DIRO
IFT 6150

TRAITEMENT D'IMAGES

BINARISATION ET MORPHOLOGIE MATHÉMATIQUE

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BINARISATION ET MORPHOLOGIE MATHÉMATIQUE

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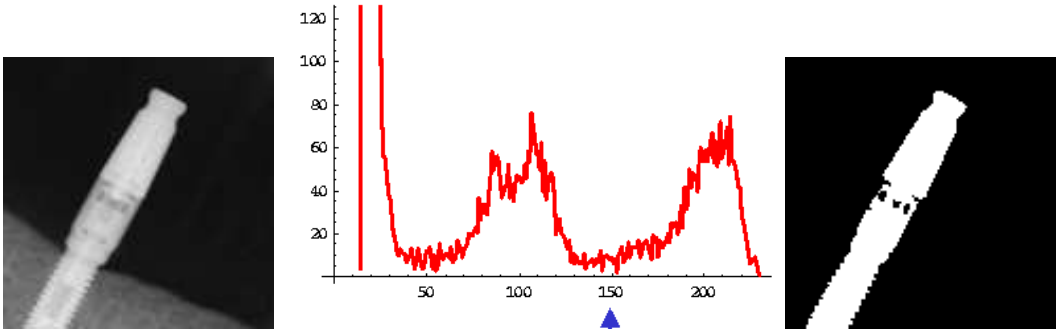
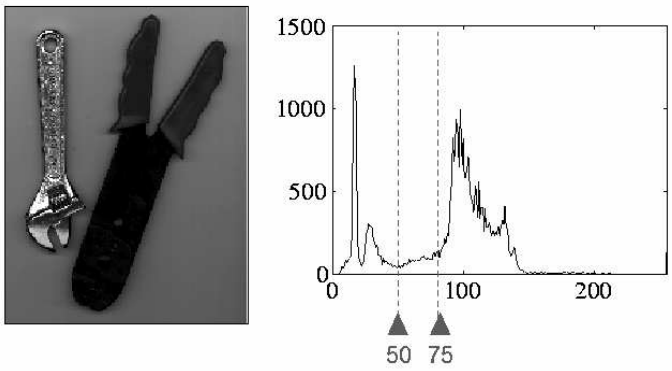
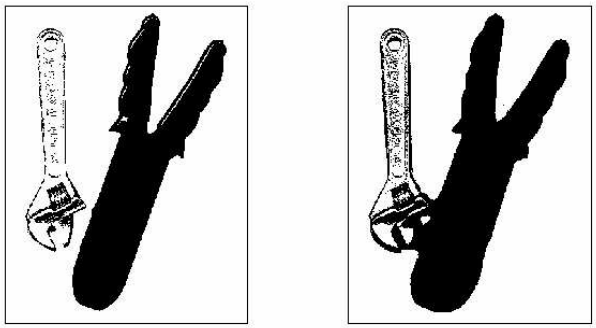
BINARISATION
SEUILLAGE PAR INSPECTION (1)

Seuillage par inspection de l'histogramme



Original Histogram



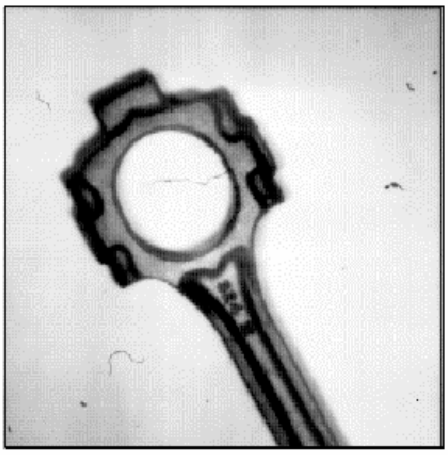


Threshold = 50 Threshold = 75

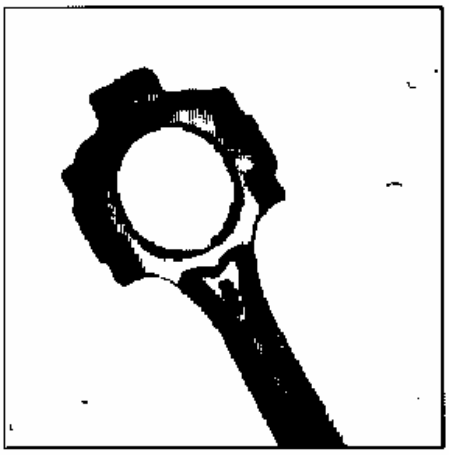
BINARISATION
SEUILLAGE PAR INSPECTION (2)

Seuillage par inspection de l'histogramme

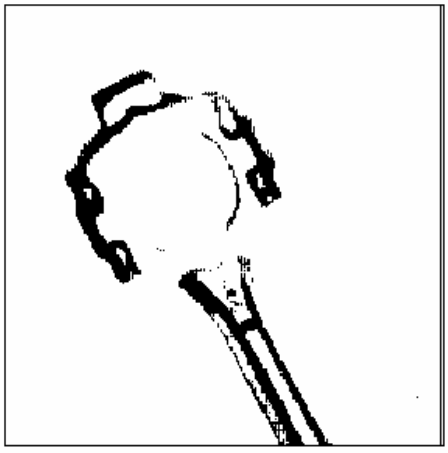
Original Image



Binary Image



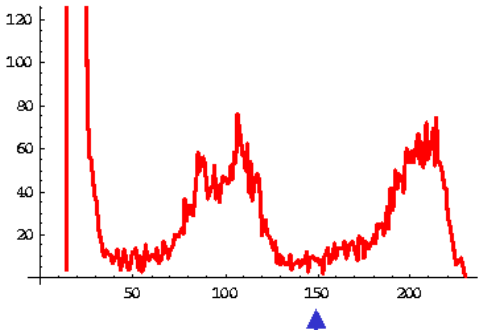
Threshold too low



Threshold too high

BINARISATION
SEUILLAGE OPTIMAL (1)

Seuillage optimal



Soit deux régions (arrière-plan et objet)
présent dans une image

La probabilité $P(z)$ d'avoir une valeur de niveau de gris z ($z \in [0, 255]$ par ex.) dans l'image est donnée par

$$p(z) = P_1\, p_1(z) + P_2\, p_2(z)$$

P_i : prop. de pixels appartenant à la région i ($P_1 + P_2 = 1$)
$p(z)$: Histogramme de l'image
$p_i(z)$: proba. d'un pixel $\in$ région i d'avoir un ng $= z$

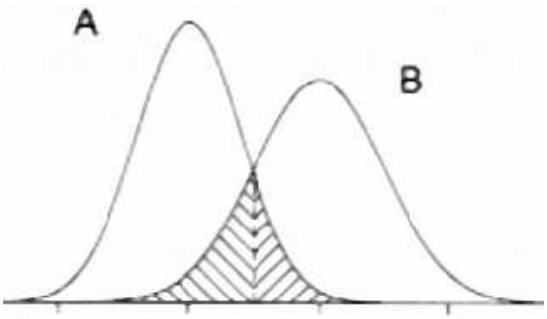
Si nous supposons que $p_1(z)$ et $p_2(z)$ sont distribués selon une loi normale, nous obtenons

$$p_1(z) = \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{(z-\mu_1)^2}{2\sigma_1^2}\right)$$
$$p_2(z) = \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left(-\frac{(z-\mu_2)^2}{2\sigma_2^2}\right)$$

BINARISATION
SEUILLAGE OPTIMAL (2)

$p(z)$ devient alors

$$p(z) = P_1 \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{(z-\mu_1)^2}{2\sigma_1^2}\right) + P_2 \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left(-\frac{(z-\mu_2)^2}{2\sigma_2^2}\right)$$



Soit E_1, la probabilité de classer un pixel dans la classe 1 lorsqu'il appartient à la classe 2 et E_2, la proba. de classer un pixel dans la classe 2 lorsque celui-ci appartient à la classe 1.

Les probabilités d'erreur sont données par

$$E_1(T) = \int_{-\infty}^{T} p_2(z)\,dz \quad \text{et} \quad E_2(T) = \int_{T}^{\infty} p_1(z)\,dz$$

La probabilité d'erreur totale est alors donnée par

$$E(T) = P_2 E_1(T) + P_1 E_2(T)$$

Cherchons une valeur de T qui minimise $E(T)$

$$\begin{aligned} E'(T) &= -P_1\,p_1(T) + P_2\,p_2(T) = 0 \\ P_1\,p_1(T) &= P_2\,p_2(T) \end{aligned}$$

BINARISATION
SEUILLAGE OPTIMAL (3)

Après simplification, nous obtenons une expression de la forme

$$AT^2 + BT + C = 0$$

Avec

$$\begin{aligned} A &= \sigma_1^2 - \sigma_2^2 \\ B &= 2\,(\mu_1\,\sigma_2^2 - \mu_2\,\sigma_1^2) \\ C &= \mu_2^2\,\sigma_1^2 - \mu_1^2\,\sigma_2^2 + 2\sigma_1^2\,\sigma_2^2 \ln\left(\frac{\sigma_2\,P_1}{\sigma_1\,P_2}\right) \end{aligned}$$

- Lorsque les variances sont égales

$$\begin{aligned} \sigma_1^2 &= \sigma_2^2 = \sigma^2 \\ T &= \frac{\mu_1 + \mu_2}{2} + \frac{\sigma^2}{\mu_1 - \mu_2} \ln\left(\frac{P_2}{P_1}\right) \end{aligned}$$

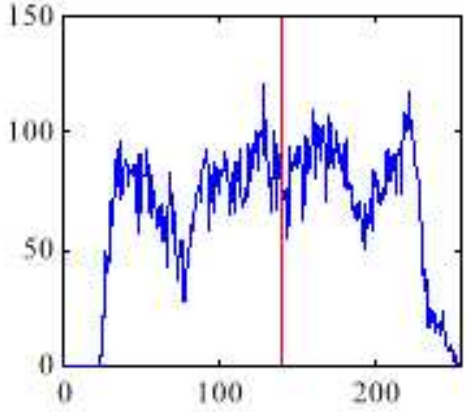
- Lorsque les variances et les proportions sont égales

$$\begin{aligned} P_1 &= P_2 \\ T &= \frac{\mu_1 + \mu_2}{2} \end{aligned}$$

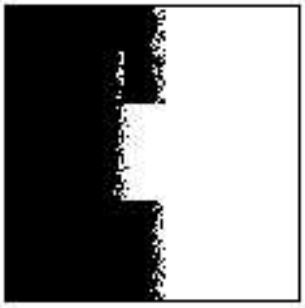
BINARISATION
SEUILLAGE LOCALE (1)

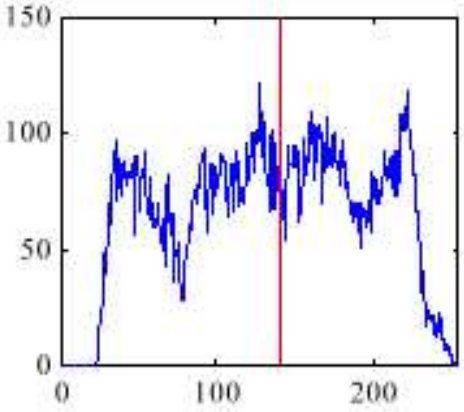
Original

Histogram



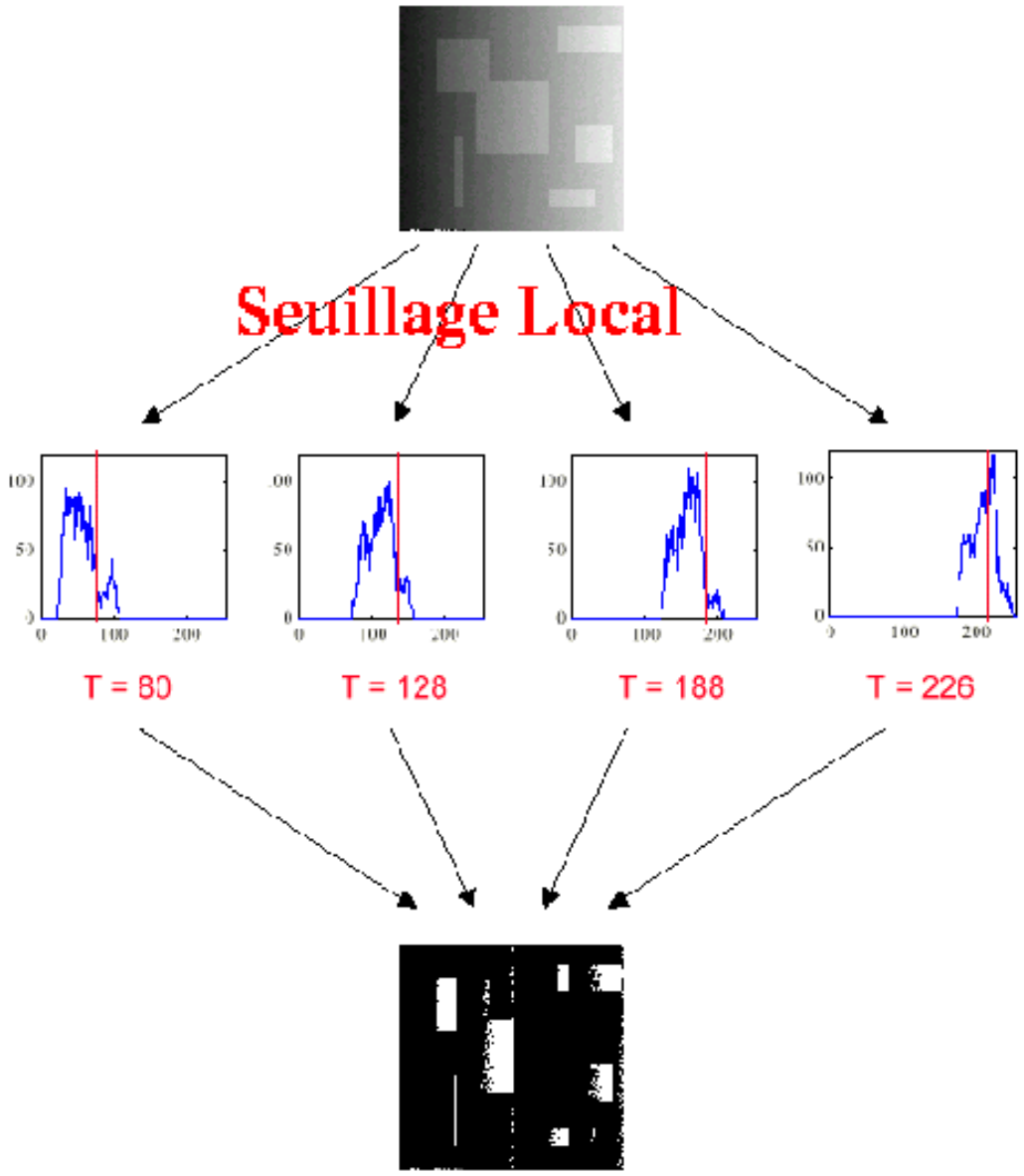
Single Global Threshold



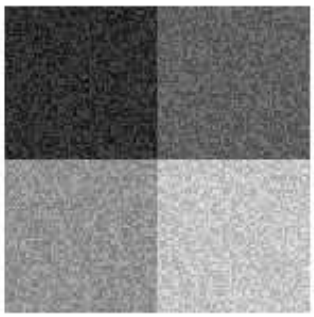


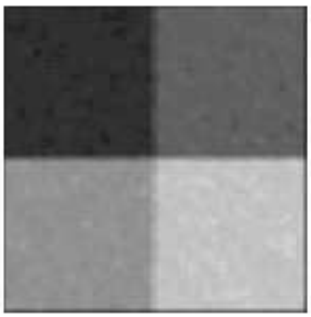
T = 128

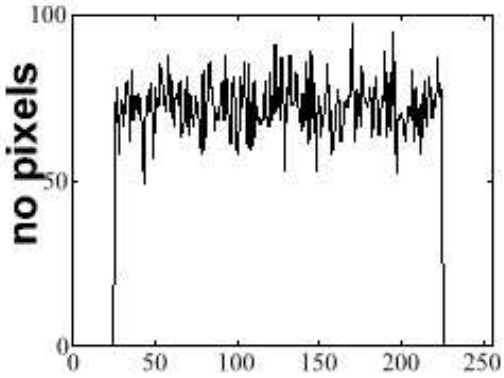
BINARISATION
SEUILLAGE LOCALE (2)

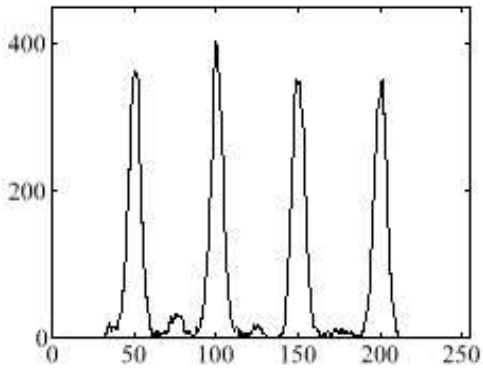


BINARISATION
SEUILLAGE & FILTRAGE









Le bruit complique la sélection d'un seuil

▶ Filtrage passe-bas

MORPHOLOGIE MATHÉMATIQUE
THÉORIE DES ENSEMBLES

▶ Traitement d'image basé sur la théorie des ensembles

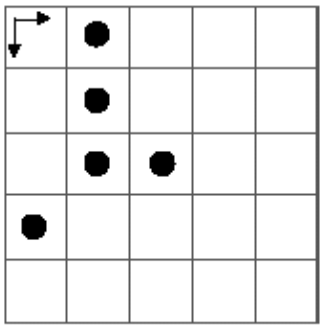
Quelques images binaires

A, B	=	Ens. des pixels = Ens. des coordonnées	
$A \cup B$	=	$\{ x \mid x \in A$ ou $x \in B \}$	Union
$A \cap B$	=	$\{ x \mid x \in A$ et $x \in B \}$	Intersection
A^C	=	$\{ x \mid x \notin A \}$	Complément
$A - B$	=	$\{ x \mid x \in A, x \notin B \}$	Différence
$(A)_C$	=	$\{ x \mid x = a + c,\ a \in A\}$	Translation
$\hat{A}$	=	$\{ x \mid -x \in A\}$	Inversion

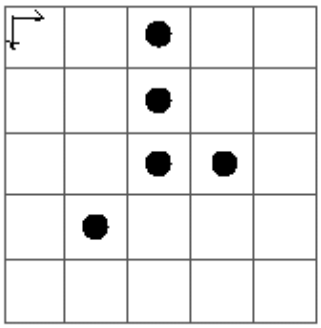
MORPHOLOGIE MATHÉMATIQUE
TRANSLATION

Translation

$A = \{(1,0), (1,1), (1,2), (2,2), (0,3)\}$
$x = (1,0)$
$(A)_x = \{(2,0), (2,1), (2,2), (3,2), (1,3)\}$



A



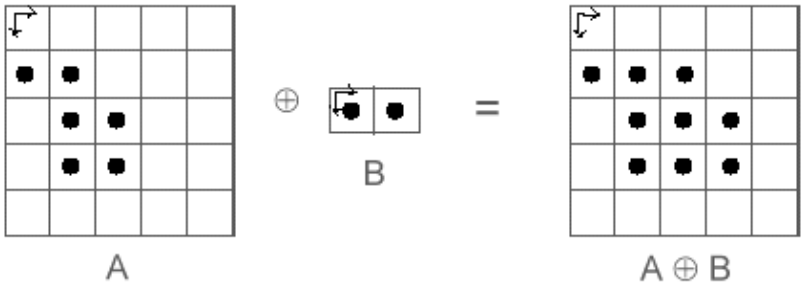
$(A)_x$

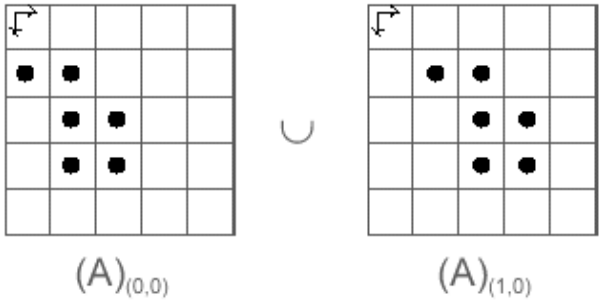
MORPHOLOGIE MATHÉMATIQUE
DILATATION (1)

Dilatation

$$A \oplus B = \{ x \mid x=a+b,\ a \in A,\ b \in B \}$$

$$A \oplus B = \bigcup_{b \in B} (A)_b$$

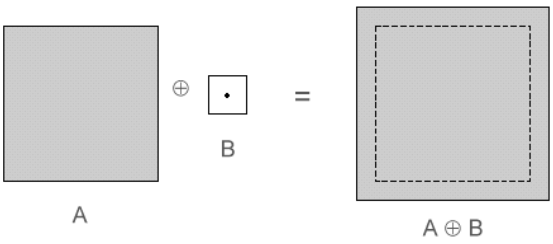


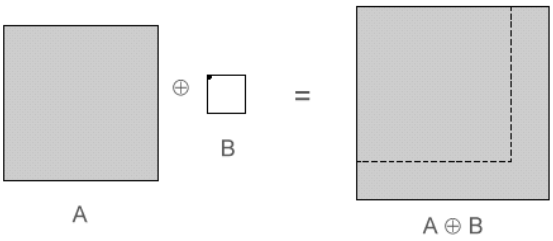


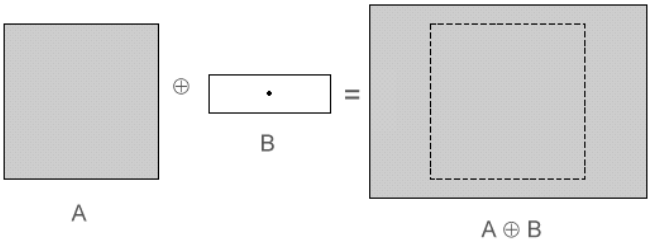
B : Élément structurant

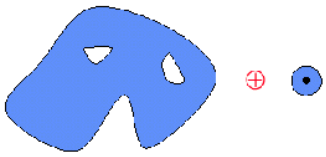
MORPHOLOGIE MATHÉMATIQUE

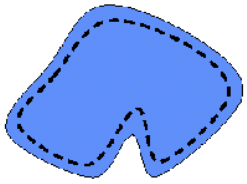
DILATATION (2)











MORPHOLOGIE MATHÉMATIQUE
DILATATION (3)

Propriétés de la dilatation

$A \oplus B = B \oplus A$ — Commutative

$A \oplus (B \oplus C) = (A \oplus B) \oplus C$ — Associative

$A \subseteq B \Rightarrow C \oplus A \subseteq C \oplus B$ — Monotone

$(A \cup B) \oplus C = (A \oplus C) \cup (B \oplus C)$ — Distributive

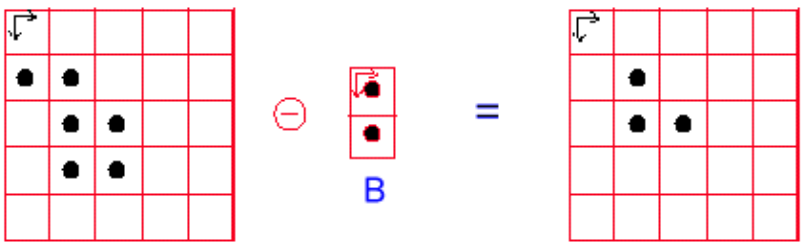
$(A \cap B) \oplus C \subseteq (A \oplus C) \cap (B \oplus C)$

Érosion

$$A \ominus B = \{ x \mid x+b \in A, \ \forall b \in B \}$$

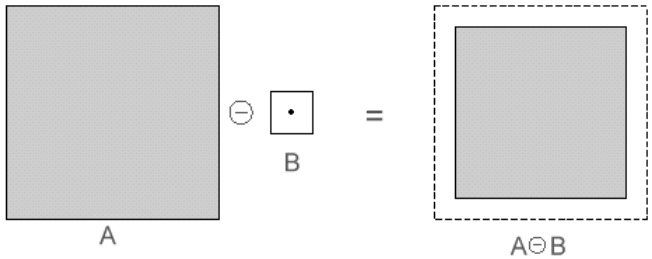
$$A \ominus B = \{ x \mid (B)_x \subseteq A \}$$

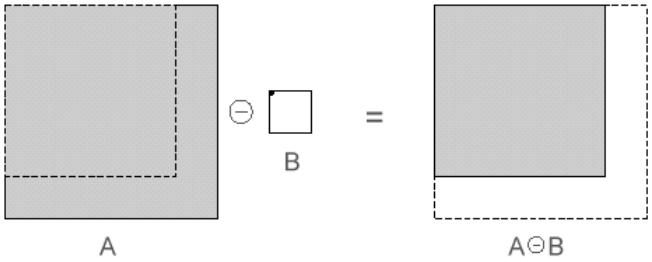
$$A \ominus B = \bigcap_{b \in B} (A)_{-b}$$

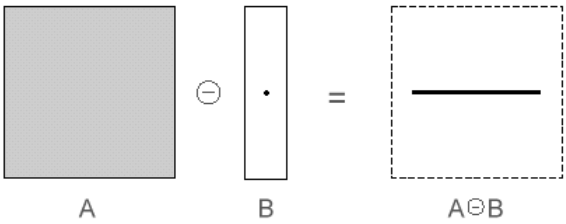


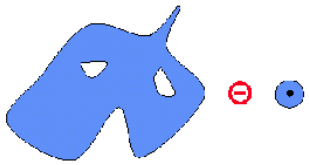
MORPHOLOGIE MATHÉMATIQUE

ÉROSION (2)









MORPHOLOGIE MATHÉMATIQUE
ÉROSION (3)

Propriétés de l'érosion

$A \subseteq B \Rightarrow A \ominus C \subseteq B \ominus C$ Monotone

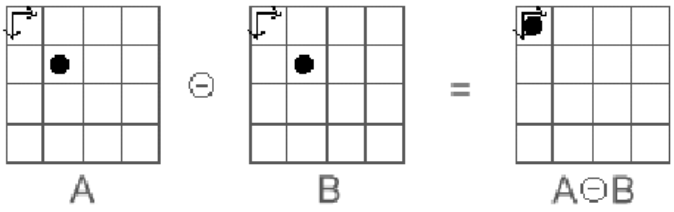
$A \supseteq B \Rightarrow C \ominus A \subseteq C \ominus B$

$(A \ominus B)^c = A^c \oplus \hat{B}$ Erosion vs Dilation

Note:

$(A \ominus B) \subseteq A$?

Non! Exemple:



MORPHOLOGIE MATHÉMATIQUE
DÉTECTION DE CONTOURS

Original

Eroded

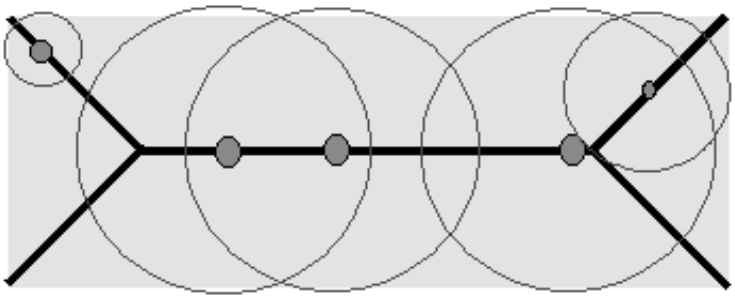
Edges = Original - Eroded

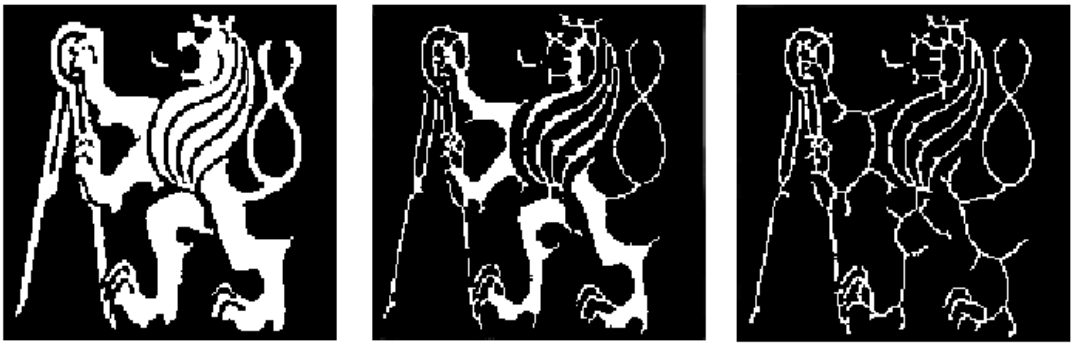
MORPHOLOGIE MATHÉMATIQUE
ESTIMATION DE SQUELETTE

Squelette d'un objet

▶ Estimation de l'axe intermédiaire d'un objet

(Méthode des feux de prairies ou des disques circulaires)



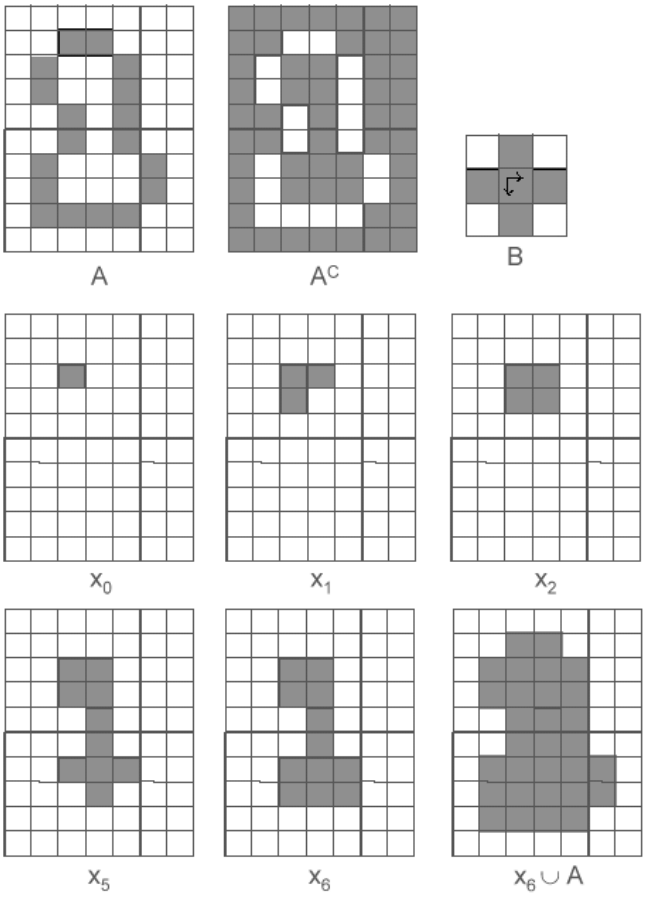


Original Erosion Multiple Erosion = Skeleton

Si on conserve les derniers résultats juste avant
la disparition du point ou du segment

MORPHOLOGIE MATHÉMATIQUE

REMPLISSAGE

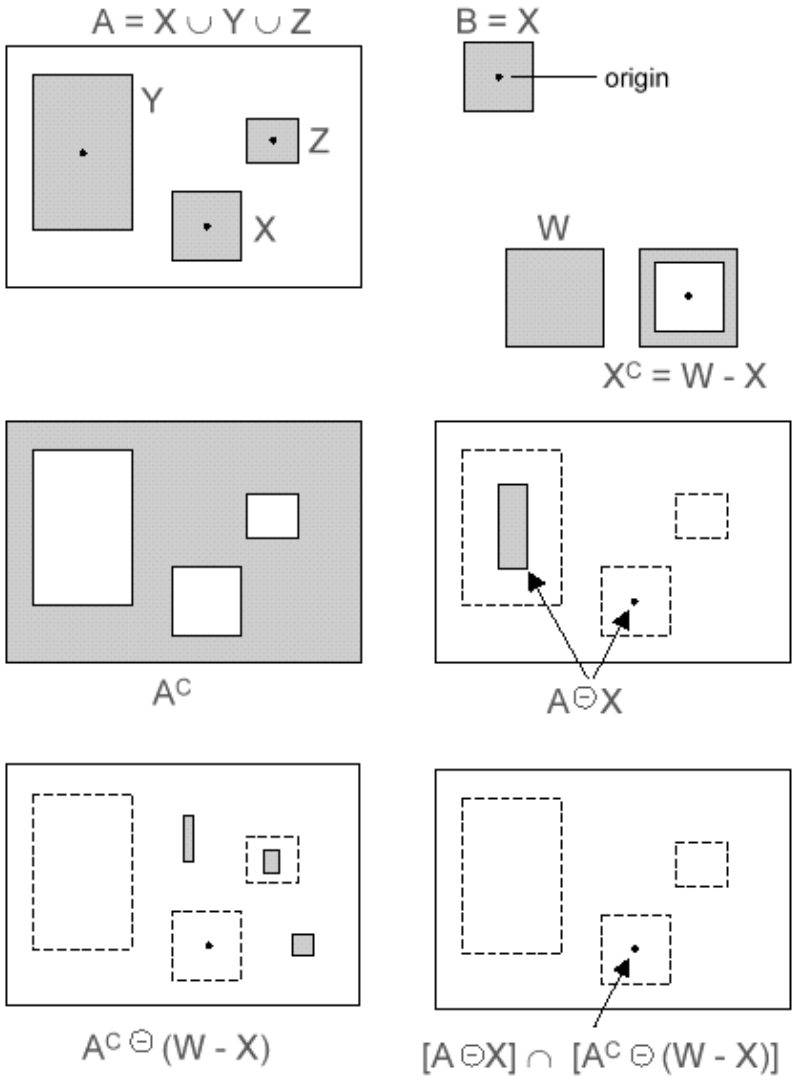


$$x_k = (x_{k-1} \oplus B) \cap A^C$$

- Répéter jusqu'à $x_k = x_{k-1}$
- Contour rempli : $x_k \cup A$
- Germe x_0 : point à l'intérieur

MORPHOLOGIE MATHÉMATIQUE

DÉTECTION (HIT OR MISS)

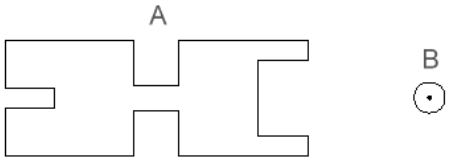


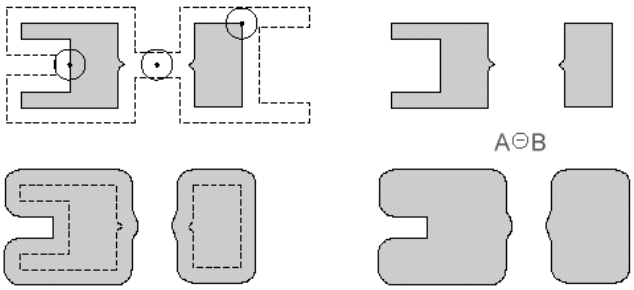
$$[A \ominus X] \cap [A^C \ominus (W - X)]$$

W : fenêtre englobant l'objet X à détecter

MORPHOLOGIE MATHÉMATIQUE

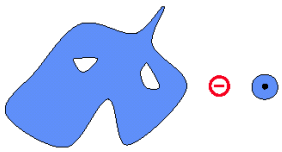
OUVERTURE





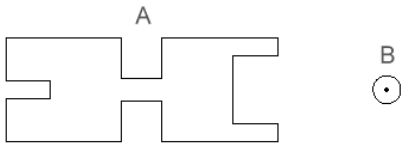
$A \circ B = (A \ominus B) \oplus B$

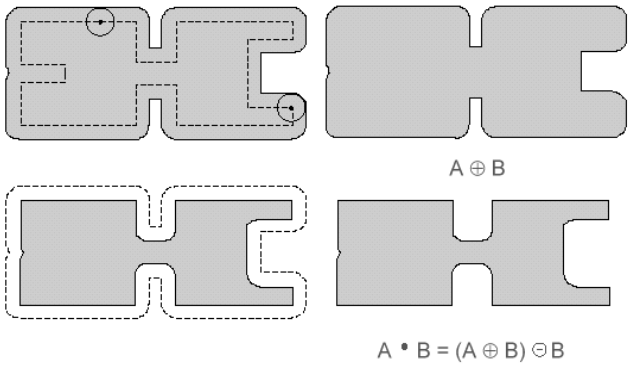
$$A \circ B = (A \ominus B) \oplus B$$



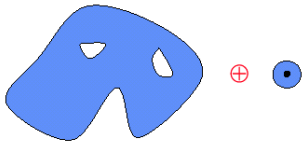
MORPHOLOGIE MATHÉMATIQUE

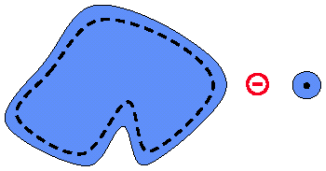
FERMETURE

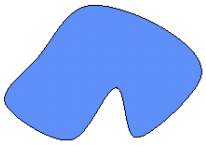




$$A \bullet B = (A \oplus B) \ominus B$$







MORPHOLOGIE MATHÉMATIQUE

PROPRIÉTÉS DE L'OUVERTURE ET FERMETURE

$$(A \bullet B)^C = (A^C \circ \hat{B})$$

$$A \circ B \subseteq A$$

$$A \subseteq B \Rightarrow A \circ C \subseteq B \circ C$$

$$(A \circ B) \circ B = A \circ B$$

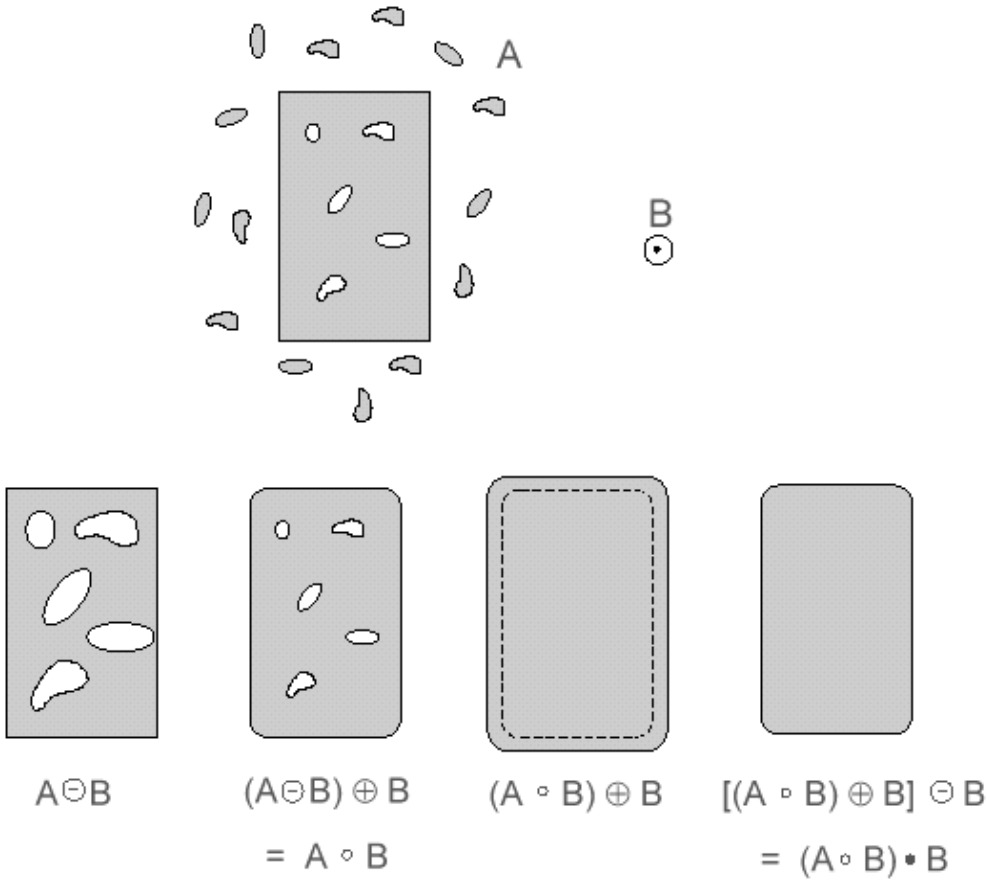
$$A \bullet B \supseteq A$$

$$A \subseteq B \Rightarrow A \bullet C \subseteq B \bullet C$$

$$(A \bullet B) \bullet B = A \bullet B$$

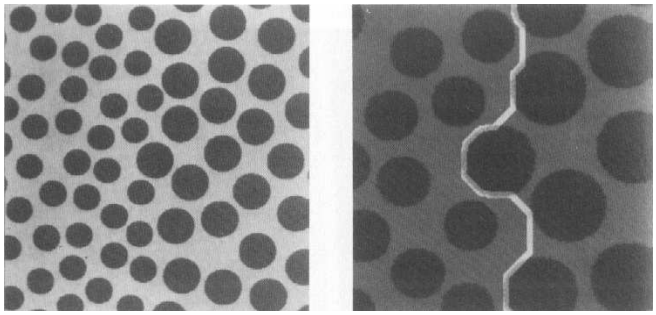
MORPHOLOGIE MATHÉMATIQUE

FILTRAGE MORPHOLOGIQUE (BRUIT)



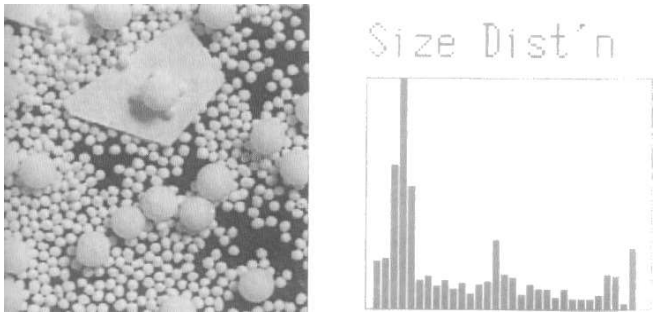
MORPHOLOGIE MATHÉMATIQUE
AUTRES APPLICATIONS

Segmentation



- Ouverture avec un élément structurant qui fait disparaître les petits disques noirs
- Fermeture qui fusionne les gros disques
- Détection du contour

Granulométrie



- Ouverture avec des éléments structurant de taille croissante pour faire disparaître les granules blancs successivement en fonction de leur taille
- Moyenne de la différence avec l'image précédente pour obtenir le nombre d'occurence

MORPHOLOGIE MATHÉMATIQUE

MORPHOLOGIE EN NIVEAUX DE GRIS

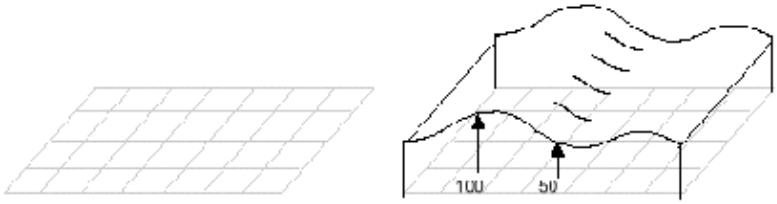
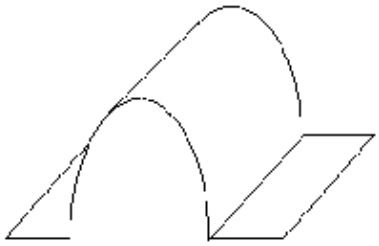
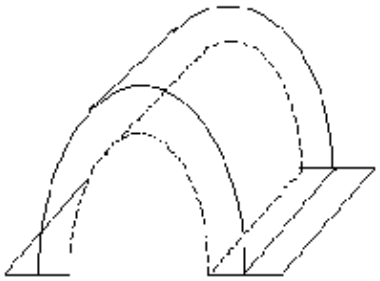


Image
NdeG

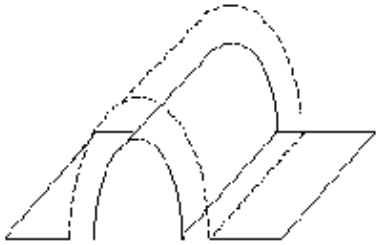


A

B



A ⊕ B



A⊖B

MORPHOLOGIE MATHÉMATIQUE
EXEMPLES (1)

Original

Dilation

Erosion

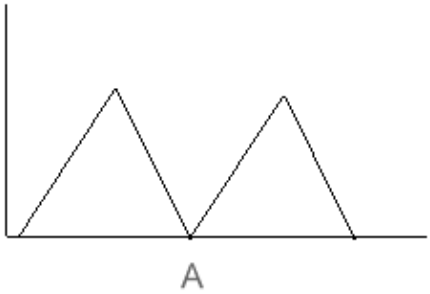
Opening

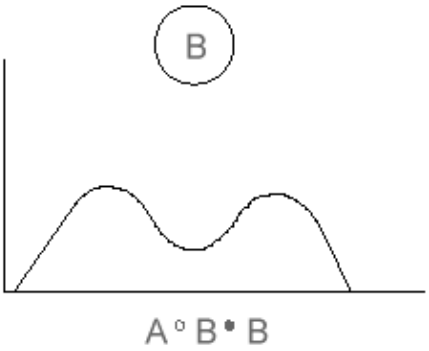
Closing

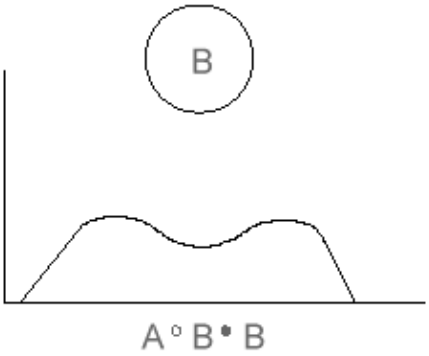
MORPHOLOGIE MATHÉMATIQUE

EXEMPLES (2- FILTRAGE PASSE-BAS)

Bruit Poivre & Sel

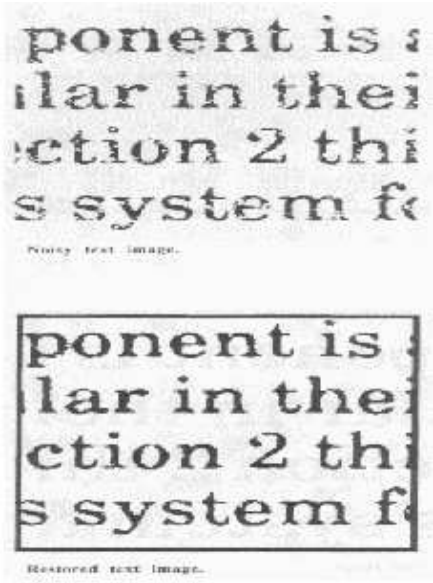






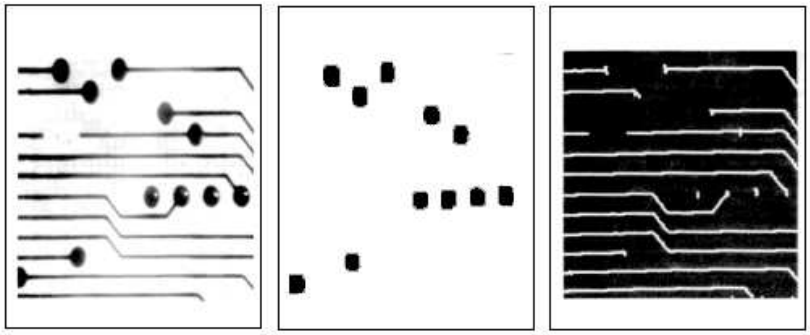
MORPHOLOGIE MATHÉMATIQUE
EXEMPLES (3)

Restauration de documents imprimés



$A \circ B \bullet B$

Segmentation de Circuit imprimés

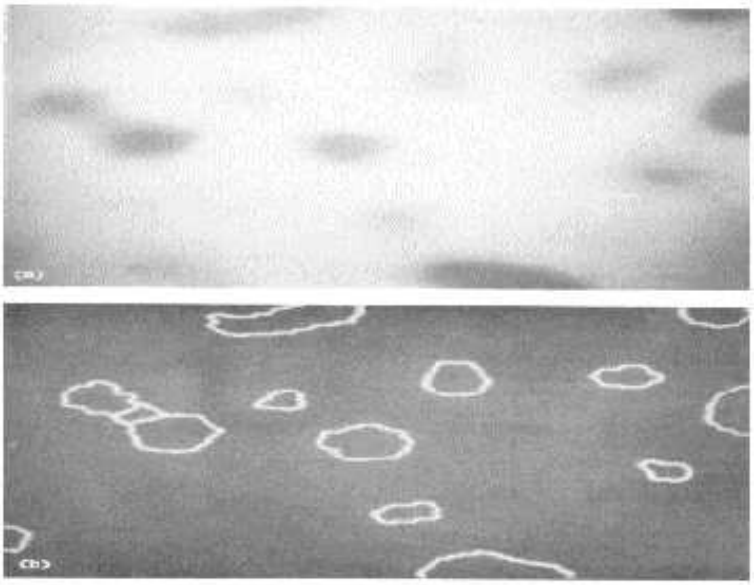


Circuit Imprimé $A \circ B$ Squelette

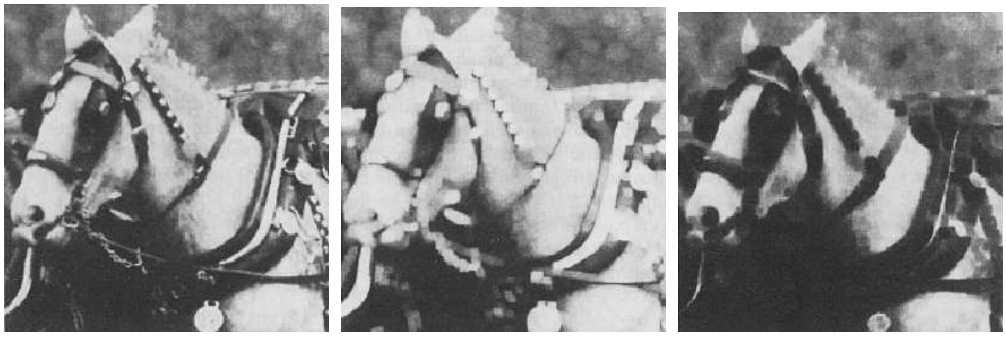
MORPHOLOGIE MATHÉMATIQUE

EXEMPLES (4)

Segmentation



$A \bigoplus B - A \ominus B$



Original $A \bigoplus B$ $A \ominus B$

MORPHOLOGIE MATHÉMATIQUE
EXEMPLES (5)

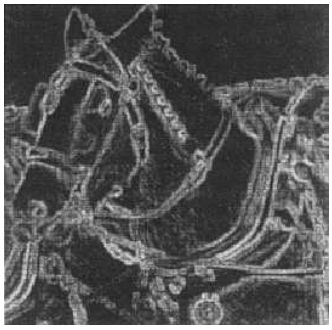
Original

$A \circ B$

$A \bullet B$

Original

$A \circ B \bullet B$



$A \bigoplus B - A \ominus B$