# A Multi-Objective Decision Making Approach for Solving the Image Segmentation Fusion Problem

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Abstract-Image segmentation fusion is defined as the set of methods which aim at merging several image segmentations, in a manner that takes full advantage of the complementarity of each one. Previous relevant researches in this field have been impeded by the difficulty in identifying an appropriate single segmentation fusion criterion, providing the best possible, i.e., the more informative, result of fusion. In this paper, we propose a new model of image segmentation fusion based on multi-objective optimization which can mitigate this problem, to obtain a final improved result of segmentation. Our fusion framework incorporates the dominance concept in order to efficiently combine and optimize two complementary segmentation criteria, namely, the global consistency error and the F-measure (precision-recall) criterion. To this end, we present a hierarchical and efficient way to optimize the multi-objective consensus energy function related to this fusion model, which exploits a simple and deterministic iterative relaxation strategy combining the different image segments. This step is followed by a decision making task based on the so-called "technique for order performance by similarity to ideal solution". Results obtained on two publicly available databases with manual ground truth segmentations clearly show that our multi-objective energy-based model gives better results than the classical mono-objective one.

*Index Terms*—Color textured image segmentation, combination of multiple segmentations, energy-based model, F-measure (precision-recall) criterion, global consistency error, multi-objective optimization, multi-criteria decision making, segmentation ensemble.

#### I. INTRODUCTION

MAGE segmentation is one of the most crucial components of image processing and pattern recognition system whose aim is to represent the image content into different regions of coherent properties with homogeneous characteristics such as texture, color, movement and boundary continuity [1]. This pre-treatment is crucial because the resulting segments form the basis for the subsequent classification, which may be based on spectral, structural, topological, and/or semantic features [2], [3].

In order to solve the difficult unsupervised segmentation problem, different strategies have been proposed in

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the past [4], [5]. Among them, one can mention the region based segmentation which in fact assumes that neighboring pixels within the same region should have similar values [6] and more precisely segmentation models exploiting directly clustering schemes [7], [8] using Gaussian mixture modeling, fuzzy clustering approaches [9], [10] or fuzzy sets [11], region growing strategies [12], compression models [13], wavelet transform [14] or watershed transformation [15], Bayesian [16], or texton-based approaches [17], graph-based [18]–[20], deformable surfaces [21], or active contour model [22] or genetic algorithm [23] and spectral clustering [24], just to mention a few.

Another line of work has recently become the focus of considerable interest, which suggests that an improved segmentation result can be achieved through the combining of multiple, quickly estimated and weak segmentation maps of the same scene. To the best of our knowledge, Jiang and Zhou [25] was the first to investigate this merging strategy based on a defined criterion, but this approach has suffered from a constraint related to the initial segmentations which should include the same regions number. Afterward, this approach has also been implemented without this restriction, with an arbitrary number of regions [26], [27].

Fusion of segmentation has been extensively studied, in particular with respect to a single criterion. However, an inherent weakness of the mono-criterion based fusion model comes from the facts that, the segmentation is inherently an ill-posed problem related to the large number of possible partitioning solutions for any image, and also, by the fact that a single criterion cannot model all the geometric properties of a segmentation solution or otherwise said, the single criterion optimization process is only dedicated to exploring a subset or a specific region of the search space.

Thus, a key problem with much of the literature on the fusion of segmentation consists in choosing the most appropriate criterion able to generate the best segmentation result. Motivated by the above observations, in this work, we focus on proving that a fusion model of segmentation, expressed as a multi-objective optimization problem, with respect to a combination of different and complementary criteria, is an interesting approach that can overcome the limitations of a single criterion and give a competitive final segmentation result for different images with several distinct texture types. In addition, the proposed strategy can be also viewed as a general framework for combining several *a priori* energy terms in any energy-based models or several prior distributions in a possible Bayesian multi-objective framework.

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The remainder of this paper is organized as follows. In Section II we discuss the literature review concerning the fusion models of segmentations. In Section III we describe our proposed fusion model; we start by introducing basic concepts about multi-objective optimization in the first part of the section, in the second part we define the two criteria used in our model, in the third part we present the multiobjective function relating to this novel fusion framework, in the fourth part we describe the optimization strategy used to minimize our multi-objective function and in the fifth part we outline the decision making method adopted for the selection of the best solution from an ensemble of non-dominated solutions. In Section IV we describe the generation of the segmentation set to be combined by our model. In Section V we illustrate a set of experimental results and comparisons with existing segmentation algorithms. In this section, our strategy of segmentation is validated on two publicly available databases. Finally, we conclude the paper in Section VI.

# II. LITERATURE REVIEW

In the literature, there are several examples of new fusion algorithms, which all solve the segmentation problem based on a single criterion. Here we only give a brief review of some popular criteria.

One of the first implementation of the fusion of regionbased segmentations of the same scene was carried out by Mignotte [26], who proposed the merging of the initial input segmentations in the within-cluster variance sense, since the obtained segmentation result was achieved by exploiting a fusion scheme based on K-means algorithm. This fusion framework remains simple and fast, however, the final segmentation result closely depends on the distance choice and the value of K used in the final K-means based fusion procedure. Following this strategy, we can also mention the fusion model suggested by Harrabi and Braiek [28], which adopted the same approach, but for the set of local *soft* labels estimated with a multilevel thresholding scheme and for which the fusion procedure is thus provided in the sense of the weighted within-cluster inertia, with the same disadvantages of the previous method while requiring more computational time for estimating the mass functions of the information's to be combined.

Another widely used criterion is the Rand index [29] (RI) which was first used in [30], with the idea of evidence accumulation in a hierarchical agglomerative clustering model, for combining the results of multiple conventional clusterings. This RI measure of agreement can be also used in the case of two segmentations, by encoding the set of constraints, in terms of pairs of pixel labels (identical or not), achieved by each of the segmentations to be fused. This idea has been first proposed in [27] with a random walking stochastic approach and associated with an estimator based on mutual information to estimate the optimal regions number, and later by Ghosh *et al.* [31] with an algebraic optimization based fusion model using non-negative matrix factorization. The penalized version of the RI criterion has also been used in [32], by adding a global constraint on the fusion process, which

restricts the size and the number of the regions, within a Markovian framework and an analytical optimization method and by Alush and Goldberger [33] exploiting a constrained version of this RI criterion by an expectation maximization (EM) algorithm applied on super-pixels preliminary provided by an over-segmentation process. The main drawback of the Rand Index criterion is due to its quadratic complexity in terms of data set size since it uses all pairs of pixel, and in terms of algorithm complexity of the fusion model.

Fusion of segmentation maps can also be accomplished with the entropy, or more precisely in the variation of information (VoI) sense [34] with an energy-based model optimized by exploiting an iterative steepest local energy descent strategy combined with a connectivity constraint. This criterion is interesting but some studies have shown than it is less correlated with human segmentation in term of visual perception compartively to the RI or the least square or withincluster inertia criterion. It is also important to mention the fusion scheme proposed by Ceamanos et al. [35], which is based on the maximum-margin hyperplane sense and in which the hyperspectral image is segmented according to the decision fusion of multiple and individual support vector machine classifiers that are trained in different feature subspaces emerging from a single hyperspectral data set. Similarly, Song and Li [36] presented a recent Bayesian fusion procedure for satellite image segmentation, in which class labels obtained from different segmentation maps are fused by the weights of an evidence model which estimates each final class label with the maximum logit posterior odd. Recently, Khelifi and Mignotte [37] proposed the fusion of multiple segmentation maps according to the global consistency criterion (GCE). In this metric sense, which measures the extent to which one segmentation map can be viewed as a refinement of another segmentation, a perfect correspondence is obtained if each region in one of the segmentation is a subset or geometrically similar to a region in the other segmentation.

It is important to mention, that all these above-described studies treat the image segmentation fusion problem with a single criterion. However, the major problem of the monocriterion based fusion model comes from the fact that, the segmentation is inherently an ill-posed problem related to the large number of possible partitioning solutions for any image, and also, that a single criterion cannot model all the geometric properties of a segmentation solution or otherwise said, the single criterion optimization process is only dedicated to exploring a subset or a specific region of the search space.

The fusion model outlined in this work is called multiobjective optimization based-fusion model (MOBFM). The motivation of using multi-objective optimization is to design a new segmentation fusion model that takes advantage of the complementarity of different objectives to achieve a final better segmentation. Besides, in order to better constrain and to improve the optimization process, we resort to the iterative conditional modes (ICM) algorithm applied on pre-estimated super-pixel to be labeled. To this end, we have incorporated, in the ICM-based optimization strategy, the dominance concept in order to combine and optimize different segmentation criteria; namely the (region-based) global consistency error (GCE) criterion and the (contour-based) F-measure (precision-recall) criterion. This strategy allows us to find a consensus segmentation resulting from the fusion of different and complementary criteria to enhance the quality of the final segmentation result.

# **III. PROPOSED FUSION MODEL**

# A. Multi-Objective Optimization

In this work, we take advantage of the multi-objective optimization concept, also called vector optimization or multicriteria optimization [38], [39], by regarding the segmentation problem from different points of view, in terms of different, complementary or contradictory criteria to be simultaneously satisfied with aim of achieving a better segmentation result.

As shown in the preliminary work [40], a mono-objective approach aims to optimize a single objective function with respect to a set of parameters. Otherwise, in the multi-objective case, there are several, often conflicting objectives to be simultaneously maximized or minimized [41]. Mathematically, in the case of minimization, the problem is generally formulated as follows:

$$\min \overline{f}(\overrightarrow{x}) \text{ (k functions to be optimized)}$$
s.t  $\overrightarrow{g}(\overrightarrow{x}) \leq 0$ 
 $\overrightarrow{h}(\overrightarrow{x}) = 0$ 

$$(1)$$

where  $\vec{x} \in \Re^n$ ,  $\vec{f}(\vec{x}) \in \Re^k$ ,  $\vec{g}(\vec{x}) \in \Re^m$ ,  $\vec{h}(\vec{x}) \in \Re^p$ . Note that the vectors  $\vec{g}(\vec{x})$  and  $\vec{h}(\vec{x})$  describe, respectively, *m* inequality constraints and *p* equality constraints. This set of constraints delimits a restricted subspace to be searched for the optimal solution [42]. In our case the number of functions *k* to be optimized is equal to 2 and without any inequality or equality constraints (i.e., m = 0 and p = 0).

The resolution of this problem consists of minimizing or maximizing these k objective functions without degradation of the optimal values obtained comparing with those obtained from a mono-objective optimization achieved objective by objective. Generally, approaches solving this problem are divided into three popular classes or types [42]. The first is the scalarization approach, also known as the weightedsum; according to this approach, a multi-objective problem is solved by assigning a numerical weight to each objective and combining its multiple objectives by adding all weighted criteria into a single composite function [43]. In addition to the scalarization technique, another alternative approach is the progressive preference technique. Here, the user refines his choice of the compromise during the progress of the optimization. A further important approach, which is increasingly used, includes a posteriori preference method. Thus, instead of transforming a multi-objective problem into a monoobjective problem, we can define a dominance relationship, where the overarching goal is to find the best compromise between objectives. Hence, several dominance relationships have already been presented, but the most famous and the most commonly used is the Pareto dominance, called also the Pareto Approach (PTA). This domination concept that will be used in our study is defined by:

Definition 1: The solution  $x^{(i)} \in S$  dominates another solution  $x^{(i)} \in S$ , denoted  $x^{(i)} \prec x^{(j)}$  (in case of minimization),

Fig. 1. Pareto frontier of a multi-objective problem in case of a minimization.

if and only if:  $f_l(x^{(i)}) \le f_l(x^{(j)})$  for all  $l \in \{1, 2, ..., k\}$  and,  $f_l(x^{(i)}) < f_l(x^{(j)})$  for some  $l \in \{1, 2, ..., k\}$ .

where S denotes the search space and  $f_l(.)$  represents the *l*-th objective function. In Fig. 1, we present the Pareto frontier (i.e., the set of solutions that dominate all other solutions) of a multi-objective problem in case of minimization.

# B. Segmentation Criteria

1) The F-Measure Criterion: The F-measure is, a combination of two complementary measures; precision and recall, which are commonly used by information retrieval theorists and practitioners [44]. In the contour-based image segmentation case, these two scores represent, respectively, the fraction of detections of the true boundaries and the fraction of true boundaries detected [45]. On the one hand, a low precision value is typically the result of over-segmentation<sup>1</sup> and indicates that a large number of boundary pixels have poor localization. On the other hand, the recall measure is low when there is significant under-segmentation<sup>1</sup>, or when there is a failure to capture the salient image structure.

Mathematically, let  $S_{\rm T} = \{R_{\rm T}^1, R_{\rm T}^2, \ldots, R_{\rm T}^{Nb_T}\}$  &  $S_{\rm M} = \{R_{\rm M}^1, R_{\rm M}^2, \ldots, R_{\rm M}^{Nb_M}\}$  represent, respectively, the segmentation test result to be measured and the manually segmented image with  $Nb_T$  being the number of segments or regions (R) in  $S_{\rm T}$  and  $Nb_M$  the number of regions in  $S_{\rm M}$ . Let us now suppose that  $B(R_{\rm T})$  denotes the set of pixels that belongs to the boundary of the segment  $R_{\rm T}$  in the segmentation  $S_{\rm T}$  and let us also consider that  $B(R_{\rm M})$  is the ensemble of pixels belonging to the boundary of the segments  $R_{\rm M}$  in the ground truth segmentation  $S_{\rm M}$ . The precision (Pr) and recall (Re) are then respectively defined as follows:

$$Pr = \frac{|B(R_{\rm T}) \cap B(R_{\rm M})|}{|B(R_{\rm T})|}, \quad Re = \frac{|B(R_{\rm T}) \cap B(R_{\rm M})|}{|B(R_{\rm M})|}$$
(2)

Here,  $\cap$  represents the intersection operator and |X| denotes the cardinality of the set of pixel X. While the precision assesses the amount of noise in the output of a detector, the recall evaluates the amount of ground-truth detected. An interesting measure that considers both the precision and the recall is called the F-measure. This combined measure



<sup>&</sup>lt;sup>1</sup>In the over-segmentation: An object is partitioned into multiple regions after the segmentation and in the under-segmentation case: multiple objects are presented by a single region after the segmentation process [46].



Fig. 2. Four images from the BSDS300 and their ground truth boundaries. The images shown in the last column are obtained by our MOBFM fusion model.

aims to estimate a compromise between these two quantities and a specific application can determine a trade-off  $\alpha$  between these two measures, describing the harmony between Pr and Re [47]. Then, the F-measure between the segmentations  $S_{\rm T}$ and  $S_{\rm M}$  can be evaluated as follows:

$$F_{\alpha}(S_{\mathrm{T}}, S_{\mathrm{M}}) = \frac{Pr \times Re}{\alpha \times Re + (1 - \alpha) \times Pr} \quad \text{with } \alpha \in [0, 1] \quad (3)$$

Where the  $F_{\alpha}$  is in the interval of [0, 1], and the value of 1 proves that similar edges exists between the two segmentations, on the contrary, a value of 0 indicate the opposite situation.

2) The GCE Criterion: The global consistency error (GCE) [48] computed the extent to which one regionbased segmentation map can be viewed as a refinement of another segmentation. This segmentation error measure is particularly useful in evaluating the agreement of a segmentation machine with a given ground truth segmentation (see Fig. 2) since different experts can segment an image at different levels of details.

Formally, let *n* be the number of pixels in the image and let  $S_{\rm T} = \{R_{\rm T}^1, R_{\rm T}^2, \ldots, R_{\rm T}^{Nb_T}\}$  &  $S_{\rm M} = \{R_{\rm M}^1, R_{\rm M}^2, \ldots, R^{Nb_M}\}$  be, respectively, the segmentation test result to be measured and the manually segmented image and  $Nb_T$  being the number of segments or regions (*R*) in  $S_{\rm T}$  and  $Nb_M$  the number of regions in  $S_{\rm M}$ . Let now  $p_i$  be a particular pixel and the couple  $(R_{\rm T}^{< p_i >}, R_{\rm M}^{< p_i >})$  be the two segments including this pixel (respectively in  $S_{\rm T}$  and  $S_{\rm M}$ ). The local refinement error (LRE) can be computed at pixel  $p_i$  as:

$$LRE(S_{T}, S_{M}, p_{i}) = \frac{|R_{T}^{< p_{i} >} \setminus R_{M}^{< p_{i} >}|}{|R_{T}^{< p_{i} >}|}$$
(4)

where  $\setminus$  represents the operator of difference and |R| denotes the cardinality of the set of pixels *R*. Thus, a measure of 0 expresses that the pixel is practically included in the refinement area, and an error of 1 means that the two regions overlap in an inconsistent manner [48].

As it has been reported in [48], the major drawback of this segmentation measure, is that it encodes a measure of refinement in only one direction, i.e, not symmetric. To solve this issue, an interesting and straightforward way is to combine the LRE at each pixel into a measure for the whole image and for each sense. The combining result is the so-called global consistency error (GCE), which forces all local refinement to be in the same direction; in this manner, every pixel  $p_i$  must be computed twice, once in each sense, in the following manner:

$$GCE(S_{T}, S_{M}) = \frac{1}{n} \left\{ \sum_{i=1}^{n} LRE(S_{T}, S_{M}, p_{i}) + \sum_{i=1}^{n} LRE(S_{M}, S_{T}, p_{i}) \right\}$$
(5)

with this above representation, there is still considerable ambiguity, since we can find two degenerate segmentation cases; one pixel per region and one region per image giving a GCE value equal to 0. To avoid these two problems, we can propose the new measure GCE<sup>\*</sup> as follows [40]:

Since the GCE<sup> $\star$ </sup> ranges in the interval of [0, 1], the GCE<sup> $\star$ </sup> reaches its best value at 0, this value expresses a perfect match between the two segmentations to be compared. However, it reaches the worst value at 1, this value represents a maximum difference between the two segmentations.

#### C. Multi-Objective Function Based-Fusion Model

Suppose now that we have a family of *J* segmentations  $\{S_j\}_{j \leq J} = \{S_1, S_2, \ldots, S_J\}$  associated with a same scene to be combined for providing a final improved segmentation result, and let also  $S_I$  be a selected segmentation map belonging to the set  $\{S_j\}_{j \leq J}$ . The two complementary criteria; namely the contour-based F-measure and the region-based GCE measure (see section III-B), can be used directly, as cost functions,



Fig. 3. A set of initial segmentations and the final fusion result achieved by MOBFM algorithm. From top to bottom; Four first rows; *K*-means clustering results for the segmentation model detailed in Section IV. Fifth row: Natural image from the BSDS500 and final segmentation map resulting of our fusion algorithm.

in an energy-based model. In this context, the consensus segmentation is simply obtained from the solution of the following multi-optimization problem:

$$\text{MOBJ}(S_{I}, \{S_{j}\}_{j \leq J}) = \begin{cases} \arg \max \overline{F}_{\alpha}(S_{I}, \{S_{j}\}_{j \leq J}) \\ \bigcap \\ \arg \min \overline{\text{GCE}}^{\star}(S_{I}, \{S_{j}\}_{j \leq J}) \end{cases}$$
(7)

with  $\overline{X}(S_I, \{S_j\}_{j \leq J}) = \frac{1}{J} \sum_{j=1}^{J} X(S_I, S_j)$ . To improve the accuracy of our segmentation result, we have made a modification in the multi-objective function (as proposed in [45]), by weighting the importance of each segmentation of  $\{S_j\}_{j \leq J}$ . This strategy allows us to penalize outliers and consequently aims to increase the robustness of our fusion model. So, we have weighted the first member (F-measure criterion), by a coefficient  $z_j$  proportional to its mean F-measure  $\overline{F}_{\alpha}(S_I, \{S_j\}_{j \leq J})$ . This coefficient is defined as:

$$z_j = \frac{1}{H} \exp\left(\frac{\overline{F}_a(S_I, \{S_j\}_{j \le J})}{d}\right)$$
(8)

where *d* is a parameter controlling the decay of the weights, and *H* is a normalizing constant ensuring  $\sum_j z_j = J$ . This modification allows us to ensure the robustness of our model when facing a possible bad segmentation map belonging to  $\{S_j\}_{j \leq J}$  far away from the fused segmentation result. In addition, for the second member (GCE criterion), we have added a regularization term, allowing the incorporation of knowledge concerning the types of resulting fused segmentation, *a priori* defined as acceptable solutions. This term is defined as:

$$T_{\text{Reg}}(S_j) = \left| -\sum_{k=1}^{Nb_j} \left[ \frac{|R_j^k|}{n} \log \frac{|R_j^k|}{n} \right] - \overline{\mathcal{Q}} \right|$$
(9)

with  $S_j = \{R_j^k\}_{k \le Nb_j}$  and  $Nb_j$  is the number of regions in the segmentation map  $S_j$  and where  $\overline{Q}$  is an internal parameter

of our regularization term that represents the mean entropy of the *a priori* defined acceptable segmentation solutions. Thus, if the current segmentation solution has an entropy lower than  $\overline{Q}$ , this  $T_{Reg}$  term favors splitting. On the contrary, if the current segmentation solution has an entropy greater than  $\overline{Q}$ ,  $T_{Reg}$  favors merging. Also, we have added a parameter  $\gamma$ to allow for weighting the relative contribution of the region splitting/merging term. Finally, with these two modifications in the multi-objective function, a penalized likelihood solution of our fusion model is thus given by the resolution of this following function:

$$MOBJ(S_{I}, \{S_{j}\}_{j \leq J}) = \begin{cases} \arg \max \left\{ \overline{F}_{\alpha} \left( S_{I}, \{z_{j}\}, \{S_{j}\}_{j \leq J} \right) \right\} \\ \bigcap \\ \arg \min \left\{ \overline{GCE}^{\star} \left( S_{I}, \{S_{j}\}_{j \leq J} \right) + \gamma \ T_{Reg}(S_{I}) \right\} \end{cases}$$
(10)

## D. Optimization Algorithm of the Fusion Model

In our work, the fusion model of multiple segmentations in the bi-criteria sense (F-measure and GCE) is presented as a multi-objective optimization problem with a complex energy function. To solve this consensus function, several optimization algorithms can be efficiently used, such as the stochastic simulated annealing or the genetic algorithms, which are both insensitive to initialization and are guaranteed to find the optimal solution but with the drawback of a huge computational load. Another alternative is to perform the optimization step by an iterative conditional modes (ICM) proposed by Besag [49], i.e.; a Gauss-Seidel relaxation where pixels (superpixels<sup>2</sup> in our hierarchical approach) are updated one at a time. This iterative search technique is simple and deterministic, however, it can converge towards a bad local minima in case of an initialization by the segmentation map far from the optimal one. To solve this problem, we can choose for the first iteration of the optimization procedure, among the J segmentation to be combined, the one ensuring the minimal consensus energy of our fusion model, in the  $\overline{\text{GCE}}_{\gamma}^{\star}$  sense. This segmentation map  $\hat{S}_{\overline{cev}}^{\star[0]}$  can be defined as:

$$\hat{S}_{\overline{\text{GCE}}_{\gamma}^{\star}}^{[0]} = \arg\min_{S \in \{S_j\}_{j \le J}} \overline{\text{GCE}}_{\gamma}^{\star} \left(S_I, \{S_j\}_{j \le J}\right)$$
(11)

In the mono-objective case, the ICM aims to accept a new solution for each pixel if this one is better than the current solution or decreases the energy function. On the contrary, in our multi-objective case, this iterative algorithm amounts to simultaneously obtain, for each (super)-pixel to be labeled, the minimum value of  $\overline{\text{GCE}}_{\nu}^{\star}$  and the maximum value of  $\overline{F}_{\alpha}$ . For this purpose, we have incorporated into the ICM a domination function (defined in section III-A); Concretely, in each iteration, the modified ICM practically accepts a new solution to enter on the list of non-dominated solutions  $(L_{NDS})$  only if this one is not dominated by any other solution contained in this  $L_{NDS}$  list and then updates the  $L_{NDS}$  by deleting solutions dominated by the new solution. Afterward, when the maximum number of iterations  $(T_{max})$  is attained (and/or a sufficient number of solutions have been explored) and that no more non-dominated solution can not be found, the algorithm stops in a Pareto local optimum, and this set of non-dominated solutions is then given as input to TOPSIS technique (see Section III-E). Finally, our MOBFM algorithm with the iterative steepest local energy descent strategy and the Pareto domination is presented in pseudo-code in Algorithm 1.

### E. Decision Making With TOPSIS

As soon as the generation of the Pareto frontier has been carried out [i.e., the output of Algorithm 1 (see Fig. 4)], one solution must be chosen, and consequently, we are faced to a multi-criteria decision making (MCDM) problem. To solve this issue we resort to a useful and efficient technique called TOPSIS (technique for order performance by similarity to ideal solution [50]). The TOPSIS technique is based on the selection of the alternative (solution) that is the closest to the ideal solution and the farthest from the negative ideal solution (see Figs. 5 and 6). The ideal solution is the one that maximizes the benefit criterion, i.e., criterion with larger value is better, and minimizes the cost criterion, i.e., criterion with smaller value is better, on the contrary, the negative ideal solution minimizes the benefit criterion and maximizes the cost criterion [51]. Let us note that these two ideal and negativeideal solutions are, in fact, two virtual solutions or two virtual 2D points in the cost-benefit criterion space of the set of the non-dominated solutions since they are not associated with a non-dominated segmentation. Nevertheless, these two virtual solutions will be exploited by the TOPSIS technique in order to find the optimal solution according to this multi-criteria decision strategy.

A	lgorithm	1	MO	-Based	Fusion	N	lode	l a	Igorit	thm
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Mathematical n	otation:
$\underline{GCE}_{\gamma}$	Penalized mean GCE
$F_{\alpha}$	Mean F-Measure
$\{S_j\}_{j\leq J}$	Set of $J$ segmentations to be fused
$\{z_j\}_{j\leq J}$	Set of weights
$\{b_i\}$	Set of superpixels $\in \{S_i\}_{i \leq J}$
È	Set of region labels in $\{S_i\}_{i \leq J}$
$L_{NDS}$	List of non-dominated segmentations
1100	(Pareto set of solutions)
$S_{I}$	Solution $\in L_{NDS}$
$T_{-}$	Maximal number of iterations $(=11)$
⊥ max	Regularization parameter
α α	E-Measure compromise parameter
Innuts (C)	r-measure compromise parameter
<b>Input:</b> $\{S_j\}_{j \leq J}$	
A. Initializati	on:
1:	$S_I^{[0]} \longleftarrow \arg\min_{S \in \{S_i\}_{i < I}} \overline{\operatorname{GCE}}_{\gamma}^{\star}(S, \{S_j\}_{j \leq J})$
B. Steepest L	ocal Energy Descent:
2: while $p < T_{\text{max}}$	
3: for each $b_1$	superpixel $\in \{S_j\}_{j \leq J}$ do
4: Draw a	new label x according to the uniform distribution
in the s	$\mathcal{E}$
5: Let $S_I^{\mu}$	the new segmentation map including $b_j$ with
the reg	ion label x
6: Compu	te $\overline{\operatorname{GCE}}^{\star}_{\gamma}(S_I^{[p],\operatorname{new}}, \{S_j\}_{j \leq J})$
7: Compu	te $\overline{\mathbf{F}}_{\alpha}(S_{I}^{[p],\text{new}}, \{z_{i}\}, \{S_{i}\}_{i \leq J})$
8: <b>if</b> $S_{\tau}^{[\hat{p}],i}$	<sup>ww</sup> dominates $S_{1}^{[p]}$ (see Definition 1) then
۰. <b>if</b>	$\nexists$ $S_L \in L_{NDG}$ in which $S_L$ dominates $S_{L}^{[p],new}$ then
10:	$\frac{1}{GCE^*} \leftarrow \frac{1}{GCE^{*,new}}$
10.	$\overline{\mathbf{E}}$ $\langle \overline{\mathbf{E}}^{\text{new}} \rangle$
11:	$\Gamma_{\alpha} \leftarrow \Gamma_{\alpha}$ C[p] = C[p], new
12:	$S_{I}^{*} \leftarrow S_{I}^{*}$
13:	Update $L_{NDS}$ (see Algorithm 2)
14: en	<b>a</b> II $\alpha^{[p],\text{new}}$ <b>a</b> $\alpha^{[p]}$ <b>b</b> $\alpha^{[p]}$
15: else II	$S_I^{uv}$ not dominates $S_I^{uv}$ and $S_I^{uv}$ not dominates
$S_I^{(p),m}$	then [w]
16: <b>if</b>	$\nexists S_L \in L_{NDS}$ in which $S_L$ dominates $S_I^{[p],\text{new}}$ then
17:	Update $L_{NDS}$ (see Algorithm 2)
18: <b>en</b>	d if
19: end if	
20: end for	
21: $p \longleftarrow p +$	1
22: end while	

# Algorithm 2 L<sub>NDS</sub>-Updating Algorithm

Mathematic	al notation:					
$S_I^{[p],\text{new}}$	A new solution generated at iteration number p					
-	(see Algorithm 1)					
$L_{NDS}$	List of non-dominated segmentations (Pareto set					
	of solutions)					
$S_L$	Solution $\in L_{NDS}$					
\	Private operator					
U	Union operator					
Input: $L_{ND}$	$S, S_I^{[p],new}$					
Output: $L_N$						
1: Add the solution $S_{I}^{[p],\text{new}}$ to the list $L_{NDS}$						
$L_{NDS} \longleftarrow L_{NDS} \stackrel{\circ}{\bigcup} S^{[p], {\sf new}}_{J}$						
2: for each solution $S_L \in L_{NDS}$ do						
3: if $S_{L}^{[p],\text{new}}$ dominates $S_{L}$ (see Definition 1) then						
4: Delete the solution $S_L$ from the list $L_{NDS}$						
$L_{NDS} \longleftarrow L_{NDS} \setminus S_L$						
5: end if						
6: end for						

As others have highlighted [52], [53], one of the advantages of this technique is its simple competition process, which allows for solving many real-problems in the research operation field (see paper [53] for more examples). Finally, the TOPSIS

<sup>&</sup>lt;sup>2</sup>Superpixels are given in our application by the set of regions given by each individual segmentations to be combined.



Fig. 4. First row; a natural image ( $n^0$ 176035) from the BSDS500. Second row; the Pareto frontier generated by the MOBFM algorithm (cf. Algorithm 1).



Fig. 5. Graphical representation of TOPSIS (technique for order performance by similarity to ideal solution).

method is described in pseudo-code in Algorithm 3 and its graphical representation is presented in Fig. 5.

#### **IV. SEGMENTATION ENSEMBLE GENERATION**

The initial segmentations used by our fusion framework are simply acquired, in our application, by a *K*-means [54] clustering algorithm, with 12 different color spaces, namely; P1P2, YIQ, HSV, LUV, i123, YCbCr, LAB, TSL, RGB, HSL, h123, XYZ. The class number of the *K*-mean algorithm (K) is computed for each input image of the BSDS300 by using a metric measuring the complexity, in terms of its number of distinct texture classes within the image. This metric, defined in [55] ranges in [0, 1], where a value close to 0 means that we have an image with a low number of texture patterns, and a value close to 1 if we have an image with several different texture types (see Fig. 7). Mathematically, the value of *K* is

## Algorithm 3 TOPSIS Method

Mathen	natical notation:
n	Number of criteria
m	Number of alternatives (solutions)
J	Set of benefit criteria (larger is better)
$J^{'}$	Set of cost criteria (smaller is better)
$W_j$	The relative weight of the j-th criterion,
-	and $\sum_{i=1}^{n} W_i = 1$
$L_{NDS}$	List of non-dominated segmentations
	(Pareto set of solutions)
$S^{bst}$	Best solution (segmentation)
Input:	$L_{NDS}$ (output of Algorithm 1)

Output:  $S^{bst}$ 

- 1: Construct the decision matrix  $X_{ij}$ ; i = 1, 2, ..., m j = 1, 2, ..., n
- 2: Calculate the normalized decision matrix
- (using vector normalization)

$$N_{ij} = \frac{X_{ij}}{\sqrt{\sum_{i=1}^{m} X_{ij}^2}}; i = 1, 2, .., m \ j = 1, 2, .., n$$

3: Calculate the weighted normalized decision matrix (in our case,  $W_1 = 1/3$  and  $W_2 = 2/3$ )

$$W_{ij} = N_{ij} * W_j \; ; \; i = 1, 2, .., m \; \; j = 1, 2, .., m$$

4: Determine the ideal solution  $A^+$  and the negative ideal solution  $A^ A^+ = \{V_1^+, V_2^+, \dots, V^+\}$ 

$$= \{ (max_iV_{ij} \mid j \in J), (min_iV_{ij} \mid j \in J') \}$$
  

$$A^- = \{ V_1^-, V_2^-, ..., V_n^- \}$$
  

$$= \{ (min_iV_{ij} \mid j \in J), (max_iV_{ij} \mid j \in J') \}$$

 5: Calculate the separation measure from the ideal solution(E<sub>i</sub><sup>+</sup>) and the negative ideal solution(E<sub>i</sub><sup>-</sup>) (using Euclidean distance)

$$\begin{array}{l} \text{defan distance)} \\ E_i^+ = \sqrt{\sum_{j=1}^n (V_{ij} - V_j +)^2} \ ; \ i = 1, 2, ..., m \\ E_i^- = \sqrt{\sum_{j=1}^n (V_{ij} - V_j -)^2} \ ; \ i = 1, 2, ..., m \end{array}$$

6: Calculate the relative closeness  $\overline{C_i^*}$  of each alternative to the ideal solution

$$\overline{C_i^*} = \frac{E_i}{E_i^+ + E_i^-} ; \ 0 \le \overline{C_i^*} \le 1$$
  
Choose an alternative with maximum of  $\overline{C_i^*} (S^{bst})$ 

written as:

$$K = \text{floor}\left(\frac{1}{2} + \left[K^{\max} \times \text{ complexity value}\right]\right)$$
(12)

where floor(x) is a function that gives the largest integer less than or equal to x and  $K^{\max}$  is an upper-bound of the number of classes for a very complex natural image. In our framework, we use three different values of  $K^{\max}$ , namely  $K_1^{\max} = 11$ and  $K_2^{\max} = K_1^{\max} - 2$  and  $K_3^{\max} = K_1^{\max} - 8$ . More details about the complexity value of an image are given in [34], but we can mention that the complexity in our case is simply the absolute deviation measure ( $L_1$  norm) of the normalized histograms set or feature vectors for each overlapping, fixedsize squared ( $N_w$ ) neighborhood included within the input image.

Besides the points listed above, as input multidimensional descriptor of feature, we exploited the ensemble of values (estimated around the pixel to be labeled) of the requantized histogram (with equal bins in each color channel). In our framework, this local histogram is re-quantized, for each color channels, in a  $N_b = q_b^3$  bin descriptor, estimated on an overlapping, squared fixed-size ( $N_{uv} = 7$ ) neighborhood, centered around the pixel to be classified with three different seeds for the *K*-means algorithm and with two different values of  $q_b$ , namely  $q_b = 5$  and  $q_b = 4$  for



Fig. 6. The ordered set of solutions, i.e., segmentations, belonging to the Pareto-front; The boxes marked in blue, black and yellow indicate, respectively, the solution which has the minimum  $\overline{\text{GCE}}_{\gamma}^{*}$  score, the solution which has the maximum  $\overline{F}_{\alpha}$  score and the best solution chosen automatically by TOPSIS among these different solutions belonging to the Pareto frontier (cf, Fig. 4).



Fig. 7. Complexity values obtained on five images of the BSDS300 [48]. From left to right, value of complexity = 0.450, 0.581, 0.642, 0.695, 0.796 corresponding to the number of classes (*k*) (with the three different value of  $K^{\text{max}}$ :  $K_1^{\text{max}}$ ,  $K_2^{\text{max}}$  and  $K_3^{\text{max}}$ ) of the *k*-means clustering algorithm respectively to (5, 4, 2), (6, 5, 2), (7, 6, 2), (8, 6, 2), (9, 7, 3) in the *k*-means segmentation model.

a total of  $(3 + 2) \times 12 = 60$  input segmentations to be combined.

# V. EXPERIMENTAL RESULTS AND DISCUSSION

# A. Initial Tests

It is important to recall that the proposed fusion model [see (10)] has been experimented from a segmentation ensemble  $\{S_j\}_{j \le J}$  with J = 60 initial segmentations acquired with the simple *K*-means based procedure, as indicated in Section IV (see Fig. 3). In this case, the convergence properties of our iterative optimization procedure has been tested by considering as initialization of the ICM based iterative steepest local energy descent algorithm, respectively, two blind initializations (image spatially divided by k = 5 rectangles with *k* different labels), the input segmentation which has the J/6 = 10 th minimal (i.e. best)  $\overline{\text{GCE}}_{\gamma}^{\star}$  score, the J/3 = 30 th best score, the worst score, i.e., maximal, and the best score (see Fig. 8). It is clearly that the multi-objective cost function is certainly non-convex and complex with many local minima (see Fig. 9 and Fig. 10). Also, it is worth mentioning that



Fig. 8. Fusion convergence result on six different initializations for the Berkeley image  $n^0247085$ . Left: initialization and Right: result after 11 iterations of our MOBFM fusion model. From top to bottom, the original image, two blind initialization, the input segmentation which have the J/6 = 10 - th best  $\overline{\text{GCE}}_{\gamma}^{\star}$  score, the input segmentation which have the J/2 = 30 - th best  $\overline{\text{GCE}}_{\gamma}^{\star}$  score and the two segmentations which have the worst and the best score  $\overline{\text{GCE}}_{\gamma}^{\star}$ .

the strategy, consisting of initializing the ICM procedure by the segmentation close to the optimal solution in terms of  $\overline{\text{GCE}}_{\gamma}^{\star}$  score, appears as a good initialization strategy that improves the final segmentation result. As a consequence, the combination of using the superpixels of  $\{S_j\}_{j \leq J}$  with a good



Fig. 9. First row; a natural image  $(n^{0}134052)$  from the BSDS300. Second and third row; evolution of the resulting segmentation map (0-th, 1-st, 2-nd, 4-th, 6-th, 8-th, 11-th, 20-th, 40-th, 80-th) (from lexicographic order) along the iterations of the relaxation process starting from a blind initialization.





Fig. 10. First and second row; evolution of the resulting segmentation map (0-th, 1-st, 2-nd, 4-th, 6-th, 8-th, 11-th, 20-th, 40-th, 80-th), from lexicographic order along the iterations of the relaxation process starting from the initial segmentation which have the best  $\overline{\text{GCE}}_{y}^{x}$  score. Third row; evolution of the Mean GCE value and the F-Measure value along iterations.

initialization strategy [see (11)] allows us to ensure the good convergence properties of our fusion model.

# B. Evaluation of the Performance

For an objective comparison with other segmenters, we compare the use of different segmentation algorithms, with or without a fusion model strategy, evaluated on two segmentation datasets; the BSDS300 [48] and the BSDS500 [68]. In addition, to provide a basis of comparison for the MOBFM model, we quantitatively evaluate the performance of the segmentation from two levels, namely, region level with the PRI [56], the GCE [48] and the VoI [57] and boundary level with the BDE [58]. It is important to mention that, in our application, all color images are normalized to have the longest side equal to 320 pixels. The segmentation results are then super-sampled in order to obtain segmentation images with

TABLE I BENCHMARKS ON THE BSDS300. RESULTS FOR DIVERSE SEGMENTATION ALGORITHMS (WITH OR WITHOUT A FUSION MODEL STRATEGY) IN TERMS OF: THE VOI, THE GCE (THE LOWER VALUE IS THE BETTER) AND THE PRI (THE HIGHER VALUE IS THE BETTER) AND A BOUNDARY MEASURE: THE BDE (THE LOWER VALUE IS THE BETTER)

LOWER VALUE IS THE BETTER)

	BSDS300					
	$\mathrm{VoI}\downarrow$	$\text{GCE}\downarrow$	$\mathrm{PRI}\uparrow$	BDE $\downarrow$		
HUMANS	1.10	0.08	0.87	4.99		
With Multi	-Criteria	-Criteria Fusion Model				
MOBFM	1.98	0.20	0.80	8.25		
With Mono	Criterio	n Fusion	Model			
GCEBFM [67]	2.10	0.19	0.80	8.73		
FMBFM [45]	2.01	0.20	0.80	8.49		
PRIF [32]	1.97	0.21	0.80	8.45		
FCR [26]	2.30	0.21	0.79	8.99		
SFSBM [55]	2.21	0.21	0.79	8.87		
With	out Fusi	on Model				
CTM [60]	2.02	0.19	0.76	9.90		
Mean-Shift [61] $(in [60])$	2.48	0.26	0.75	9.70		
$\mathrm{FH}$ [19] (in [60])	2.66	0.19	0.78	9.95		
DGA-AMS [65]	2.03	-	0.79	-		
LSI [64]	-	-	0.80	-		
CRKM [66]	2.35	-	0.75	-		

the original resolution  $(481 \times 321)$  before the estimation of the performance metrics.

1) BSDS300 Tests: The BSDS300 is a dataset of natural images that have been segmented by human observers. It contains 300 natural images divided into a training set of 200 images, and a test set of 100 images. This dataset serves as a benchmark for comparing different segmentation and boundary finding algorithms. First, in terms of region performance measures, the obtained final scores are: GCE = 0.20, VoI = 1.98 (for which a lower value is better) and PRI = 0.80; this value indicates that, on average, 80 %of pairs of pixel labels are correctly labeled in the results of segmentation. It is worth noticing that our segmentation procedure gives a very competitive PRI score compared to the state-of-the-art segmentation methods recently proposed in the literature (see Table I). Fig. 11 outlines, respectively, the distribution of the PRI measure and the number and size of segments provided by our MOBFM algorithm over the BSDS 300. These results show us that the average number of regions estimated by our algorithm is close to the average value given by humans (24 regions) and that the PRI distribution shows us that few segmentations exhibit a bad PRI score even for the most difficult segmentation cases. Second, for the boundary performance measures, our MOBFM model performs well, with a BDE score at 8.25 (see Table I). We can also observe (see Figs. 12 and 13) that the PRI, VoI, BDE and GCE performance measures are better when the number of segmentations to be fused J is high. It can be mentioned from this result that our performance scores are perfectible if the segmentation set is completed by other segmentation maps of the same image.

2) BSDS500 Tests: This new dataset is an extension of the BSDS300. It consists of 500 natural images divided into a training set of 300 images and a test set of 200 images,



Fig. 11. From top to bottom, distribution of the PRI measure, the number and the size of regions over the 300 segmented images of the BSDS300 database.



Fig. 12. Example of fusion results using respectively J = 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60 input segmentations (i.e., 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 color spaces).

and each image was segmented by five different subjects on average. On the BSDS500, in terms of region-based metrics we obtained these following scores; GCE = 0.20, VoI = 2.05and PRI = 0.80. Also, for the boundary performance measure the obtained final score is BDE = 8.05 (see Table II). These results prove the effectiveness and the scalability of our segmentation algorithm against different natural images and segmentation datasets.

# C. Sensitivity to parameters

To ensure the integrity of the evaluation, the internal parameters of our segmentation algorithm, namely  $K_1^{max}$  required for the segmentation ensemble generation (see Section IV), and those required for the fusion step;  $\overline{Q}$  [see (9)],  $\gamma$  [see (10)] and  $\alpha$  [see (3)] was chosen after trial and error

#### TABLE II

BENCHMARKS ON THE BSDS500. RESULTS FOR DIVERSE SEGMENTATION ALGORITHMS (WITH OR WITHOUT A FUSION MODEL STRATEGY) IN TERMS OF: THE VOI, THE GCE (THE LOWER VALUE IS THE BETTER) AND THE PRI (THE HIGHER VALUE IS THE BETTER) AND A BOUNDARY MEASURE: THE BDE (THE LOWER VALUE IS THE BETTER).

	BSDS500				
	$\mathrm{VoI}\downarrow$	$\text{GCE}\downarrow$	$\mathrm{PRI}\uparrow$	BDE $\downarrow$	
HUMANS	1.10	0.08	0.87	4.99	
With Multi	i-Criteria	a Fusion M	Iodel		
MOBFM	2.05	0.20	0.80	8.05	
With Mono	-Criterio	n Fusion	Model		
GCEBFM [67]	2.18	0.20	0.80	8.61	
FMBFM [45]	2.00	0.21	0.80	8.19	
PRIF [32]	2.10	0.21	0.79	8.88	
VOIBFM [34]	1.95	0.21	0.80	9.00	
FCR [26]	2.40	0.22	0.79	8.77	
With	out Fusi	on Model			
$CTM~[60]~{\scriptstyle (in~[62])}$	1.97	-	0.73	-	
Mean-Shift [61] $(in [62])$	2.00	-	0.77	-	
FH [19] (in [62])	2.18	-	0.77	-	
WMS $[63]$ (in [62])	2.10	-	0.75	-	



Fig. 13. From lexicographic order, evolution of the PRI (higher is better) and VoI, GCE, BDE measures (lower is better) as a function of the number of segmentations (J) to be combined for our MOBFM algorithm. More precisely for  $J = 1, 5, 10, 15, 20, \ldots, 60$  segmentations, by considering first, one *K*-mean segmentation and then by considering five segmentations for each color space and  $1, 2, 3, \ldots, 12$  color spaces.

with a grid-type search approach applied on the train image set of the BSDS300 database.

The parameter  $K_1^{max}$  allows to refine the final segmentation map and allows, to a certain extent, to avoid some over-segmented (especially when  $K_1^{max}$  is high) and undersegmented (when  $K_1^{max}$  is low) partition maps results. In order to quantify the influence of parameter  $K_1^{max}$ , we have compared the performance measures obtained with our method using three different values of  $K_1^{max}$  (see Table III). Also, we have tested the role of the parameters  $\alpha$  and  $\overline{Q}$  on the obtained segmentation solutions. Figs. 14 and 15 show clearly that  $\alpha$ and  $\overline{Q}$  efficiently act as two regularization parameters of our fusion model. The parameter  $\alpha$  favors over segmentation for



Fig. 14. Example of segmentation solutions obtained for different values of  $\alpha$ , from top to bottom and left to right,  $\alpha = \{0.55, 0.70, 0.86, 0.99\}$ .



Fig. 15. Example of segmentation solutions obtained for different values of  $\overline{Q}$ , from top to bottom and left to right,  $\overline{Q} = \{0.2, 1, 2, 4.2\}$ .

TABLE III INFLUENCE OF THE VALUE OF PARAMETER  $K_1^{max}$  (Average Performance on the BSDS300)

	BSDS300			
MOBFM $(K_1^{max})$	$\mathrm{VoI}\downarrow$	$\text{GCE}\downarrow$	$\mathrm{PRI}\uparrow$	$\text{BDE}\downarrow$
10	1.95	0.20	0.80	8.21
11	1.98	0.20	0.80	8.25
12	2.03	0.20	0.80	8.19
16	2.28	0.18	0.79	8.42
22	2.42	0.16	0.79	8.77

value close to 0 and merging for value close to 1. Contrary,  $\overline{Q}$  favors under-segmentation, for low value and consequently splitting, for a higher value. In addition, tests show that the fusion method is sensitive to the number of segmentations to be fused (*J*), in the sense that the performance measures are all the more better than *J* is high (see Fig. 13).

Finally, we can notice that  $K_1^{max} = 10$  or 11,  $\overline{Q} = 4.2$ ,  $\gamma = 0.01$  and  $\alpha = 0.86$  is a good set of internal parameters leading to a very good PRI score of 0.80 and a good consensus score for the other metrics (see Table I). Further, it is important to note that we have used the same values of parameters both

with the BSDS300 and BSDS500 and we have found similar values of performance measures. These results show that the parameters required for the fusion step of our algorithm do not depend on the used database and consequently that the proposed fusion model does not overfit and generalizes well. However, as the MOBFM fusion method's performance strongly depends on the level of diversity and complementarity existing in the initial ensemble of segmentations to be fused, this makes necessarily the four internal parameters of the MOBFM method highly sensitive to the pre-segmentation method (used to generate the segmentation ensemble).

# D. Other Results and Discussion

Since the ICM algorithm depends on the choice of the initialization, a good initialization strategy should be used. In this context, we have used an initial segmentation based on  $\overline{\text{GCE}}_{\gamma}^{\star}$  score [see (11)] and we have found that this choice leads to the scores mentioned above. In addition, we have tested our approach with an initialization based on the F-Measure ( $\overline{F}_{\alpha}$ ) with the same internal parameters of our algorithm, and we have found that this strategy leads to the following performance measures: PRI = 0.79, VoI = 1.88, GCE = 0.20 and BDE = 8.62 on the BSDS300; which are slightly less better in terms of PRI and BDE than an initialization based on  $\overline{\text{GCE}}_{\gamma}^{\star}$ .

We can also see, from Table IV, that if we compare the average performances to those provided by using a single criterion, F-measure or GCE, we obtain significantly better performance rate. This shows clearly that our strategy of combining two complementary contour and region-based criteria of segmentation is effective. In order to test the robustness of our fusion approach with a third criterion, we have added to the cost function [see 10] the VoI (variation of information) objective, also used in [34] as the main and unique criterion of fusion of segmentations. This metric estimates the information shared between two partitions by measuring the amount of information that is gained or lost in changing from one clustering to an other [34]. The obtained final scores are; PRI = 0.80, VoI = 1.97, GCE = 0.19 and BDE = 8.35on the BSDS300. These results show some improvements, which can be explained by the addition of this new VoI-based criterion. But, the combination of three objectives makes our algorithm slower, with 6 minutes per image on average, and complexifies the optimization process, indicating that a high number of objectives cause additional challenges [59].

Also, as another strategy whose aim is to reduce the execution time of the algorithm, we have used the dominance function to converge directly to a solution close to the Pareto frontier, by comparing the current solution with new solutions without seeking the Pareto front; this strategy gives us the following results: PRI = 0.80, VoI = 1.99, GCE = 0.20, BDE = 8.37 on the BSDS300 and an execution time equal to 4 minutes on average. For qualitative comparison, we now illustrate an example of segmentation results (see Fig. 16) obtained by our algorithm MOBFM on four images from the BSDS300 compared to other algorithms with or without a fusion model strategy (FCR [26], GCEBFM [37] and



Fig. 16. Example of segmentation results obtained by our algorithm MOBFM on four images from the BSDS300 compared to other algorithms with or without a fusion model strategy (FCR [26], GCEBFM [37] and CTM [60].

CTM [60]). From these qualitative results, we can notice that the strength of our fusion model relies in its ability to provide an appropriate set of segments for any kind of natural images.

Based on the PRI score which seems to be among the most correlated with human segmentation in term of visual perception. The results show that application of the MOBFM on the BDSD300 gives a PRI mean equal to 0.802 and a standard deviation equal to 0.1194, i.e., a significantly better mean performance along with a lower dispersion of score values than the CTM which provides a PRI mean equal to 0.761 and a standard deviation equal to 0.1427. In our case, this leads to a Z score<sup>3</sup> equal to 3.82, meaning that the two sample results are highly significantly different according to the Z-test.

This significance of improvement is also visually and qualitatively confirmed in Fig. 16 where different segmentation results achieved by the CTM algorithm are illustrated and compared with the proposed segmentation method.

To sum up, our fusion method of simple segmentation results based on multi-objective optimization appears to be very competitive for different kinds of performance metrics and thus appears as an interesting alternative to mono-objective segmentation fusion models existing in the literature.

# E. Discussion and Future Work

Let us recall that our fusion algorithm is composed of two stages, where in the first one, our algorithm estimates the set of the non-dominated solutions, constituting the

 ${}^{3}Z = (0.802 - 0.761)/\sqrt{(0.11942^{2}/300) + (0.1427^{2}/300)}$  is the distance from the sample mean to the population mean in units of the standard error.

#### TABLE IV

THE VALUE OF VOI, GCE, PRI AND BDE AS A FUNCTION OF THE USED CRITERION; SINGLE-CRITERION (EITHER F-MEASURE AND GCE)

AND THE TOW COMBINED CRITERIA (GCE+F-MEASURE)

	BSDS300			
Our Fusion Model	$\mathrm{VoI}\downarrow$	$\text{GCE}\downarrow$	$\mathrm{PRI}\uparrow$	BDE $\downarrow$
GCE	2.11	0.20	0.79	8.86
F-measure	2.04	0.20	0.78	8.52
GCE+F-measure	1.98	0.20	0.80	8.25

so-called Pareto-front or Pareto-optimal set (see Algorithm 1 and Figs. 4 and 5). Concretely, this set of non-dominated solutions necessarily includes the solution or the segmentation map that only optimizes (at least locally, since the ICM-based algorithm 1 is deterministic) the first criterion and also the solution that uniquely satisfies the second criterion (these two solutions are represented by the blue and the black triangle symbols, respectively, at the top right and bottom left in Fig. 4). The other non-dominated solutions ( $\in L_{NDS}$ ), belonging to the Pareto-front, are, in fact, some "interesting" trade-offs or compromised solutions between the two considered criteria. Therefore, conceptually, the Pareto-front thus captures the whole set of "interesting" compromise solutions between the two considered criteria. By the word "interesting," we mean, more precisely, in fact, the set of non-dominated solutions according to the classical definition used in MCDM "a non-dominated solution is a feasible solution where there does not exist another feasible solution better than the current one in some objective function without worsening other objective function."

It is interesting also to note that this list or set of nondominated solutions, belonging to the Pareto-front, can be easily ordered into a connected path of solutions, from the solution that minimizes the first criterion to the solution that optimizes the second criterion (see Fig. 6). This "linked chain" of segmentation maps, represented by the ordered triangles from right top to bottom left in Fig. 4, could help us to visually understand how the first criterion influences and characterizes a segmentation solution, in terms of the boundaries and region properties of the segments or, more generally, in terms of geometrical, aggregative, morphometric properties, compared to the second considered criterion, and this could be useful for finding a specific criterion or a pair of criteria for a specific vision application.

In addition, it is interesting to note that the length of the Pareto curve, in average for a diversified image database, is in fact a good indicator that could help us to know how a criterion is different, complementary, conflicting or contradictory from a second given criterion. Indeed, when the Pareto-front comes down to a single point or solution, it simply means that the obtained solution is the one that simultaneously minimizes the first but also the second criterion. In this case, a monoobjective segmentation fusion model, using either the first or the second criterion, would have given the same segmentation result.

Besides, the set of plausible solutions, or candidate segmentation maps given by the Pareto-front, obtained for different given pair of criteria, could also be interestingly compared, in term of agreement, to the set of available manual segmentations estimated for each natural image, by several human observers, in the Berkeley segmentation dataset. We recall that this variability expressed by the multiple acceptable ground truth solutions associated with an image, represents, in fact, the different levels of detail and/or the possible interpretations of an image between human observers. This comparison could help us to find the pair of criteria which will give us the set of plausible solutions which would be consistent with the existing inherent variability existing between human segmenters.

Also, the Pareto-optimal set of plausible solutions could be exploited to adaptively estimate the optimal or the best compromise number of segments or regions of the segmented image.

Finally, it would be interesting to compare the length of the Pareto front, obtained for different given pair of criteria, for different segmentation ensembles (see Section IV) generated by different strategies. This measure could be a good indicator of the consistent diversity, as opposed to a noisy diversity, of the segmentation ensemble which is indispensable for a good fusion result.

# F. Algorithm

The execution time takes, on average, between 4 and 5 minutes for an Intel<sup>®</sup> 64 Processor core i7-4800MQ, 2.7 GHz, 8 GB of RAM memory and non-optimized code running on Linux. More accurately, the first step in our segmentation procedure, i.e., estimations of the J = 60 weak segmentations to be fused, takes on average, 1 minute. The second step, i.e.,

TABLE V Average CPU Time for Different Segmentation Algorithms on the BSDS300

ALGORITHMS	CPU time (s)	On [image size]					
With Multi-Criteria Fusion Model							
MOBFM	$\simeq 240$	$[320\times214]$					
With Mono-Criterion Fusion Model							
GCEBFM [67]	$\simeq 180$	$[320\times214]$					
FMBFM [45]	$\simeq 90$	$[320\times214]$					
SFSBM [55]	$\simeq 60$	$[320\times214]$					
FCR [26]	$\simeq 60$	$[320 \times 200]$					
PRIF [32]	$\simeq 80$	$[320\times214]$					
VOIBFM [34]	$\simeq 60$	$[320\times214]$					
Without Fusion Model							
CTM [60]	$\simeq 180$	$[320 \times 200]$					
FH [19]	$\simeq 1$	$[320 \times 200]$					
Mean-Shift [61] $_{in [62]}$	$\simeq 80$	$[320 \times 200]$					
WMS $[63]$ in $[62]$	$\simeq 2$	$[320 \times 480]$					

minimization of our fusion procedure, takes approximately 3 or 4 minutes for the fusion step and for a  $320 \times 214$  image. Our segmentation method has acceptable computation time in comparison with some results given in the literature (see Table V. However, improvements can be made, since these two steps can be easily computed in parallel by using the parallel abilities of any graphic processor unit (GPU). Moreover, the whole implementation was developed using the C++ language and the source code, data and all that is necessary for the reproduction of results and the ensemble of segmented images are available at this address; http://www etud.iro.umontreal.ca/~khelifil/ResearchMaterial/mobfm.html in order to make possible comparisons with future segmentation algorithms.

## VI. CONCLUSION

In this paper, we have proposed a novel and efficient fusion model based on multi-objective optimization (MOBFM), whose goal is to combine multiple segmentation maps with multiple different criteria to achieve a final improved segmentation result. This model is based on two complementary (contour and region-based) criteria of segmentation. To optimize our fusion model, we used a modified ICM algorithm, including a dominance function that allowed us to find a compromise between these different segmentation criteria. Besides that, we have used an efficient technique of decision making called TOPSIS, allowing us to find the most preferred solution from a given set of non-dominated solutions. Applied on the BSDS300-500, the proposed segmentation model gives competitive results compared to other segmentation models, which proves the effectiveness and the robustness of our bicriteria fusion approach.

To sum up, we have shown that the strategy of fusion of different segmentations remains simple to implement and perfectible by incrementing the number and the complementarity of the segmentations to be fused. We have also shown that a fusion model of segmentations, expressed as a multi-objective optimization problem, with respect to a combination of different and complementary criteria, is an interesting approach that can overcome the limitations of a single criterion based fusion procedure. It gives a competitive final segmentation result for different images with several distinct texture types. Besides, the Pareto-optimal set of plausible segmentations given by this MO fusion strategy can help to understand ambiguous natural scene, by providing different and plausible segmentations of an image in a similar way than the neural mechanisms of visual perception, which also provides many competing organizations making possible several conflicting interpretations of the same image. In our case, this set of multiple distinct segmentations, which corresponds to interesting compromise solutions between the two considered criteria, can be advantageously used in a last stage of computation for a specific higher level vision task.

In addition, this new multi-objective optimization strategy based on multiple different and complementary criteria remains enough general to be applied to other energy-based models, until now based on a single criterion, and extensively used in image processing, image understanding and computer vision applications. This idea is currently under investigation, especially for energy-based restoration models, denoising and deconvolution, where a fusion of different and complementary regularization terms could be appealing in order to better constrain the optimization process or to better incorporate (complementary or contradictory) knowledge or beliefs concerning the types of restorations a priori defined as being acceptable solutions in the associated inverse (illposed) optimization problem. Similarly, classification procedures, such as energy-based semantic interpretation model (scene parsing), consisting in semantically labeling every pixel in the segmented image, is also under investigation since it can also be efficiently done in a fusion framework with several complementary criteria, and on the basis of a training or learning set of segmentation with pre-interpreted classes.

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