

# An Adaptive Segmentation-based Regularization Term For Image Restoration

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## Introduction

In image restoration with Bayesian methods, the solution is regularized by introducing a *a priori* constraints [1]. Expressed as a prior distribution  $P_X(x)$  of the unknown image  $x$  or analytically encoded through an energy function  $\Omega(x)$  added to the likelihood term, this prior or energy term aims at reflecting the knowledge or beliefs concerning the types of images acceptable as estimates. Most of the proposed prior models assume that the original image to be recovered is smooth. However, this is a global requirement and therefore not very effective in terms of local smoothness.

Most of the recent adaptive edge-preserving methods in image restoration propose to manipulate “edge-variables” in the regularization term. An alternative approach to apply smoothness while preserving discontinuities, proposed in this work, is to use “label variables” and more precisely, to apply a smoothness constraint only on constant areas of the image to be recovered. This problem of identifying constant regions in a given image is called a segmentation problem. A simple, alternative and adaptive local quadratic smoothness term would then consist in penalizing solutions exhibiting a luminance distribution with large variance within each regions (i.e., the set of connected pixels belonging to the same class). This regularization strategy, which assume as prior model that the image is piecewise smooth over pre-estimated regions is the main contribution of this work.

**In this new inhomogeneous Bayesian restoration model, regularization is achieved during the iterative restoration process with a segmentation-based a priori term. This adaptive edge-preserving regularization term applies a local smoothness constraints to pre-estimated constant areas of the image to be recovered. These constant-valued regions (the segmentation map) of the target image are obtained by an unsupervised Markovian segmentation on a preliminary Wiener deconvolution estimate. To compute the MAP estimate associated to the restoration, we use a simple steepest descent procedure resulting in an efficient iterative process converging to a globally optimal restoration.**

## Adaptive Prior Model

In order to impose local smoothness on constant areas of the image to be restored, we propose the following quadratic regularization term, based on a partition into regions, and whose goal is to penalize solutions exhibiting a luminance distribution with large variance within each region

$$\Omega(x) = \|\rho(x)\|^2 = \|x - \Gamma(x)\|^2$$

$\Gamma(x_s)$  designates the operator that gives the mean of grey level values of the region of belonging to the pixel at location  $s$ .

In this context, the restoration problem of  $y (= h * x + n)$  is thus defined as the search of the global *minima* of the following energy function

$$E(x) = \left\{ \|y - h * x\|^2 + \frac{\alpha}{2} \|x - \Gamma(x)\|^2 \right\}$$

This search of the global *minima* is simply performed in our application by a steepest descent procedure which moves the estimates iteratively in the negative gradient direction, as follows

$$\hat{x}^{[n+1]} = \hat{x}^{[n]} - \gamma \nabla E(x)$$

where  $\gamma$  is the step size. A large step size  $\gamma$  is needed for fast convergence, but a too large value may destabilize the iterative algorithm.  $\nabla E(x)$  with  $E(x)$  above-defined can be easily defined and allows to obtain the following iterative procedure of restoration

$$\hat{x}^{[n+1]} = \hat{x}^{[n]} + \gamma \left( h^{\#*} (y - h * \hat{x}^{[n]}) - \alpha \rho'(\hat{x}^{[n]}) \rho(\hat{x}^{[n]}) \right)$$

where  $h^{\#}(i, j) = h(-i, -j)$  (for  $h$  symmetric, we have  $h^{\#} = h$ ),  $*$  is the linear convolution operator, and, in this form of notation, the multiplication (between  $\rho'$  and  $\rho$ ) is done pixel-by-pixel. Besides, if  $\mu$  denotes the mean of the grey level values of the region to which the pixel at location  $s$  belongs, we have,

$$\frac{\partial \rho(x)}{\partial x_s} = 1 - \frac{1}{\mathcal{N}}$$

where  $\mathcal{N}$  is the number of pixels of the region of belonging to pixel at location  $s$ . The successive approximation of the solution according to the minimization of the cost function  $E(x)$  results in initializing  $\hat{x}^{[0]}$  with the restoration result given by the Iterative Wiener filter [2], and by using the iterative process until some convergence criterion is met.

## Parameter Estimation Step

### A. Iterative Wiener Filter

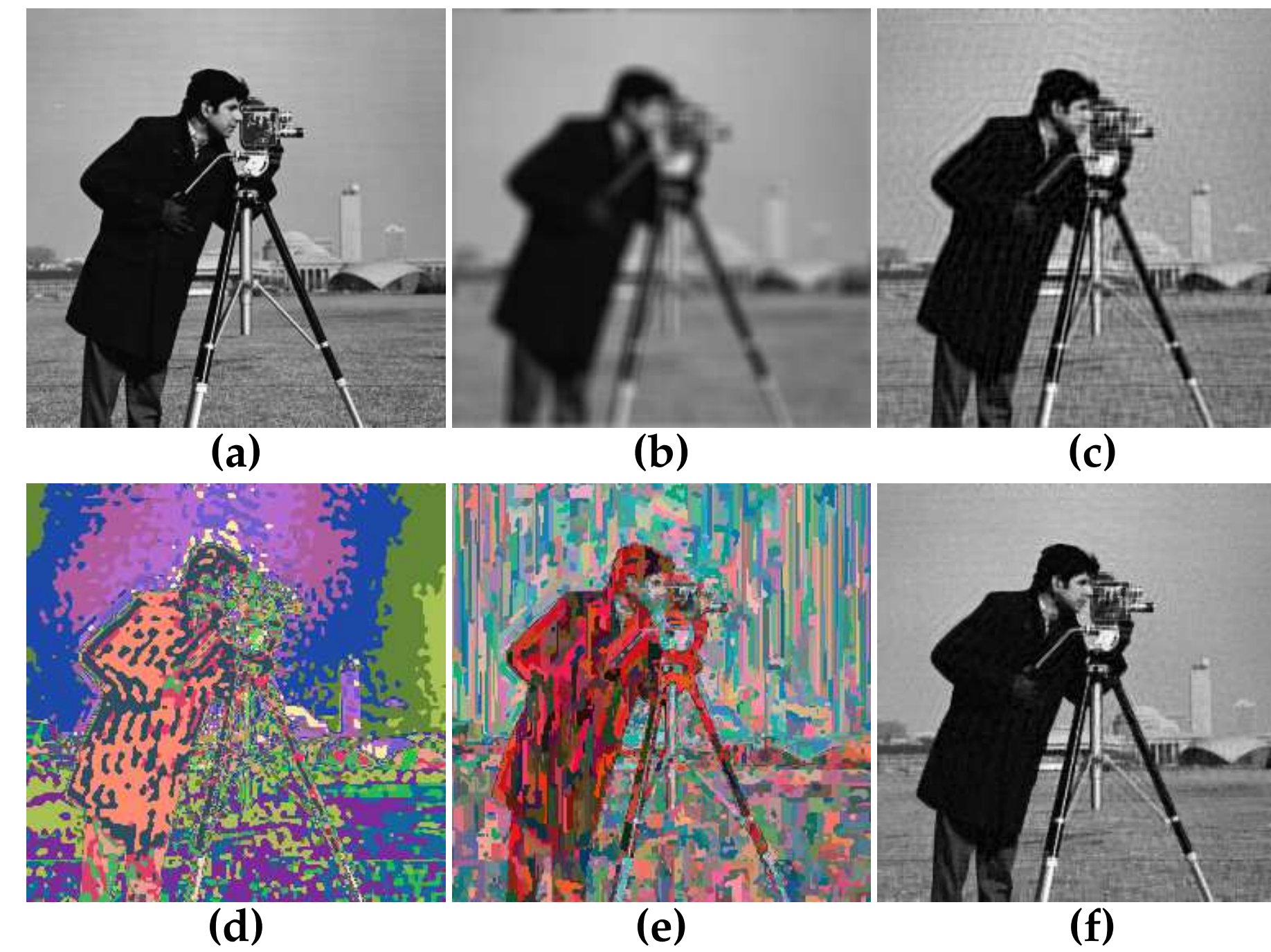
The proposed restoration algorithm assumes knowledge of an oversegmentation of the original image  $x$  into homogeneous regions. Since this image (before degradation) is unknown, the first step of our algorithm consists of obtaining an approximation of the true image  $x$ , with an unsupervised iterative Wiener filtering [2]. The quality of the restored image by this procedure is sufficient to automatically estimate a Markovian segmentation that will be used in our restoration model.

### B. Unsupervised Markovian Segmentation

To this end, we have adopted the monoscale version of the Markovian segmentation model described in [3], and already successfully applied to noisy sonar images, with Gaussian law, as degradation model to describe the luminance distribution within each class. In our application, we take  $K = 20$  classes.

### C. Partition Into Regions

We now exploit this over-segmentation in order to get a reliable partition  $\mathcal{R}$  of the image into homogeneous regions. To this end, we simply search the set of disjoint regions (i.e., the set of connected pixels belonging to the same class). In order to limit regions with a large number of pixels, which could produce an undesirable “staircase” or quantization effects by our regularization/prior term which tends to favor piecewise smooth restorations, we subdivide all the regions with more than 100 pixel size.



(a) Original image, (b) Noisy-blurred image with uniform blur of size  $9 \times 9$ , and AWGN with  $\sigma^2 = 0.308$  (BSNR=40 dB), (c) Iterative Wiener filter [2] (ISNR=5.7 dB), (d) Unsupervised twenty-class segmentation, (e) Partition into regions of the segmentation presented in (d), (f) Restored image using the proposed approach (ISNR=8.04 dB).

## Experimental Results

We have taken for all the following experiments  $K = 20$  classes for the segmentation step and  $\zeta = 0.2$  for the restoration step.

	Blur	$\sigma^2$	BSNR
Exp1	$9 \times 9$ uniform	0.308	40
Exp2	$h_{ij} = (1 + i^2 + j^2)$ $i, j = -7, \dots, 7$	2	32
Exp3	$[1, 4, 6, 4, 1]^t [1, 4, 6, 4, 1]/256$	49	18

Blur, noise variance and BSNR (dB) for each experiment

	ISNR (dB)		
Method	Exp1	Exp2	Exp3
Proposed algorithm	8.04	7.23	1.34
Figueiredo & Nowak [4]	7.59	6.93	2.94
Neelamani <i>et al.</i> [5]	7.30	-	-
Banham & katsaggelos [6]	6.70	-	-
Jalobeanu <i>et al.</i> [7]	-	6.75	-
Liu & Moulin [8]	-	-	1.08

ISNR (dB) of the proposed algorithm and of the methods [4], [5], [6], [7], [8] for the three experiments



From top to bottom: Noisy-blurred image Exp2 (see Table I) and restored image using the proposed approach (ISNR=7.23 dB) (see Table II).

### Several Segmentations

An improvement of this model is to consider several partitions into regions, allowing to minimize the effects of the dependence of a bad segmentation map on the restoration result  $\dagger$ .

<sup>†</sup> M. Mignotte. A segmentation-based regularization term for image deconvolution. *IEEE Trans. on image processing*, accepted for publication.

Segmentation-Based Deconvolution Algorithm (Mig-1)	
$N_{\text{seg}}$	Number of segmentations/partitions
$K$	Number of classes of the segmentation
$\sigma^2$	Variance of the noise
<b>1. <math>\hat{x}^{[0]} \leftarrow</math> Iterative Wiener Filtering of <math>y</math></b>	
<b>2. Unsupervised Segmentations/Partitions</b>	
<b>for <math>k = 1</math> to <math>N_{\text{seg}}</math> do</b>	
<ul style="list-style-type: none"> <li>• K-means clustering of each pixel of <math>\hat{x}^{[0]}</math> into <math>K</math> clusters, initially randomly chosen (SEED = <math>k</math>)</li> <li>• <math>\Phi_k^{[0]} \leftarrow</math> ML fitting of Gaussian law for each cluster</li> <li>• <math>z_{\text{ML}}^{[0]} \leftarrow</math> ML segmentation of <math>\hat{x}^{[0]}</math> based on <math>\Phi_k^{[0]}</math></li> <li>• <math>\Phi \leftarrow</math> ICE algorithm on <math>\hat{x}^{[0]}</math> (initialized with <math>z_{\text{ML}}^{[0]}</math> and SEED = <math>k</math>)</li> <li>• <math>z_{\text{ML}} \leftarrow</math> ML segmentation of <math>\hat{x}^{[0]}</math> based on <math>\Phi</math></li> <li>• <math>z_{\text{MAP}} \leftarrow</math> ICM segmentation (initialized with <math>z_{\text{ML}}</math>) and based on <math>P_{Z X, \Phi}(\cdot   \hat{x}^{[0]}, \Phi)</math></li> <li>• <math>z_k \leftarrow</math> Partition into regions (of less than 100 pixel size) of <math>z_{\text{MAP}}</math></li> </ul>	
$\alpha \leftarrow \zeta \frac{\ y - h * \hat{x}^{[0]}\ ^2}{\Omega(\hat{x}^{[0]})}$	
<b>3. Segmentation-Based Deconvolution</b>	
<b>while <math>[E(\hat{x}^{[n+1]}) - E(\hat{x}^{[n]})]/E(\hat{x}^{[n+1]}) \leq 10^{-5} \sigma^2</math></b>	
<b>do</b>	
<ul style="list-style-type: none"> <li>• <math>\hat{x}^{[n+1]} \leftarrow \hat{x}^{[n]} + \left( h^{\#*} (y - h * \hat{x}^{[n]}) - \alpha \rho'(\hat{x}^{[n]}) [\hat{x}^{[n]} - \frac{1}{N_{\text{seg}}} \sum_{k=1}^{N_{\text{seg}}} \Gamma_k(\hat{x}^{[n]})] \right)</math></li> <li>• Compute the MAP</li> </ul>	
$E(\hat{x}^{[n+1]}) \leftarrow \left\{ \ y - h * \hat{x}^{[n+1]}\ ^2 + \frac{\alpha}{2} \ \hat{x}^{[n+1]}\ ^2 - \frac{1}{N_{\text{seg}}} \sum_{k=1}^{N_{\text{seg}}} \Gamma_k(\hat{x}^{[n+1]}) \right\}^2$	
$n \leftarrow n + 1$	

## References

- [1] S. Geman and D. Geman. Stochastic relaxation, Gibbs distributions and the Bayesian restoration of images. 6(6):721–741, 1984.
- [2] A. D. Hillery and R. T. Chin. Iterative wiener filters for image restoration. 39:1892–1899, 1991.
- [3] M. Mignotte, C. Collet, P. Pérez, and P. Boutheymy. Sonar image segmentation using a hierarchical MRF model. 9(7):1216–1231, 2000.
- [4] M. A. T. Figueiredo and R. D. Nowak. An EM algorithm for wavelet-based image restoration. 12:906–916, 2003.
- [5] R. Neelamani, H. Choi, and R. G. Baraniuk. Wavelet-domain regularized deconvolution for ill-conditioned systems. In *IEEE International Conference on Image Processing -ICIP'99*, volume 1, pages 204–208, October 1999.
- [6] M. R. Banham and A. K. Katsaggelos. Spatially adaptive wavelet-based multiscale image restoration. 5(4):619–634, 1996.
- [7] A. Jalobeanu, N. Kingsbury, and J. Zerubia. Image deconvolution using hidden markov tree modeling of complex wavelet packets. In *IEEE International Conf. Image Proc. -ICIP-2001*, October 2001.
- [8] J. Liu and P. Moulin. Complexity-regularized image restoration. In *IEEE International Conference on Image Processing -ICIP'98*, pages 555–559, October 1998.