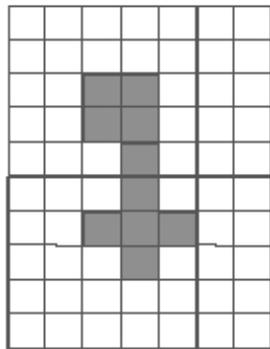
x_0



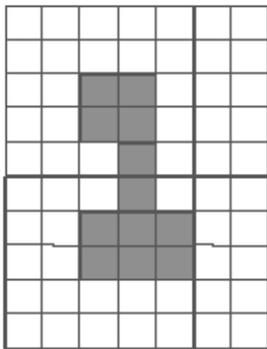
x_1



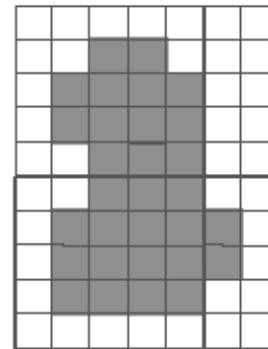
x_2



x_5



x_6



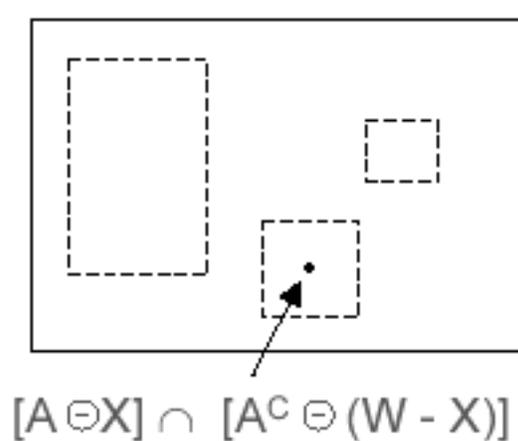
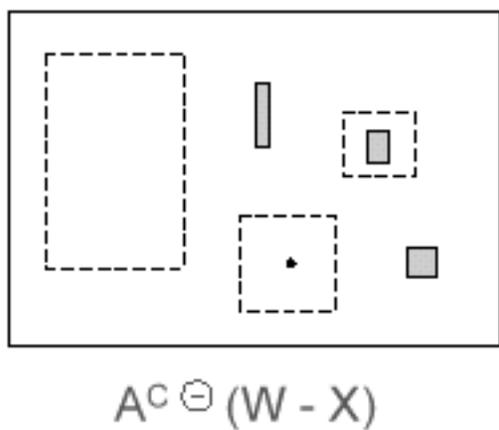
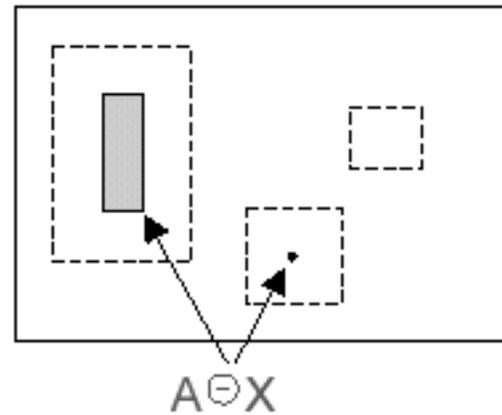
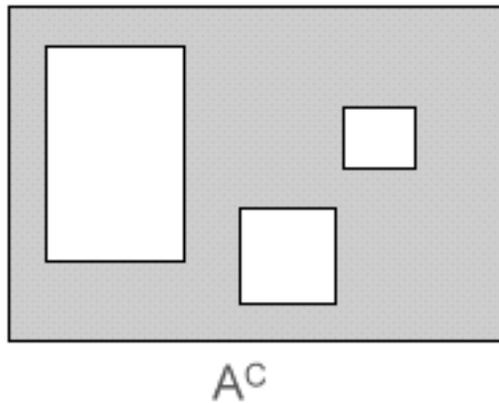
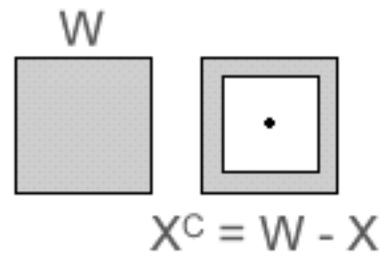
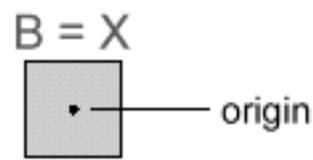
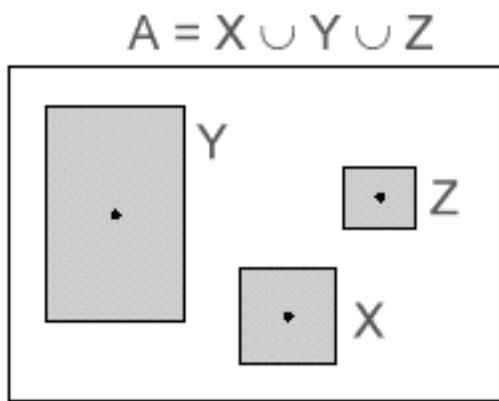
$x_6 \cup A$

$$x_k = (x_{k-1} \oplus B) \cap A^c$$

- Répéter jusqu'à $x_k = x_{k-1}$
- Contour rempli : $x_k \cup A$
- Germe x_0 : point à l'intérieur

MORPHOLOGIE MATHÉMATIQUE

DÉTECTION (HIT OR MISS)

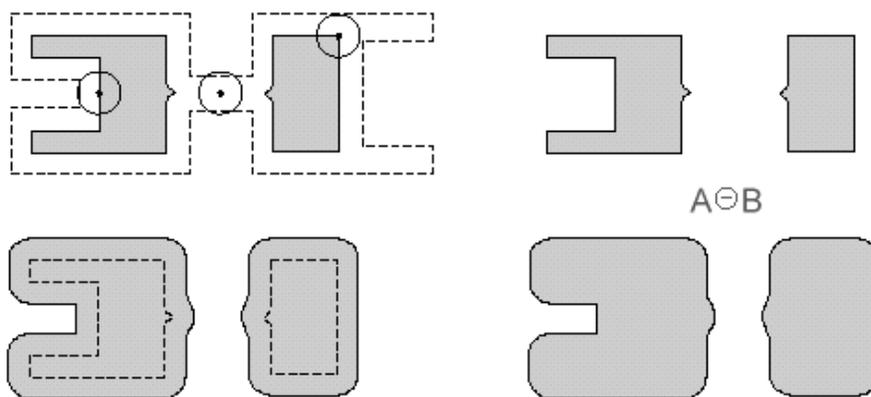
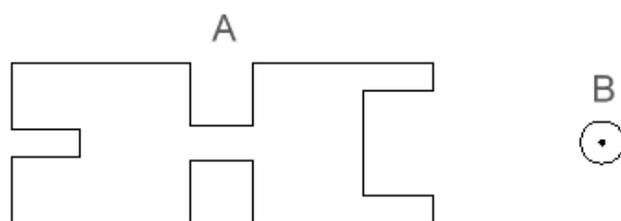


$$[A \ominus X] \cap [A^c \ominus (W - X)]$$

W : fenêtre englobant l'objet X à détecter

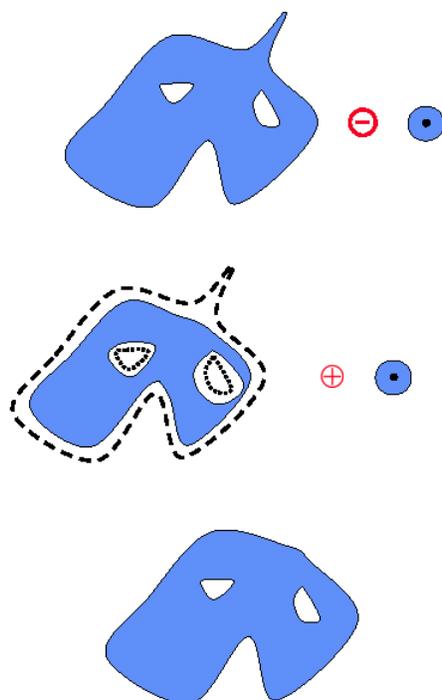
MORPHOLOGIE MATHÉMATIQUE

OUVERTURE



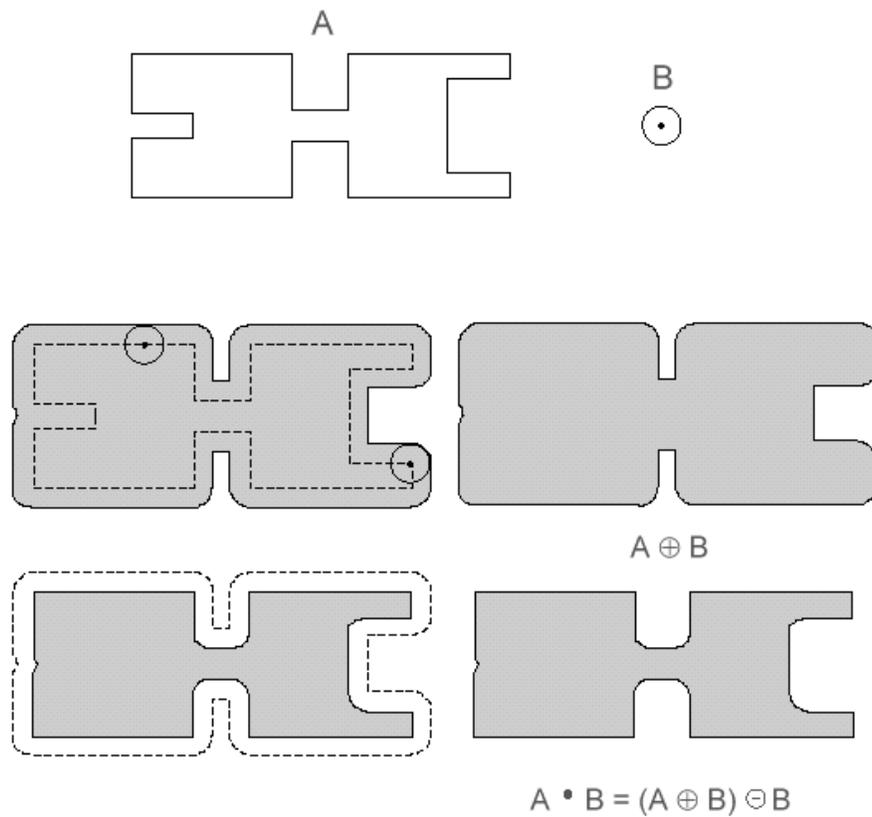
$$A \circ B = (A \ominus B) \oplus B$$

$$A \circ B = (A \ominus B) \oplus B$$

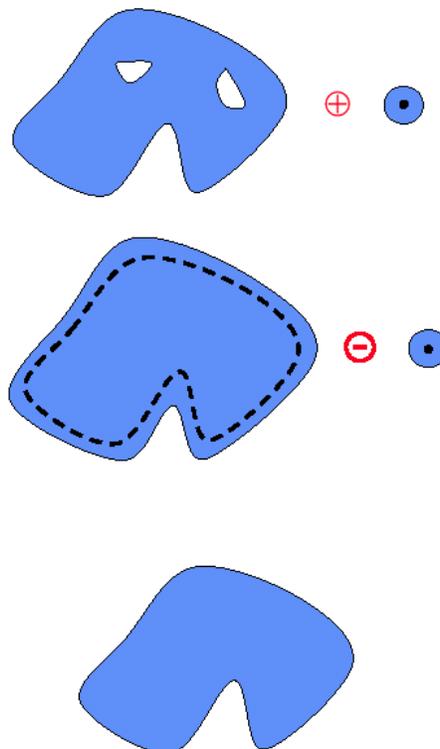


MORPHOLOGIE MATHÉMATIQUE

FERMETURE



$$A \bullet B = (A \oplus B) \ominus B$$



MORPHOLOGIE MATHÉMATIQUE

PROPRIÉTÉS DE L'OUVERTURE ET FERMETURE

$$(A \bullet B)^c = (A^c \circ \hat{B})$$

$$A \circ B \subseteq A$$

$$A \subseteq B \Rightarrow A \circ C \subseteq B \circ C$$

$$(A \circ B) \circ B = A \circ B$$

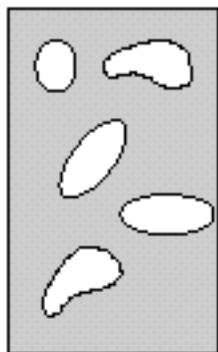
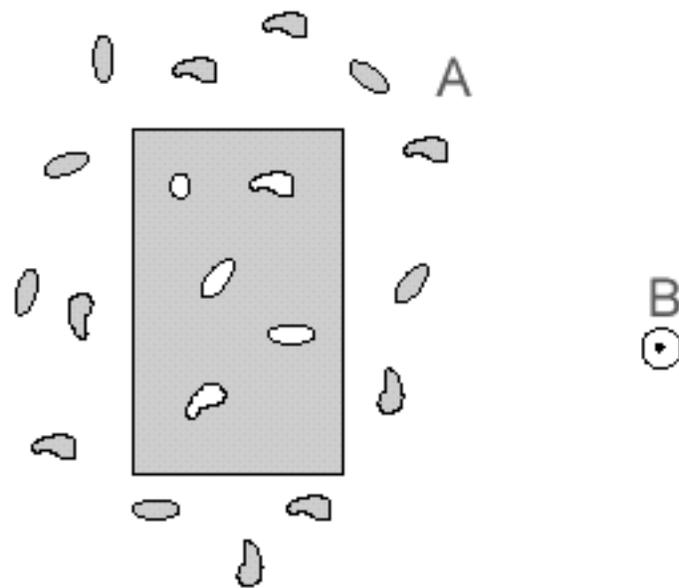
$$A \bullet B \supseteq A$$

$$A \subseteq B \Rightarrow A \bullet C \subseteq B \bullet C$$

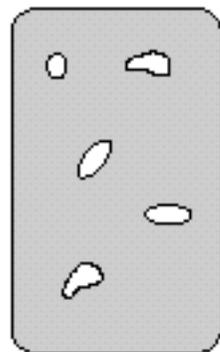
$$(A \bullet B) \bullet B = A \bullet B$$

MORPHOLOGIE MATHÉMATIQUE

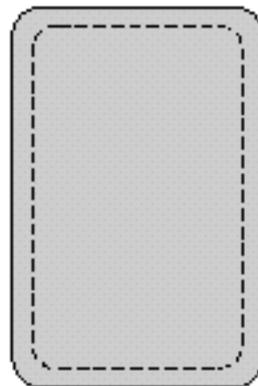
FILTRAGE MORPHOLOGIQUE (BRUIT)



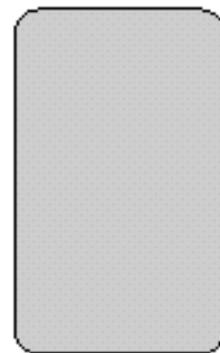
$$A \ominus B$$



$$(A \ominus B) \oplus B \\ = A \circ B$$



$$(A \circ B) \oplus B$$

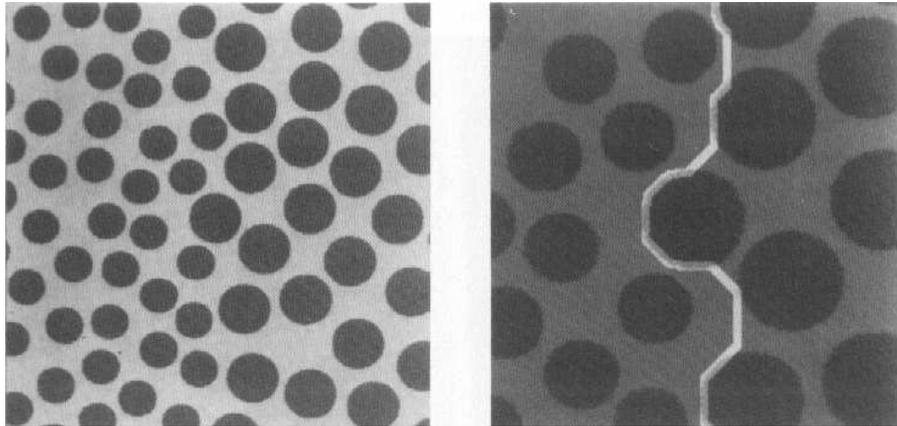


$$[(A \circ B) \oplus B] \ominus B \\ = (A \circ B) \bullet B$$

MORPHOLOGIE MATHÉMATIQUE

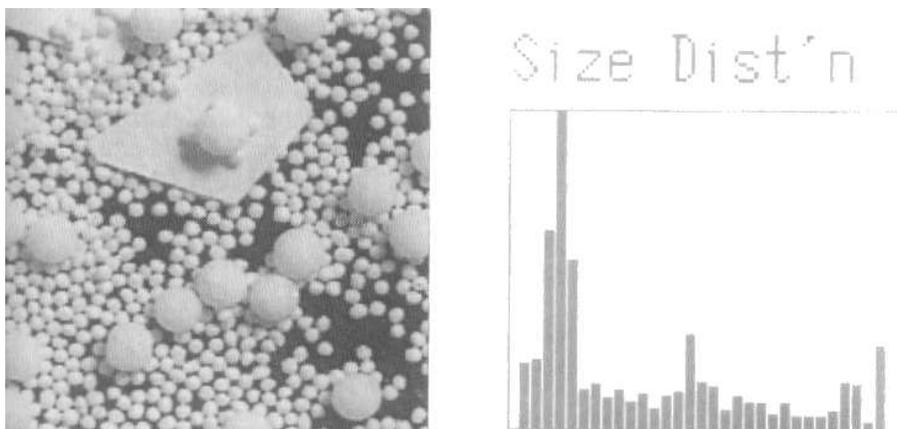
AUTRES APPLICATIONS

Segmentation



- Ouverture avec un élément structurant qui fait disparaître les petits disques noirs
- Fermeture qui fusionne les gros disques
- Détection du contour

Granulométrie



- Ouverture avec des éléments structurant de taille croissante pour faire disparaître les granules blancs successivement en fonction de leur taille
- Moyenne de la différence avec l'image précédente pour obtenir le nombre d'occurrence

MORPHOLOGIE MATHÉMATIQUE

MORPHOLOGIE EN NIVEAUX DE GRIS

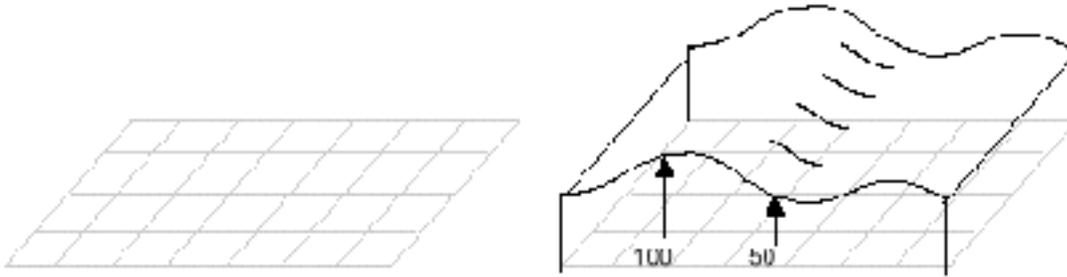
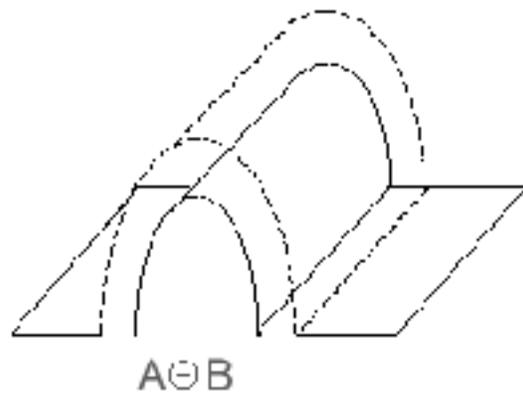
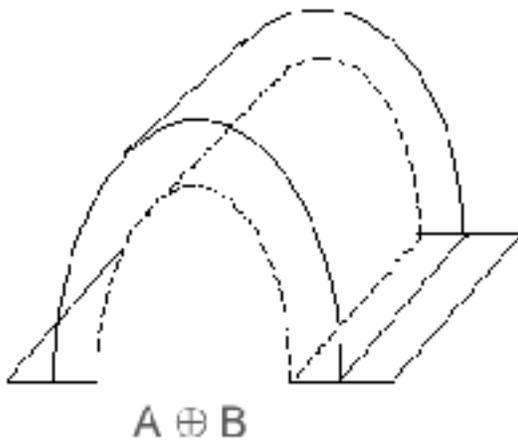
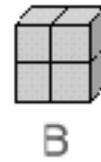
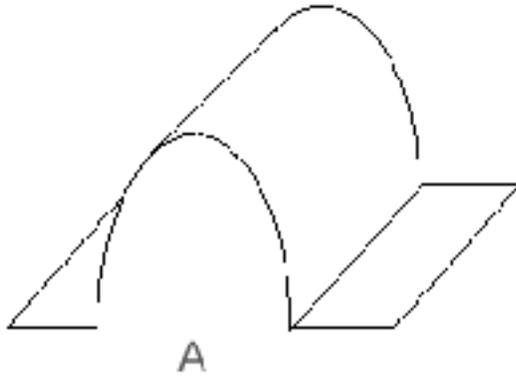


Image
NdeG



MORPHOLOGIE MATHÉMATIQUE

EXEMPLES (1)



Original



Dilation



Erosion



Opening

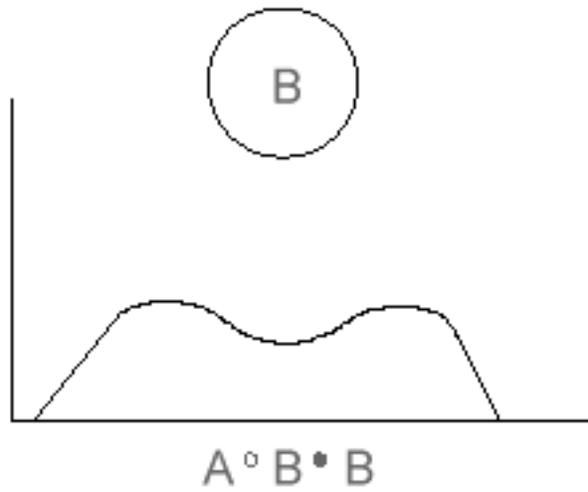
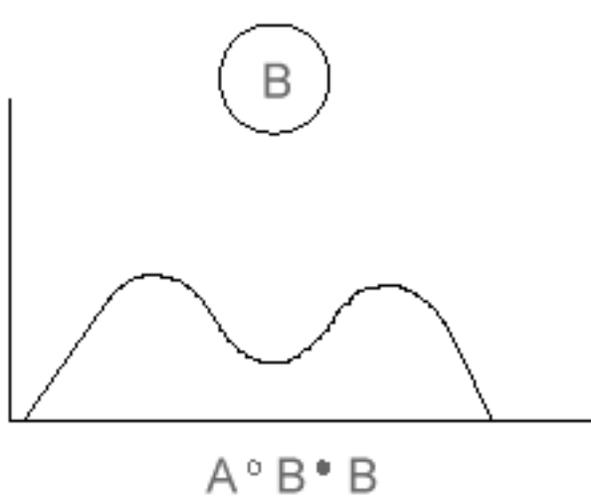
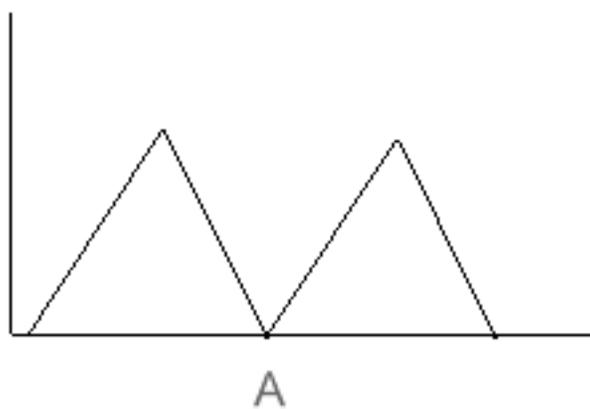


Closing

MORPHOLOGIE MATHÉMATIQUE

EXEMPLES (2- FILTRAGE PASSE-BAS)

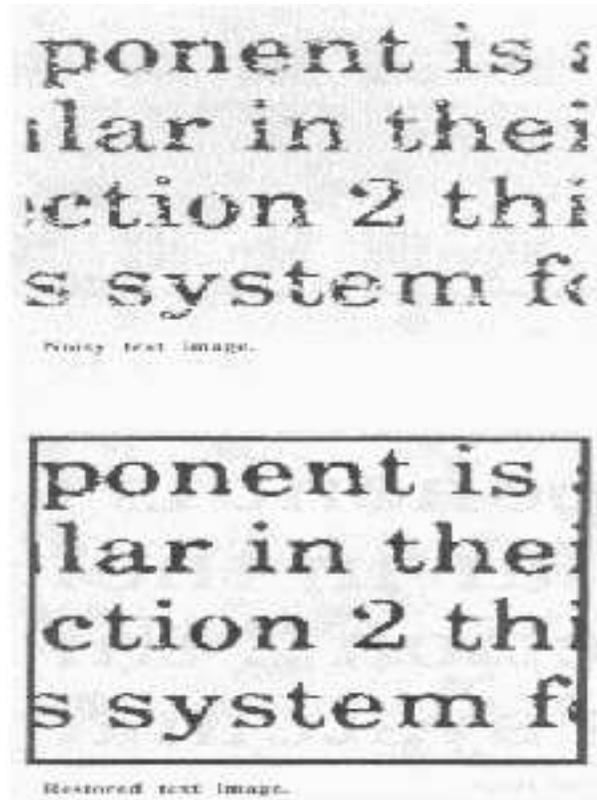
Bruit Poivre & Sel



MORPHOLOGIE MATHÉMATIQUE

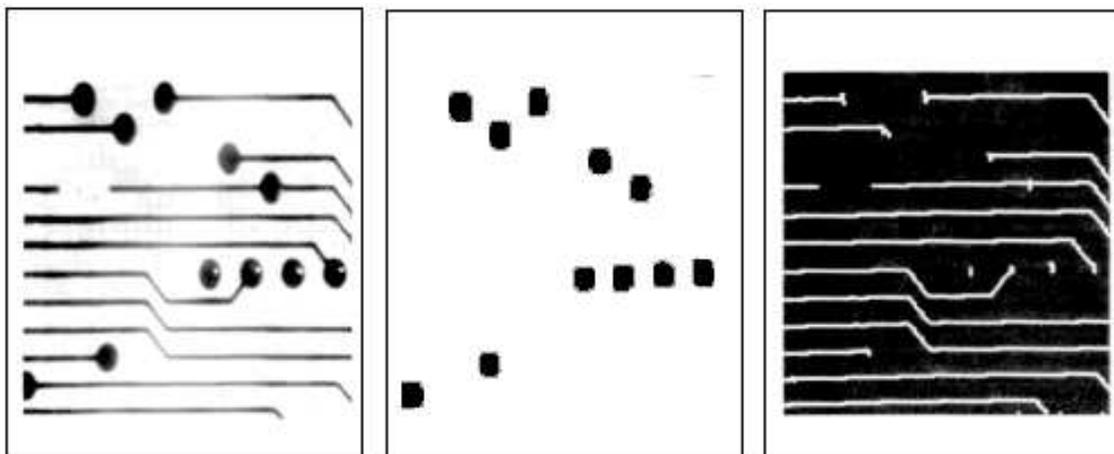
EXEMPLES (3)

Restauration de documents imprimés



$$A \circ B \bullet B$$

Segmentation de Circuit imprimés



Circuit Imprimé

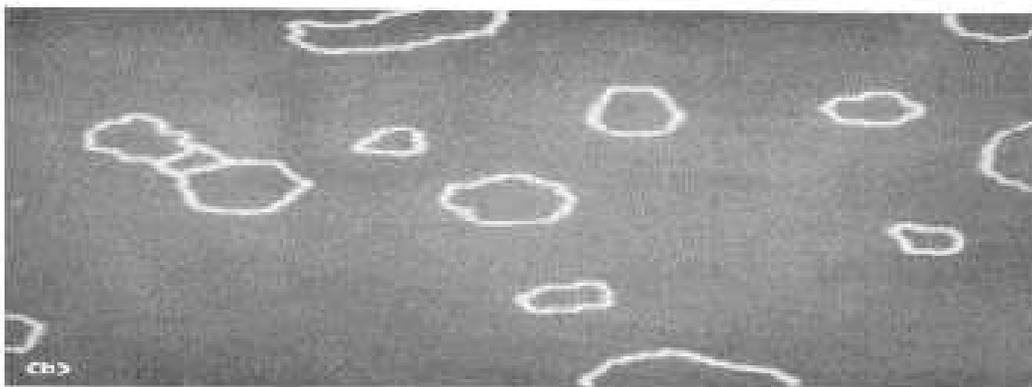
$A \circ B$

Squelette

MORPHOLOGIE MATHÉMATIQUE

EXEMPLES (4)

Segmentation



$$A \oplus B - A \ominus B$$



Original

$A \oplus B$

$A \ominus B$

MORPHOLOGIE MATHÉMATIQUE

EXEMPLES (5)



Original



$A \circ B$



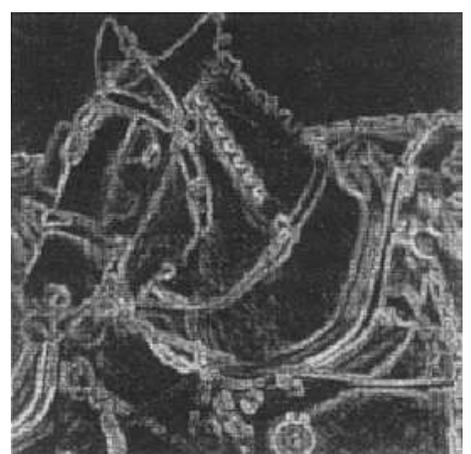
$A \bullet B$



Original



$A \circ B \bullet B$



$A \oplus B - A \ominus B$