

$$du dv = \begin{vmatrix} \frac{\partial u}{\partial w} & \frac{\partial u}{\partial \theta} \\ \frac{\partial v}{\partial w} & \frac{\partial v}{\partial \theta} \end{vmatrix} dw d\theta$$

$$= \underbrace{\begin{vmatrix} \cos \theta & -w \sin \theta \\ \sin \theta & w \cos \theta \end{vmatrix}}_{|w|} dw d\theta = |w| dw d\theta$$