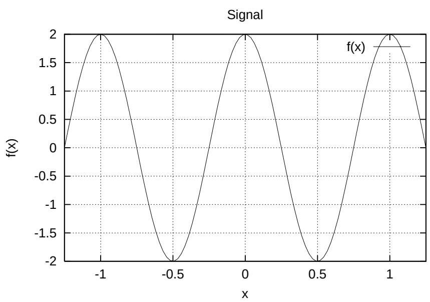
TRANSFORMÉE DE FOURIER

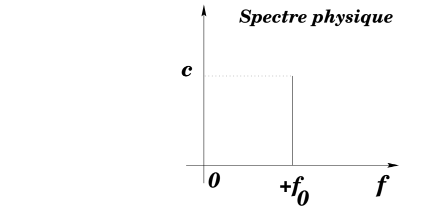
NOTION DE SPECTRE & SÉRIE DE FOURIER (1)

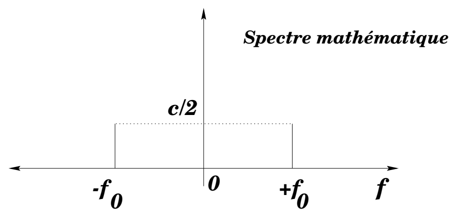
NOTION DE SPECTRE

- Signal sinusoïdal de fréquence f_0 ▶ $f(x) = c\cos(2\pi f_0 x)$
  (c est l'amplitude du signal et $T = 1/f_0$, sa période)

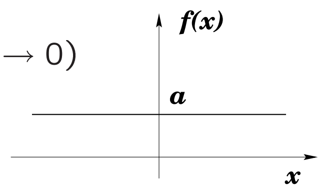
Notation complexe ▶ $f(x) = \frac{c}{2}\big(\exp(2\pi j f_0 x) + \exp(-2\pi j f_0 x)\big)$

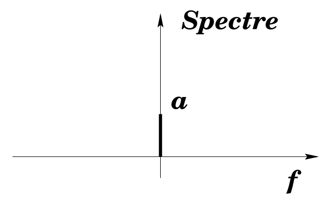




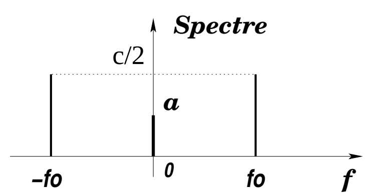


- Si $T \rightarrow \infty$, $(f_0 \rightarrow 0)$





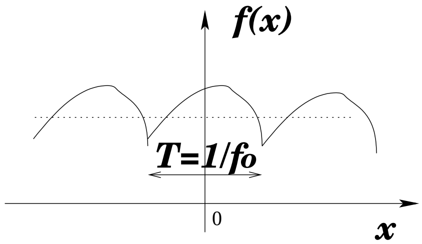
- $f(x) = a + \frac{c}{2}\big(\exp(2\pi j f_0 x) + \exp(-2\pi j f_0 x)\big)$

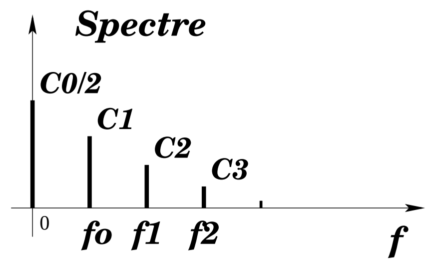


TRANSFORMÉE DE FOURIER
NOTION DE SPECTRE & SÉRIE DE FOURIER (2)

SÉRIE DE FOURIER

Soit $f(x)$, un signal périodique de période $T = 1/f_0$





$$
\begin{aligned}
f(x) &= \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos(2\pi n f_0 x) + \sum_{n=1}^{\infty} B_n \sin(2\pi n f_0 x) \\
A_n &= \frac{2}{T}\int_0^T f(x)\cos(2\pi n f_0 x)\,dx \quad \text{n>0} \\
B_n &= \frac{2}{T}\int_0^T f(x)\sin(2\pi n f_0 x)\,dx \quad \text{n>0}
\end{aligned}
$$

$$
\Longleftrightarrow f(x) = \sum_{n=1}^{\infty} C_n \cos(2\pi n f_0 x + \theta_n) + C_0/2
$$

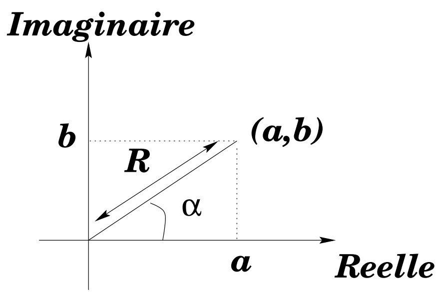
$$
\text{où} \quad C_n = \sqrt{A_n^2 + B_n^2} \quad \text{et} \quad \theta_n = \arctan\left(\frac{B_n}{An}\right)
$$

Spectre de raies

- $C_0/2$ ◀▶ Niveau moyen du signal
- f_0 ◀▶ Le fondamental
- nf_0 ◀▶ Les harmoniques

TRANSFORMÉE DE FOURIER

NOMBRES COMPLEXES (RAPPEL)



$$(a,b) \blacktriangleleft\blacktriangleright\ a + jb\ \blacktriangleleft\blacktriangleright\ R\cos\alpha + jR\sin\alpha$$

$$\boxed{\textbf{Notation} : \exp(j\alpha) = \cos\alpha + j\sin\alpha, \quad j^2 = -1}$$

$$a + jb = R\cos\alpha + jR\sin\alpha = R\exp(j\alpha)$$

$$\begin{array}{rcll} R & = & \sqrt{a^2+b^2} & \text{Amplitude} \\ \alpha & = & \arctan(b/a) & \text{Phase} \end{array}$$

Propriétés

$$\begin{array}{rl} a + jb = R\exp(j\alpha) \quad a - jb = R\exp(-j\alpha) & \text{Conjugué} \\ R_1\exp(j\theta) * R_2\exp(j\alpha) = R_1R_2\exp\big(j(\theta+\alpha)\big) & \text{Multiplication} \\ \frac{\mathrm{d}}{\mathrm{d}\alpha}\big(R\exp(j\alpha)\big) = jR\exp(j\alpha) & \text{Dérivée} \end{array}$$

Notations Complexes des Fonction sinus et cosinus

$$\begin{array}{rcl} \cos x & = & \frac{1}{2}\big(\exp(jx) + \exp(-jx)\big) \\ \sin x & = & \frac{-j}{2}\big(\exp(jx) - \exp(-jx)\big) \end{array}$$

TRANSFORMÉE DE FOURIER

CONVOLUTION 1D -FONCTIONS CONTINUES-

$$\boxed{(f * g)(x) = \int_{-\infty}^{+\infty} f(t)g(x-t)\,dt}$$