

An Energy Based Model For The Image Edge Histogram Specification Problem

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Abstract—In this correspondence, we present an original energy-based model that achieves the edge histogram specification of a real input image and that thus extends the exact specification method of the image luminance (or grey level) distribution recently proposed by Coltuc et al. Our edge histogram specification approach is stated as an optimization problem in which each edge of a real input image will tend iteratively towards some specified gradient magnitude values given by a target edge distribution (or a normalized edge histogram possibly estimated from a target image). To this end, a hybrid optimization scheme combining a global and deterministic conjugate gradient-based procedure and a local stochastic search using the Metropolis criterion is herein proposed to find a reliable solution to our energy-based model. Experimental results are presented and several applications follow from this procedure.

Index Terms—Conjugate gradient, edge histogram specification, energy based model, gradient magnitude, local stochastic search, Metropolis algorithm.

I. INTRODUCTION

Image histogram, by its ability to represent the intensity levels distribution of the image pixels, remains a useful and popular statistical tool that enables information about the visual appearance of an image to be quickly and easily obtained and/or histogram-based features (such as the mode, mean, variance, entropy, energy, kurtosis, etc.) widely used in region-based image segmentation, indexing or local enhancement techniques to be computed. Amongst the classical algorithms exploiting this intensity level distribution, histogram specification (also called histogram matching) refers to a class of image transforms which changes the histogram of a given image to another desired one. It is an important and well-known technique that can be used, for example, to watermark an image [1], enhance the contrast in only some specific regions (of interest) of the image (by modifying the dynamic range of the pixel values) [2]–[4] or to normalize two images (e.g., for fusion, mosaicing, registration, etc.).

Although the histogram specification algorithm has exact solution for continuous image (thus yielding to a perfect match between the input and the desired intensity level distribution), it is generally an ill-posed problem that does not admit an exact solution in the discrete case. For example, in the case where the output distribution is uniform, the resulted histogram after

specification (or so-called *equalization*) is flattened but may be far from being uniform. This comes from the fact that since the number of pixels is usually much larger than the number of intensity levels, there are many pixels with the same intensity level and these latter cannot be separated (they can only be merged together) in order to approximate the different bins of a uniform histogram [2]. It has been finally realized that a key to achieve a discrete exact histogram specification method was to find a strict ordering relation separating each pixel of the original image with the same intensity into several subsets (in order to approximate the different bins of the desired output distribution). Practically speaking, let I be a discrete image with L grey-levels and $N \times M$ pixels $I(x_i, y_j)$ with coordinates (x_i, y_j) representing the discrete pixel locations. Let also $H = \{h_0, h_1, \dots, h_{L-1}\}$ be the non-normalized target histogram (i.e., the desired output intensity level distribution) and let \prec be a strict ordering relation on the set of pixels of I , defined as $I(x_1, y_1) \prec I(x_2, y_2)$ if the grey-level (or intensity value) of pixel $I(x_1, y_1)$ is lower or equal than the grey-level of pixel $I(x_2, y_2)$ with respect to the lexicographic order. Then the exact specification simply proceeds as follows [5] [Algorithm A]:

- Order pixels: $I(x_1, y_1) \prec I(x_2, y_2) \prec \dots \prec I(x_{NM}, y_{NM})$
- Split this pixel ordering relation from left to right in L groups, such as group j has h_j pixels.
- For all the pixels in a group j , assign gray-level j .

In this context, the structure of the image is thus distorted by enforcing the target histogram and it yields exact results if a strict ordering relation is found. In practice, several ordering relation strategies can be used. The simplest one consist of pre-processing the original image by adding a small amount of uniform noise to each pixel intensity value [6], [7] or separating randomly each pixel of the original image with the same intensity level [8]–[10]. Another solution, avoiding noise, consists of separating pixels of the same-intensity group either according to their local mean on the four horizontal and vertical neighbors [11] or to the average intensity (of the surrounding pixels) at their location [5] or finally by taking into account not only the local mean intensity but also local edge information [12] *via* a wavelet transform (which preserves edge information and produces sharper image enhancement results compared to the classical local mean model [5], [11]).

Edges are also important features of an image because they contain significant information; indeed, edges may correspond to object boundaries or to changes in surface orientation, discontinuities in depth or material properties to name a

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few. Edges also help to extract useful information and characteristics of an image. For example, edge-based features of shape and texture are important for image retrieval and indexing. Consequently edge histogram may be important to obtain information either about the visual appearance of an image (coarse or highly detailed image, structure in the image spatial configuration, spatial resolution, spatial detail statistics and fractal dimension of an image²) or its content (naturally uneven or perfectly geometrically shaped) or man-made objects. In the light of the discussion above, it is fair to think that edge histogram specification of an image may be of interest for several computer vision and image processing applications. If the statistical distribution of the intensity value of any real images varies, the statistical distribution of edges or the gradient magnitude of an image follows a (well-known in the denoising community [14]) long-tail distribution mathematically expressed by a two-parameter density function of the form $H(z) \propto \exp(-|z/c|^p)$. This is due to the intrinsic stationary property of real-world images, containing smooth areas interspersed with occasional sharp transitions, i.e., edges. The smooth regions produce small amplitude gradient magnitudes and the transitions produce sparse large-amplitude gradient magnitudes [14]. Due to this intrinsic stationary property of any real-world images, the edge histogram will be associated with a decreasing function with a unique mode (the value that occurs the most frequently) at (amplitude gradient magnitude) 0, corresponding to the numerous smooth regions existing in any real-world images. Except for this property, different informative distributions (for different parameter positive values of p and c) can be found or specified for a given input image.

In this correspondence, an approach for edge histogram specification of a real image is proposed. This approach combines the ordering relation described above but applied to the set of the gradient magnitude values of an input image (and related to each pair of pixels separated from a given distance). It allows us first to obtain the set of increasing gradient magnitudes of an input image and then to assign to each of them a specified gradient magnitude value given by a target edge distribution (or a normalized edge histogram possibly estimated from a target image). A hybrid optimization scheme combining a global and deterministic conjugate gradient-based procedure and a local stochastic search allow each pair of pixel values to tend (iteratively) towards these specified gradient magnitude levels. The remainder of this correspondence is organized as follows: Sections II and III describe respectively the proposed model and the optimization strategy. Finally, Section IV presents the set of experimental results and applications of this edge histogram specification method.

II. PROPOSED MODEL

Let us first consider the case of an edge histogram specification procedure in the first order sense, i.e., using the absolute value of the gradient magnitude with the first order

¹The fractal dimension of an image surface corresponds to the human perception of image roughness [13].

derivative. To this end, let I be an input discrete image with $N \times M$ pixels I_s located at discrete locations $s = (x_s, y_s)$. Our edge histogram specification procedure aims at finding a new luminance mapping \hat{I} in which, each $|\hat{I}_s - \hat{I}_t|$ in this input image with pair of sites (s, t) separated by a distance $d = \max\{|x_s - x_t|, |y_s - y_t|\} = 1$ pixel (i.e., with the site t located in the first nearest 8 neighbors of s) is considered as a independent random variable whose distribution follows a target distribution or a normalized histogram H with a desired shape (possibly estimated from a target image). If this mapping \hat{I} is estimated in the minimal mean square sense, then \hat{I} is the solution image that should minimize the following objective function $E(I)$:

$$\hat{I} = \arg \min_I \underbrace{\sum_{s=1}^{NM} \sum_{t \in \mathcal{N}_s^1} \left(\beta_{[1]s,t}^2 - (I_s - I_t)^2 \right)^2}_{E(I)} \quad (1)$$

where \mathcal{N}_s^1 represents the 8 nearest neighbors of s and consequently the summation is over all the pair of sites (i.e., for all sites s and for all the pair of sites including s with t belonging to the 8 nearest neighbors of s). In this case, $\beta_{[1]s,t}$ are the values given by an edge histogram specification method (of the first order magnitude gradient) with the non-normalized target distribution $H = \{h_0, h_1, \dots, h_{Z-1}\}$ (possibly *a priori* imposed or estimated from a target image) with Z its number of bins. Practically speaking, let $W = 8 \cdot N \cdot M$ be the number of absolute values of the first order difference $|I_s - I_t|$ in the original image and let \prec be a strict ordering relation, defined among the $|I_s - I_t|$, (as $|I_s - I_t| \prec |I_u - I_v|$ if the first order difference $|I_s - I_t|$ is lower or equal than the first order difference $|I_u - I_v|$ with respect to the lexicographic order), our edge histogram specification histogram method is thus a two-step procedure which proceeds as follows [Algorithm B]³

- 1. Ordering relation

- Normalize H in order that $W = \sum_{k=0}^{Z-1} h_k$
- Order the $W = 8 \cdot N \cdot M$ pairwise pixel absolute differences: $|I_s - I_t| \prec |I_u - I_v| \prec \dots \prec |I_x - I_y|$
- Split this pixel absolute difference ordering relation from left to right in Z groups, such as group j has h_j elements, i.e., h_j couples of pixels.
- For all pairs of pixels or pair of sites (s, t) whose the absolute difference is in a group j , assign $\beta_{[1]s,t} = j$.

- 2. Optimization : Optimize (1) (see Sect. III)

This model can easily be generalized in order to ensure an edge specification histogram in the $n = 2$ (for example)

²Algorithm B corresponds to the cases where the input and target images have integer luminance values ranging from $[0 : 255]$ and we also consider $Z = 256$ bins for the luminance histogram and for the target histogram of the absolute value of the first-order difference (first-order gradient magnitude). Consequently, if the target image has the same size of the input image, h_j is necessarily a natural number (ranging from $[0 : 8NM]$). Nevertheless, if the target image has not the same size of the input image, each value h_j of the distribution H , after the first step of Algorithm B (i.e., the step ensuring that this histogram integrates to $8 \times N \times M$) must be rounded to the nearest integer and this then ensures that h_j remains a natural number. Let us also note that after rounding up to the nearest integer, the new h_j don't add up W but this is not a problem in practice.

order sense (i.e., with a gradient magnitude using the second order derivative). To this end, the summation of (1) should be all the pair of sites (s, t) with $t \in \mathcal{N}_s^2$ and \mathcal{N}_s^2 designating the 16-pixel-neighborhood of s separated by a distance $d = 2$ pixels ($d = \max\{|x_s - x_t|, |y_s - y_t|\}$) and $W = 16 \cdot N \cdot M$ pairs of pixels or $\beta_{[2] s,t}$ values. In the same way, this model can easily be generalized in order to ensure simultaneously an edge specification histogram following n_t different distributions for respectively the set of n_t gradient magnitudes in the n_t order senses. To this end, let $H_{[n_t]}$ be the vector associated to the n_t non-normalized target distributions $H_{[l]} = \{h_{[l],0}, h_{[l],1}, \dots, h_{[l],Z-1}\}$ with $l \in [0 \dots n_t[$ (possibly *a priori* imposed or estimated from a target image), let \mathcal{N}_s^l represents the $W_{[l]}$ neighbors of s separated by a distance $d = l$ pixels, the procedure will commence as follows [Algorithm C]:

- **1. Specification with ordering relation**
 - For $l = 1$ to n_t
 - * $W_{[l]} = 8 \cdot l \cdot N \cdot M$
 - * Normalize $H_{[l]}$ such that $W_{[l]} = \sum_{k=0}^{Z-1} h_{[l],k}$
 - * Order the $W_{[l]}$ pairwise pixel absolute differences: $|I_s - I_t| \prec |I_u - I_v| \prec \dots \prec |I_x - I_y|$
 - * Split this pixel absolute difference ordering relation from left to right in Z groups, such as group j has $h_{[l],j}$ elements, i.e., $h_{[l],j}$ couples of pixels.
 - * For all pairs of pixels or sites (s, t) whose absolute difference is in a group j , assign $\beta_{[l] s,t} = j$.
- **2. Optimization** : Optimize (1) for $t \in \mathcal{N}_s^1 \cup \mathcal{N}_s^2 \cup \dots \cup \mathcal{N}_s^{n_t}$ (see Sect. III)

Finally, this model can easily be generalized in order to ensure simultaneously n_t edge histogram specifications (following n_t given different distributions) and an exact histogram specification of the luminance (or intensity) level. The method [Algorithm D] simply consists in alternating Algorithm C and Algorithm A until a stability criterion is reached (i.e., the output image does not change too much between two iterations). We would like to add that extending our approach to color images is straightforward:

- In the case where the input image is specified directly from a target distribution law, it consists first of representing the input image (originally expressed in the RGB color space) in a color space where one coordinate is intensity or luminance value, such as the perceptual LAB color space and processing only on the Luminance value. After treatment, let \hat{L} be the output (edge-specified) luminance map, it then continues by converting back the \hat{LAB} into the classical RGB color space.
- In the case where the input image is specified from a target image for which we want to keep its color palette, there are two different ways:
 - 1) either the histogram of the components L , A and B of the input image is specified (Algorithm A) from the components L , A and B of the target image
 - 2) or as proposed in [5], one has to define a strict ordering relation among color image pixels and a possible solution is to use the luminance or the gray value for that. In this case, the color histogram specification procedure proceeds as follows:
 - Order color pixels of the input image (I) from their

luminance or grey value:

$$I(x_1, y_1) \prec I(x_2, y_2) \prec \dots \prec I(x_{NM}, y_{NM})$$

- Order color pixels of the target image (T) from their luminance or grey value:

$$T(u_1, v_1) \prec T(u_2, v_2) \prec \dots \prec T(u_{NM}, v_{NM})$$

Note that if the size of the target image is different from the size of the input image, an up-sampling or a sub-sampling procedure should be used.

- Assign to $I(x_s, y_s)$ the color value $T(u_s, v_s)$ for all $s < NM$.

Since two luminance values can be identical for different color values, the first strategy thus seems to be more appropriate if we want to preserve the different hues of an image to be specified in the color histogram sense. Nevertheless, the second strategy seems also well suited if the target image has a dominant hue as is the case in a texture transfer procedure such as that presented in Section IV-C.

III. OPTIMIZATION STRATEGY

The objective function to be minimized E may be more or less complex according both to the shape of the target edge distribution and the edge structure of the input image (i.e., the edge distribution shape of the input image). This cost function may be sometimes nearly convex if the two edge histograms are close or very complex with several local *extrema*, if the shape of the two edge histograms are different or if one of these two edge histograms exhibits some discontinuities or an unusual shape (i.e., a shape far away from the classical density function of the form $H(z) \propto \exp(-|z/c|^p)$ put forward by Simoncelli *et al.* in [14]). In order to ensure a good minimization and thus an accurate edge specification process in all cases, we have proposed the following hybrid and adaptive optimization strategy:

- Since an analytical expression of the derivative of this function E to be optimized is easily available we first use a conjugate gradient procedure initialized with the input original image. For the conjugate gradient, the step size is fixed to γ and adaptively decreased by a factor of two if the energy to be minimized increases between two iterations. We stop the optimization procedure if a fixed number of iterations (I_{\max}^D) or the convergence is reached.
- In order to refine the estimation given by the above-mentioned deterministic optimization method, we use the previous optimization result as the initialization of a stochastic local search. To this end we use a local exploration around the current solution using the Metropolis criteria [15] and a small radius of exploration (see Algorithm 1).

After this hybrid optimization procedure, it is possible that a local minimum of the energy function E is reached (in this case, at convergence, $E > 0$ and practically speaking, the histogram to be specified is yet far from being the target histogram; the value of E being proportional to this distance). In order to avoid being trapped in a local minimum and to be closer to the global minimum, a strategy (that was empirically tested and relatively efficient) consists of alternating the specification procedure (ordering relation) and this hybrid optimization method until a given criterion is

reached such as when the value of the energy cost function E and/or the similarity between the target and output edge histograms (e.g., estimated by a Bhattacharya distance) is not too high. Let us note that a global minimum is not ensured by this strategy. For certain images, the image structure and its properties do not allow a perfect match between the input and the desired distribution (i.e., thus inducing an error $E \neq 0$) to be arrived at every time. Consequently, our strategy that consists of alternating the ordering relation and the proposed hybrid optimization method has to be stopped when a maximal number of iterations is reached.

Algorithm 1	
Local exploration with Metropolis	
E	Energy function to be minimized
T_l	Temperature at Iteration step l
a	Cooling schedule parameter
r	Radius of exploration, real $\in]0, 1[$
T_0	Initial Temperature
T_f	Final Temperature
L_{\max}^S	Maximal number of iterations
1. Initialization $a \leftarrow \left(\frac{T_f}{T_0}\right)^{\frac{1}{L_{\max}^S}}$	
2. Local Exploration	
while $l < L_{\max}^S$ do	
for each pixel with value x_s at site s do	
<ul style="list-style-type: none"> • Compute $\Delta\text{Energy} = E(y_s) - E(x_s)$ with $y_s \in [x_s - r : x_s + r]$ and y_s (pixel value) $\in [0.0 : 1.0]$ • If ($\Delta\text{Energy} < 0$) Replace x_s by y_s • Else Replace x_s by y_s with probability $\triangleright \exp\left(-\frac{\Delta\text{Energy}}{T_l}\right)$ 	
$l \leftarrow l + 1$ and $T_l \leftarrow T_0 a^l$	

IV. EXPERIMENTAL RESULTS

A. Set Up

In all the experiments, the input image is assumed to be toroidal (i.e., wrapping around at the borders; [this property only simplifies the implementation, but we can also replicate the border pixels or use a different strategy]) with colors or luminance values or magnitude gradient ranging from $[0.0 : 1.0]$. We have used 256 bins for the histogram of the luminance values and for the histogram of the magnitude gradient.

For the conjugate gradient, the step is set to $\gamma = 0.5$. The maximal number of iterations is set to $L_{\max}^D = 20$. For the local exploration search, using the Metropolis criteria, the initial temperature and the final temperature are respectively set to $T_0 = 3 \cdot 10^{-5}$ and $T_f = 5 \cdot 10^{-10}$. The radius of exploration is $r = 0.04$ and the maximal number of iterations is set to $L_{\max}^S = 100^3$. Finally, in order to obtain a final edge specified image which will be close enough to a reliable solution, we have respectively set $E_{\min} = 0.1$, $D_{B_{\max}} = 0.1$ and $L_{\max}^H = 6$.

⁴Due to the small radius of exploration, the computational complexity of this optimization step (in fact a simple local search around the gradient estimation) is considerably reduced and this explains why a low number of iterations is herein performed.

B. Edge Histogram Specification

Our initial experiment with Algorithm B was with a target distribution (for the edge histogram using the first order derivative) with a desired shape. For this experiment, it is worth recalling that the set of possible shapes for the edge histogram of an image (see Sect. I) are the set of decreasing functions with a mode at (amplitude gradient magnitude) 0 (due to the numerous smooth regions which necessarily exist in any real-world images and that induce, statistically and more generally, an original edge histogram with a density function of the form $H(z) \propto \exp(-|z/c|^p)$ [14]). We have thus considered the following three uni-modal (at 0) decreasing (envelope) distributions (\mathcal{H} denoting the Heaviside step function).

- 1) First, the semi-Gaussian function:

$$H_{\text{Target}}(z) = \frac{1}{Z_h} \exp(-20z)^2 \mathcal{H}(z)$$

- 2) Second, the semi-triangle function:

$$H_{\text{Target}}(z) = \frac{1}{Z_h} (1 - 3z) \mathcal{H}(1 - 3z) \mathcal{H}(z)$$

- 3) Third, the (non unimodal at 0) shifted Gaussian function:

$$H_{\text{Target}}(z) = \mathcal{N}(256 * z; \text{mean} = 0.1, \text{var} = 0.0001)$$

- 4) Fourth, a decreasing exponential function:

$$H_{\text{Target}}(z) = \frac{1}{Z_h} \exp(-8z) \mathcal{H}(z)$$

with Z_h , a normalizing factor ensuring that these functions integrate to one (these above-mentioned target distributions are graphically shown at bottom right of Fig. 1). The validation and the efficiency of our algorithm is then achieved qualitatively by visually comparing the output and the desired edge histogram shapes and quantitatively by estimating the Bhattacharya distance (ranging from 0 to 1):

$$D_B[H_{\text{Target}}(z), H(z)] = \left(1 - \sum_{z=0}^{Z-1} \sqrt{H_{\text{Target}}(z) H(z)}\right)^{1/2} \quad (2)$$

between the two (normalized) edge histograms before and after the specification process. Fig. 1 (and Fig. 3) shows the obtained results. We can notice that our edge histogram specification procedure is not *exact* since the output histogram shape is not a perfect match with the target histogram shape. This may derive from the fact that the edge image structure could not be geometrically more distorted in order to better match the desired target edge histogram (or equivalently, the gradient descent procedure has reached the global minimum of the energy function E and $E \neq 0$ in this case). Another possibility is that the gradient procedure is stuck in a local minimum. Nevertheless, in all tested cases, the estimation of the Bhattacharya distance shows us that the similarity between these two histogram shapes noticeably increases (or the Bhattacharya distance decreases) after our edge specification process except for the non-uni-modal at 0 shifted Gaussian distribution for which the output histogram remains far away from the target distribution and the Bhattacharya distance remains high ($D_B = 0.544$). We can notice that the resulting output images are, in these three cases, visually different with different edge statistic properties (and this will also be confirmed in the following experiments).



Fig. 1. Algorithm B. **Edge histogram specification procedure with a target distribution model**. From left to right, the original input image, the four edge histogram specification results and at bottom right, the four output edge histograms (with the target and original histogram superimposed on the output edge histogram). The Bhattacharya distance D_B is respectively for these four experiments 0.312, 0.257, 0.783 and 0.264 before the edge specification process and 0.091, 0.061, 0.544 and 0.190 after the edge specification process.



Fig. 2. Algorithm B. **Detail enhancement and detail exaggeration procedure** on the input image shown at top and bottom left. From top to bottom. Cathedral (Notre Dame, Lyon, France) and Statue images (Berkeley database) and results for two different decreasing values of the Bhattacharya distance (respectively $0.85 \times D_{B_{init}}$ and $0.78 \times D_{B_{init}}$, as stopping criterion of Algorithm B with H_{target} being the uniform distribution), namely; Cathedral image $D_{B_{init}} = 0.57$ (original image), $D_B = 0.48$ and $D_B = 0.44$. Statue image $D_{B_{init}} = 0.54$ (original image), $D_B = 0.44$ and $D_B = 0.41$.

Our edge specification process may also be efficiently used for detail enhancement of an input image or even as an original detail exaggeration procedure that goes much further than the results usually obtained with classical high-boost



Fig. 3. Algorithm B. **Edge histogram specification procedure with a target distribution model.** From left to right, magnified regions from Fig. 1 and 2.

filters for which artifacts due to the noise amplification of the high pass filter (in the case of high value of the boosting parameter) quickly appear and may degrade the image quality. In our case, this detail exaggeration procedure simply consists of the use of Algorithm B in order to realize an edge histogram equalization technique (i.e., by considering a target distribution H_{Target} simply equal to the uniform distribution). Our iterative minimization-based edge histogram specification procedure then will aim at flattening, as much as possible, the gradient magnitude distribution of the input image. In this latter procedure, the desired level of detail in the output image can also be easily controlled, for example, by estimating, at each iteration, the Bhattacharya distance between the output and the desired uniform distribution and simply by stopping our iterative procedure when this parameter reaches a given similarity value: in this supervised procedure, the desired level of detail in the output image will increase as the user increases the value for this, namely the Bhattacharya distance based similarity measure between the output edge histogram and the uniform edge distribution. Fig. 2 and Fig. 3.[f-g] show the obtained results for two different increasing values of this above-mentioned Bhattacharya (similarity) value as stopping criterion.

Our procedure of edge histogram specification may also be used in order to render an input image with different detail levels or more generally into a specified number of separate levels of detail depths. This rendering is possible if one specifies the output edge histogram with a multimodal (edge) distribution. This allows us to render an image with different classes of edge magnitude values or to enhance a specified class of detail. Fig. 4 shows the obtained image results for respectively one, two and three different classes of detail accuracy levels (thus by specifying the output edge histogram to be respectively uni-modal, bimodal and three modal).

C. Specification of Multiple Edge Histograms

Our edge histogram specification model can also be used to somewhat eliminate an effect of unequal resolution (i.e., loss of accuracy, contrast or details) possibly created by a blurring degradation (such as a motion or focal blur) between

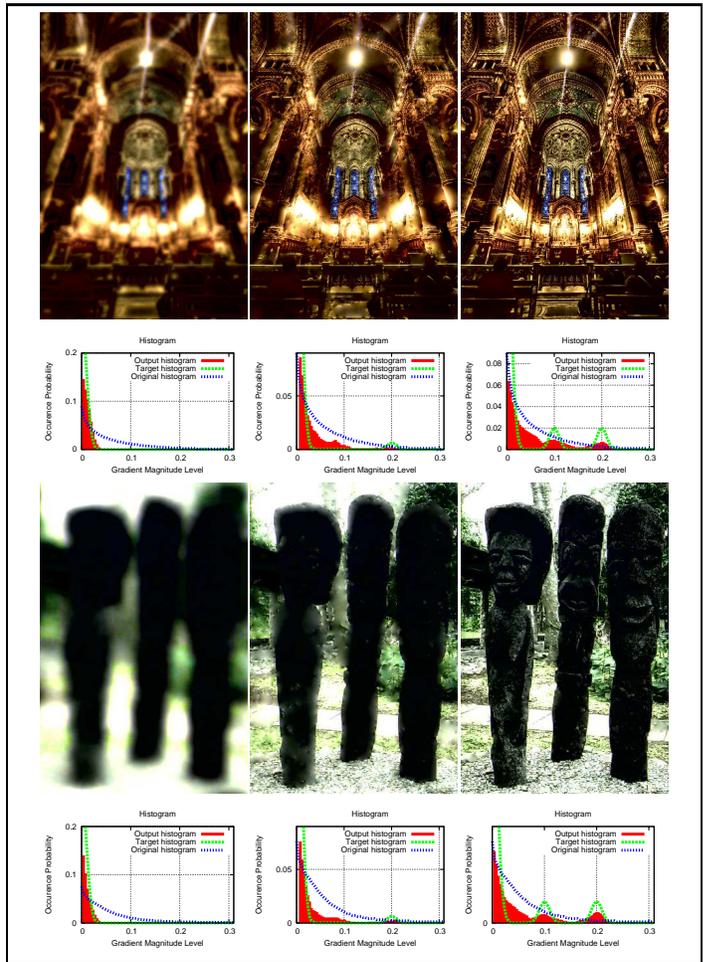


Fig. 4. Algorithm B. **Image rendering procedure with different classes of detail accuracy levels.** From top to bottom and left to right, images and edge histogram specification results (with the target histogram superimposed on the output edge histogram) for respectively one, two and three different classes of detail accuracy levels (by specifying the output edge histogram to be respectively uni-modal, bimodal and three modal). The original images are shown at Figures 1 and 2.

two images of (possibly) the same scene. This correction can be useful in order to normalize an image set (e.g., for mosaicing generation, fusion, registration, lighting correction, indexing, retrieval systems or other applications). Fig. 5 shows different views and icons of the cathedral church of Notre-Dame-de-Fourviere (Lyon, France) taken by different cameras, at different times (thus with different resolution levels and color palettes). One of these images is the cathedral image already used in the preceding experiments and considered, in this test, as the target high resolution color image on which we desire to normalize the other images in the color and resolution degree senses. The results of our edge and color histogram specification method (Algorithm D with $n_t = 2$, i.e. in the two first order senses and exploiting the first algorithm, presented at the end of Sect. II, to ensure a specification of the color histogram) on the three original images are shown in Fig. 5.

The proposed edge specification method can also be used to transform one input image into another with different edge geometric and textural properties. To this end, we have applied the Algorithm D on a portrait image exploiting the three first

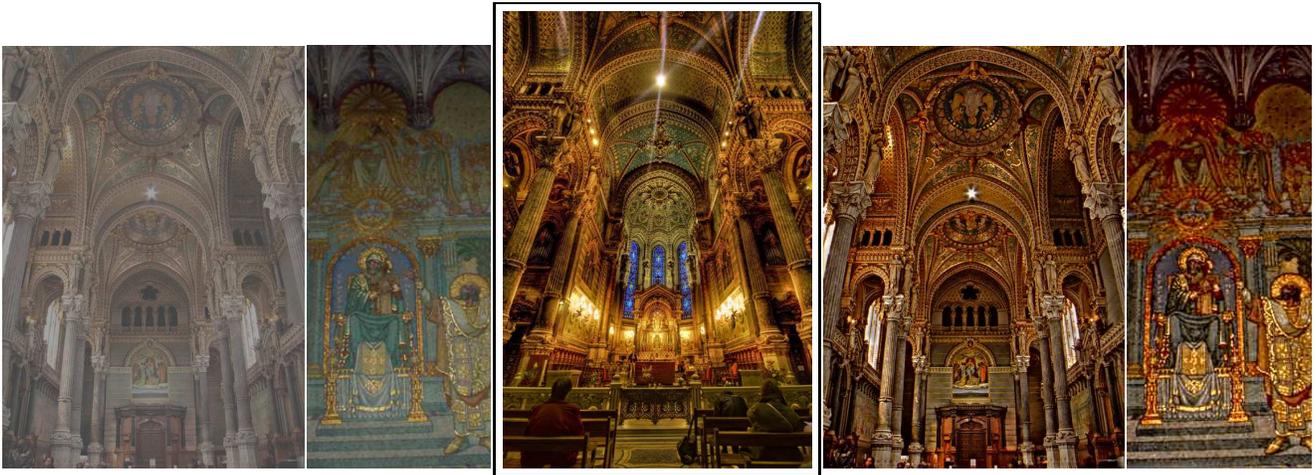


Fig. 5. Algorithm D. **Image (resolution and color) normalization procedure.** From left to right, two original images and target image, and output edge and color normalized result images obtained by our algorithm (with $n_t = 2$).

edge distributions (i.e., $n_t = 3$ in Algorithm D along with the second specification method for color histogram) estimated from a target image representing a certain drawing style. The resulting images are shown in Fig. 6.

We have compared our n_t -edge and color histogram specification method (Algorithm D) with a classical method that exploits only color information. Fig. 7 shows for a magnified region of the cathedral image (shown at Fig. 5a), that a single color histogram specification strategy (first algorithm of Sect. II, i.e., the same as that used in our algorithm D) does not allow to get an output image with the same statistical edge geometric properties and level of detail of the target image (which is more detailed than the original image). This “detail level specification” can also be quantified with the Bhattacharya distance D_B (between the edge histograms of the output and the target image) which is respectively 0.310, 0.058 and 0.534 for the original image (i.e., before any specification method), after our edge and color histogram specification method and after a classical color histogram specification. For our algorithm, the similarity of the edge histogram shapes of the resulting and target image thus noticeably increases; demonstrating that our algorithm allows to transfer, not only the color information but also the edge geometric properties of the target image (more precisely the n_t shapes of its edge distributions). Another consequence of our algorithm is that it does not distribute the different colors of the target image in the same way of our edge and color histogram specification method, since our algorithm D seems to find a compromise between a similarity between the distribution of color levels and also the distribution of gradient magnitudes of the target image. These remarks can also be confirmed in the case of a texture transfer technique only using a single color histogram specification strategy (second algorithm of Sect. II), which do not allow to copy the edge textural property of a given drawing style. This is particularly visible in the case of the pointillist style transfer technique for which its edge distributions are specific and far away from those of a natural image.

D. Sensitivity to Internal Parameters

- First, it is worth mentioning that our algorithm is relatively insensitive to high values of the step size γ because of our adaptive decreasing schedule which adaptively adjusts, and reduces this value in the conjugate gradient procedure if this parameter is set mistakenly too high.
- Second, it is also worth mentioning that our overall minimization procedure is relatively insensitive to the three parameters L_{\max}^D , L_{\max}^S and L_{\max}^H , related to the different number of iterations of the minimization procedures, since the final stopping criterion (E_{\min} and $D_{\mathcal{B}_{\min}}$) will ultimately check if the final solution is close enough to a reliable solution.
- Third, $E_{\min} = 0.1$ and $D_{\mathcal{B}_{\max}} = 0.1$ (except for Algorithm B, used as a detail enhancement or exaggeration procedure, for which $D_{\mathcal{B}_{\max}}$ has to be set by the user) must not be considered as two internal parameters of our algorithm but rather as a criterion (for example, required by the schedule of conditions) for the expected estimation accuracy of the final result.
- Fourth, T_f is easily findable in our case, since a good final temperature for a simulated annealing-like minimization procedure has to ensure that, at the end of the stochastic search, very few sites change their luminance values between two complete image sweeps. In our algorithm, this parameter has been easily found after a few trials. We have found that $T_f = 5 \cdot 10^{-10}$ was appropriate for all the experiments presented in this correspondence.
- Finally, two internal parameters are sensitive and crucial for our algorithm, namely the radius of exploration r and, in a least measure, the starting temperature T_0 of the local stochastic search. The first one was set in order to locally explore a solution whose luminance values are close to the initial solution given by the gradient minimization procedure (a value $r = 0.04$ ensures that the final solution will exhibit output luminance values, centered around the gradient estimation $\pm 0.04 * 255 = \pm 10$ luminance values for a final luminance image whose luminance values are



Fig. 6. Algorithm D. transfer procedure of the edge textural properties belonging to an image to another. From top to bottom and left to right. Original image and a set of pairs of images including a drawing style (respectively, the ink painting, sanguine, pointillist and painting style) and the obtained transfer result with Algorithm D (with $n_t = 3$).

comprised in $[0 : 255]$. T_0 is set in order to ensure that, at the beginning of the stochastic search, approximately 50% of sites change their luminance values between two complete image sweeps.

E. Algorithm

The computational times of our procedure vary greatly depending on the shape of the input and target edge histograms (i.e., between 10 and 300 seconds) for an AMD Athlon 64 Processor 3500+, 2.2 GHz, 2010.17 bogomips and non-optimized code running on Linux. Besides, it must be noted than our energy minimization can be efficiently implemented by using the parallel abilities of a graphic processor unit (GPU) (embedded on most graphics hardware currently available on the market) and can be greatly accelerated (up to a factor of 200) with a standard NVIDIA®GPU (2004) as indicated in [16]. Source code (in C++ language) and pseudo-code of our algorithm with the set of original and presented images (and some additional images) are publicly available at the following http address www.iro.umontreal.ca/~mignotte/ResearchMaterial/obehs in order to make possible eventual comparisons with future algorithms and visual comparisons.

V. CONCLUSION

In this correspondence, we have presented an original edge histogram specification model. Our approach is based both on a strict ordering relation between each pair of pixels (existing in the input image and separated by a given distance) followed by a hybrid optimization process (i.e., a deterministic global gradient followed by a stochastic local search) especially

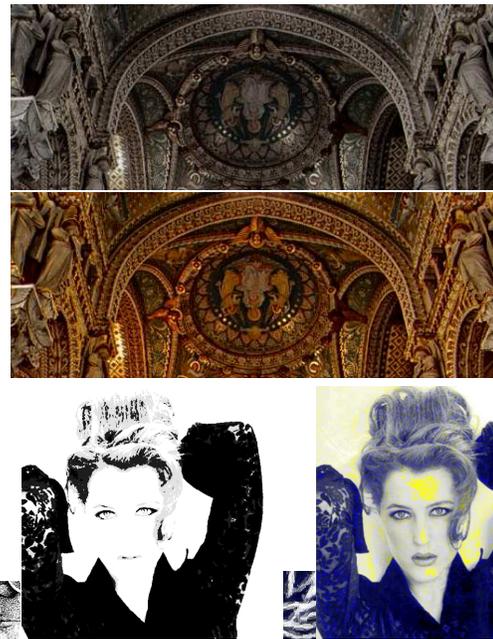


Fig. 7. From top to bottom: magnified region of the image shown at Fig. 5a for a single color histogram specification strategy and our edge and color histogram specification method. Texture transfer technique using the input image shown at top of Fig. 6 and a drawing style and exploiting only a single color histogram specification strategy (to be compared to the results shown in Fig. 6, last row).

well suited to our energy based edge histogram specification model. Concretely, this energy based model iteratively and geometrically distorts the edge structure of the input image during the minimization process, in order to transform its edge histogram, as much as possible, to another desired edge histogram. Several applications of this model, such as a detail exaggeration procedure, an edge high-boost or enhancement filter and a texture transfer technique, have been presented and discussed.

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