3D Biplanar Reconstruction of Scoliotic Vertebrae Using Statistical Models

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Abstract

This paper presents a new 3D reconstruction method of the scoliotic vertebrae of a spine, using two conventional radiographic views (postero-anterior and lateral), and a global prior knowledge on the geometrical structure of each vertebra. This geometrical knowledge is efficiently captured by a statistical deformable template integrating a set of admissible deformations, expressed by the first modes of variation in the Karhunen-Loeve expansion of the pathological deformations observed on a representative scoliotic vertebra population. The proposed reconstruction method consists in fitting the projections of this deformable template with the segmented contours of the corresponding vertebra on the two radiographic views. The 3D reconstruction problem is stated as the minimization of a cost function for each vertebra and solved with a gradient descent technique. The reconstruction of the spine is then made vertebra by vertebra. This 3D reconstruction method has been successfully tested on several biplanar radiographic images, yielding very promising results.

Keywords: *3D reconstruction model, scoliosis, 3D/2D registration, biplanar radiographies, statistical deformable model, energy function optimization.*

1. Introduction

The scoliosis is a three-dimensional deformation of the natural curve of the spinal column, including rotations and vertebral deformations. In order to analyze 3D characteristics of these deformations, which can be useful for the design, the evaluation and the improvement of orthopedic or surgical correction, several 3D reconstruction methods have been developed. First, we can cite the class of typical methods of tomodensitometric imagery's modalities (e.g. X-rays, magnetic resonance) allowing to obtain accurate 3D information of the human body anatomy. However, the high level of radiations received by the patient, the great quantity of information to be acquired and processed and the cost of its methods make them less functional [3]. We can also cite the class of methods using a limited number of projections and some simple *a priori* knowledge on the geometry of the object to be reconstructed. These last methods are interesting but are widely supervised; they require to manually identify (by an operator) a set of point of interest (landmarks) on the postero-anterior (PA_0) and lateral (LAT) radiographic images of C1 to L5 vertebrae [3][13]. Besides, these aforementioned methods are not very accurate, especially because they do not exploit all the information contained in the two radiographic images (e.g. the contours of each vertebra) or the *a priori* global geometrical knowledge of the object to be reconstructed. To this end, Bayesian inference or statistical modeling is a convenient way of taking this *a priori* information into consideration. This paradigm, in image analysis, is quite popular and has been successfully applied for the extraction of 2D objects in an image [2], an image sequence [10], 3D representation of vertebra [11] or non rigid 3D/2D registration of the knee [4][5].

In this way, we propose a new statistical 3D reconstruction model or a new 3D/2D registration model for the scoliotic vertebrae from bi-planar radiographic images. Our approach relies on the description of each vertebra by a deformable template which incorporates (statistical) knowledge about its geometrical structure and its pathological variability. The deformations of this template are expressed by the first modes of variation in the Karhunen-Loeve (KL) expansion of the pathological deformations observed on a representative scoliotic vertebra population. This prototype template, along with

the set of admissible deformations, constitute our global prior model that will be used in order to rightly constraint the ill-posed nature of our 3D reconstruction problem. In our application, the proposed method consists in fitting this template with the segmented contours of the corresponding vertebra on the two calibrated radiographic views. This matching problem leads to an optimization problem of a cost function, efficiently solved in our application by a gradient descent algorithm initialized by a rough and rigid 3D reconstruction method estimated in the least square sense.

This paper is organized as follows: Section 2 and 3 present the statistical deformable model and the proposed 3D reconstruction method. In Section 4, we show some 3D reconstruction results. Finally, conclusion is given in Section 5.

2. Statistical Deformable Model

The shape x of each vertebra is defined by a set of n control points or "landmarks", which approximate the geometrical shape of each vertebra in IR^3 [2]. Each vertebra, in the training set, is thus represented by the following $3n$ dimensional vector,

$$x = (x_1 \quad x_2 \quad \cdots \quad x_i \quad \cdots \quad x_n)^T,$$

where $x_i = (x_{x_i} \; x_{y_i} \; x_{z_i})^{\mathrm{T}}$ represents the Cartesian coordinates of each surface point. In the following, we will assume that x is a realization of a random vector that follows a normal law of mean vector $\bar{x}$ and covariance matrix C as suggested in [2].

After aligning of the training shapes, the mean shape and the covariance matrix are defined as,

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, \qquad C = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(x_i - \bar{x})^T .$$

The variabilities within the training set are characterized by the displacement vector $\tilde{x} = x - \bar{x}$ of the different surface points with respect to the mean model. A statistical analysis of this random vector makes it possible to deduce the deformation modes relative to the mean shape. The eigenvectors of the covariance matrix C of this random vector describe the variation modes in the deformation parameters space (or the information on the variability of the scoliotic deformations in the vertebra base) and the associated eigenvalues are the amplitudes of these variation modes.

An accurate description of the main variation modes may be obtained by retaining only the m eigenvectors associated to the m largest eigenvalue [2]. The model allows the generation of new instance of the shape by adding linear combinations of the m most significant variation vectors to the mean shape:

$$x = \bar{x} + \phi b, \tag{1}$$

where ϕ represents an orthogonal base of variation modes of the models of the training base, and b is the global deformation parameter vector setting the amplitudes of each deformation mode b_i. By ensuring

$$-3\sqrt{\lambda_i} \le b_i \le +3\sqrt{\lambda_i},$$

only the important deformations are extracted, discarding training data noise [2]. This low parametric representation of a vertebra constitutes our global prior model that will be used in our 3D reconstruction method (cf. Figure 1).

In theory, the ratio of an eigenvalue to the total sum of the other eigenvalues expresses the percentage of error introduced if the eigenvector associated to the corresponding eigenvalue is not selected [2]. One must thus specify a threshold f_v for the eigenvalues above which the error is considered to be sufficiently small to generate a good approximation of the initial vector. Hence, if V_T is the sum of the eigenvalues, then the number t of eigenvalues to be selected is such that:

$$\sum_{i=1}^{t} \lambda_i \ge f_v V_T .$$

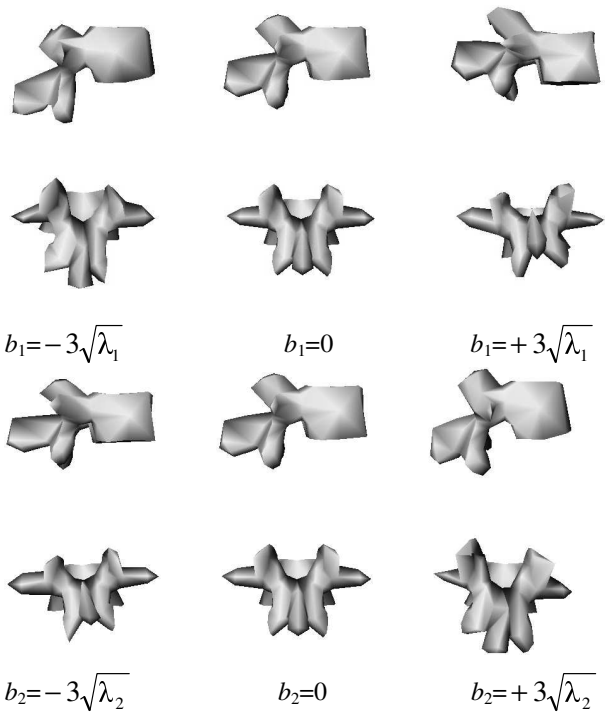


Figure 1. Visualization of the mean shape (middle row) for the sagittal (top row) and coronal views (bottom row) and two deformed shapes obtained by applying ± 3 standard deviations of the first and second deformation modes to the mean shape for L2 vertebra.

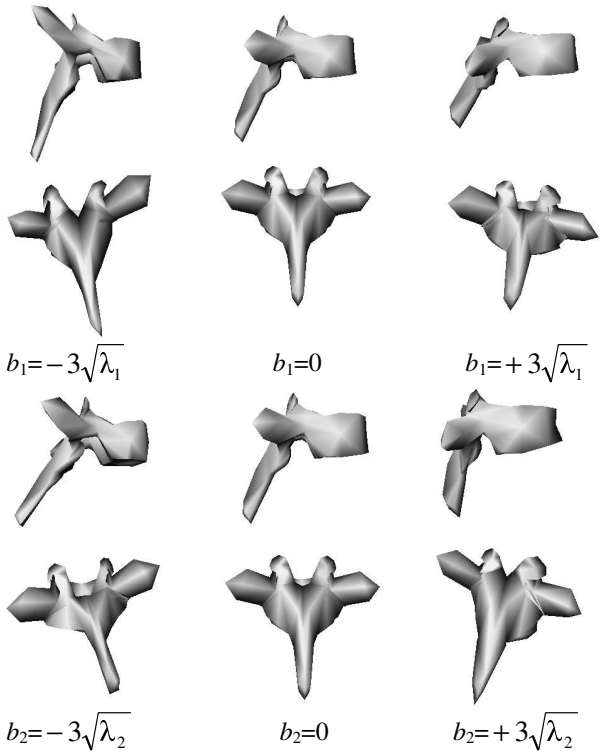


Figure 2. Visualization of the mean shape (middle row) for the sagittal (top row) and coronal views (bottom row) and two deformed shapes obtained by applying ± 3 standard deviations of the first and second deformation modes to the mean shape for T06 vertebra.

3. 3D Reconstruction

Besides the above mentioned global deformation parameters, we also consider 3D global transformations from the similarity group which finally lead to the following model for global deformations:

$$x = M(k,\theta)[\bar{x} + \phi\, b] + T\,, \qquad (2)$$

with k and θ being respectively the scale and the rotation vector (in the x, y or z axis) and T is a global translation vector.

3.1. Crude and Rigid Initial Reconstruction

In order to ensure a first crude and rigid reconstruction of each vertebra, we use the technique proposed by KAUFFMANN in [9] and VAITON in [15]. This technique requires to identify, in a preliminary step, a sequence of 8 points along the centerline of the spine from C1 cervical vertebra to L5 lumbar vertebra on the two radiographic views of the spine. These points are then exploited in order to statistically determine the position of six anatomical points (namely, the center of the superior and inferior end-plates, the upper and lower extremities of both pedicles) for each vertebra of the spine. The corresponding points on the shape of the mean vertebra being known, we can estimate, in the least square sense [7], an initial estimate of the parameter vector (k, θ, T). This leads us to a crude and rigid reconstruction for each vertebra that will be then refined by our 3D reconstruction model (cf. Figure 3).

3.2. 3D Reconstruction Model

Our reconstruction model from two radiographic views is stated as the minimization of the following cost function

$$E_\Theta(x, I_{\mathrm{PA_0}}, I_{\mathrm{LAT}}) = E(x, I_{\mathrm{PA_0}}, I_{\mathrm{LAT}}) + \beta * E(x)\,, \qquad (3)$$

where $E(x, I_{\mathrm{PA_0}}, I_{\mathrm{LAT}})$ is the likelihood energy term and $E(x)$ is the prior energy term (or the regularization term), used to constrain the ill-posed nature of this optimization problem. β is a factor that provides a relative weighting between the two penalty term and allows to control the rigidity of the statistical template [8]. Let us note that this optimization problem can also be formulated as the search of the Maximum A posteriori (MAP) of Θ, the deformation parameters of the deformed template x:

$$x^* = \underbrace{\max}_{x} P(I_{PA_0}, I_{LAT} | x) P(x)\,, \qquad (4)$$

where $P(I_{\mathrm{PA_0}},I_{\mathrm{LAT}}|x)=c_1\exp(-E(x,I_{\mathrm{PA_0}},I_{\mathrm{LAT}}))$ is the likelihood of the observations (i.e. the segmented contours on the two radiographic views) given a deformed template and $P(x)=c_2\exp(-E(x))$ is the prior probability of the deformed template (or the prior probability of a scoliotic deformation for a given vertebra). c_1 and c_2 are two constants of normalization and $\Theta=(M(\mathrm{k},\theta), T, b)$ is the deformation parameter vector of the model to be estimated.

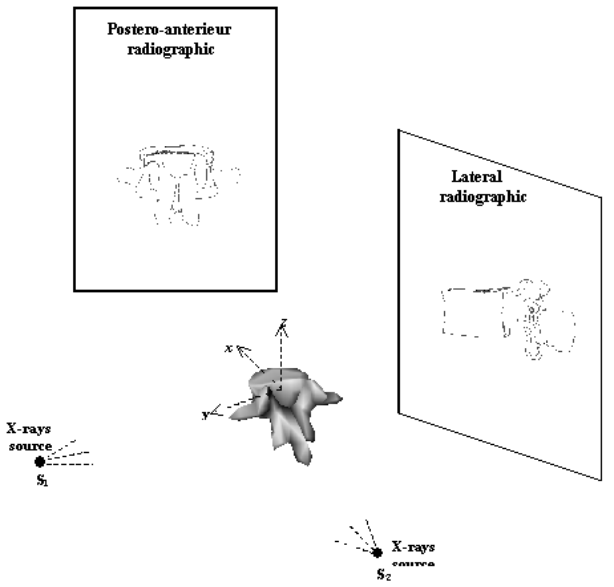
3.3. Likelihood Energy Term

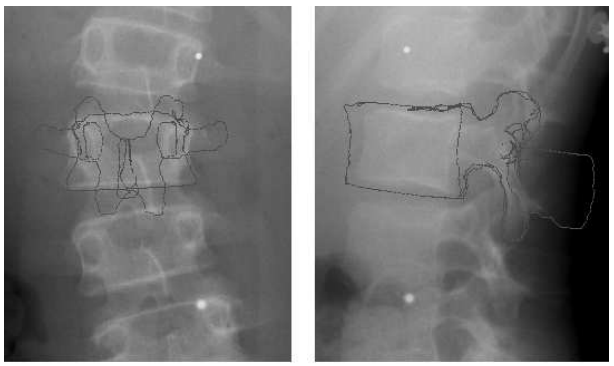
In our application, our likelihood model is expressed by a measure of similarity between the external contour of the lateral and the postero-anterior perspective projections of the deformed template and the spatial edges detected in the two radiographic views (cf. Figure 3):

$$E(x, I_{\mathrm{PA_0}}, I_{\mathrm{LAT}}) = -\frac{1}{\mathrm{Nb}_{\mathrm{PA_0}}} \sum_{\Gamma_{PA_0}} \left|\nabla I_{PA_0}\right| - \frac{1}{\mathrm{Nb}_{\mathrm{LAT}}} \sum_{\Gamma_{LAT}} \left|\nabla I_{LAT}\right|, \qquad (5)$$

where the summation of the first and second term of $E(x,I_{\mathrm{PA_0}},I_{\mathrm{LAT}})$ is over all the $Nb_{\mathrm{PA_0}}$ and Nb_{LAT} points of the external (respectively lateral and postero-anterior) contour of the deformed template projected on the two smoothed Canny edge radiographic images. This energy function attains its minimum value when there is an exact correspondence between the projected contours (of

the deformed template) and the preliminary segmented contours of the two radiographic views.





Postero-anterior View Lateral View

Figure 3. Initial estimate of the mean shape of the vertebra on the two radiographic views by the proposed crude and rigid initial reconstruction method.

3.4. Prior Energy Term

Due to the Karhunen-Loeve transform, the random variables b_i are independent and follow a normal law of a null mean and variance λ_i [2]. Thus, the law of probability of x, the deformed template can be written as [10]:

$$P(x) = \prod_{i=1}^{m} \frac{1}{\sqrt{2\lambda_i \pi}} e^{-\frac{b_i^2}{2\lambda_i}}.$$

This probability expresses the fact that the shape to be reconstructed is likely close to the mean shape. By letting $E(x)$, the prior energy term can be written as:

$$E(x) = \frac{1}{2} \sum_{i=1}^{m} \frac{b_i^2}{\lambda_i}. \quad (6)$$

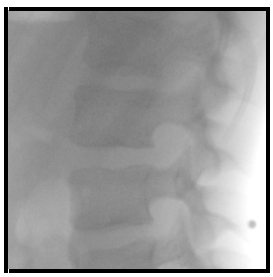
This energy function penalizes the deviation of the deformed template from the mean shape. This function does not penalize affine transformations. Equation (6) closely resembles the Mahalanobis distance. It defines a ellipsoid centered in IR^m whose principal axes are identified by $\sqrt{\lambda_i}$ when $E(x)$ is a constant.

3.5. Optimization of the Energy Function

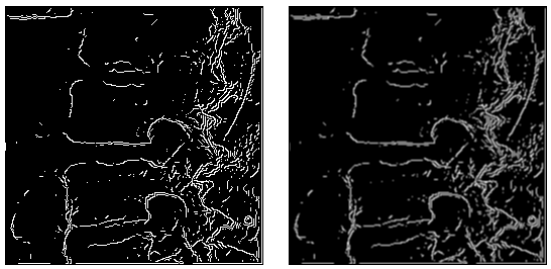
The energy function to be minimized in Equation (3) is a complex function with several local minima over the deformation parameter space. A global search is impossible due to the size of the configuration space. In our application, experiments have shown that the initial crude and rigid reconstruction technique described in Section 3.1 (i.e., the estimation of the rigid deformation parameters) can efficiently be exploited in order to initialize a gradient descent technique. This gradient descent technique is used to simultaneously refine the estimation of the rigid parameters and to estimate the non-rigid parameters.

4. Experimental Results

In our application, we use the training base described in [14].



(a)



(b) (c)

Figure 4. Preprocessing:
(a) Postero-anterior image (256x256 pixels),
(b) Edge map using a Canny edge detector,
(c) Smoothed edge map.

The mean vertebra shape of each vertebral level is calculated on sample of 30 normal vertebrae. The deformation modes of each vertebral level is calculated on a sample of 30 scoliotic vertebrae.

We have used the Canny edge detector to calculate the edge maps of the two radiographic views that will be used in the likelihood energy term (cf. Figure 4). In our application sigma=1, mask size is 5x5, and the lower and upper thresholds are given by the unsupervised estimation technique proposed in [1].

In order to constraint the deformable model to be efficiently attracted to the segmented boundaries of the vertebra by the gradient-based local optimization procedure, we need to spread the region of influence of each detected contours to a larger area. To do this, we convolve each probability map by a 2D Gaussian mask (cf. Figure 4).

In our application, we have chosen to take the number of deformation modes that allows to represent at least 90% of the admissible deformations. Table 1 shows that, for the L2 vertebra, the first 10 deformation modes integrate 91.88% of the deformations considered to be statistically admissible. For the T06 vertebra, the first 8 deformation modes represent 92.49% of the deformations.

L2 Vertebra		**T06 Vertebra**	
λ_i/V_T	λ_i/V_T **cumulated**	λ_i/V_T	λ_i/V_T **cumulated**
23.48 %	23.48 %	27.62 %	27.62 %
19.49 %	42.97 %	24.06 %	51.68 %
13.16 %	56.13 %	12.37 %	64.04 %
10.94 %	67.07 %	9.55 %	73.59 %
7.46 %	74.53 %	6.49 %	80.09 %
5.20 %	79.74 %	5.42 %	85.50 %
3.67 %	83.41 %	4.19 %	89.69 %
3.28 %	86.69 %	2.80 %	92.49 %
2.82 %	89.51 %		
2.37 %	91.88 %		

Table 1. Normalized eigenvalues computed on a training set of 178 point models of 30 Vertebrae obtained from the covariance matrix.

Besides, experiments have shown that the crude and rigid reconstruction procedure is not always a good initialization for the gradient-based optimization technique. In order to overcome this problem, our solution consists in placing the template at evenly spaced positions and in deforming it according a discretized set of translation orientation or scale (corresponding to the rigid parameters) within a range of value around the initial estimate obtained by the rigid reconstruction procedure. These deformed template configurations can then be used to initialize a deterministic gradient descent algorithm. However, the spacing between the template positions and the sampling of the transformations must be chosen judiciously: not too spaced out to cover all the significant local minima of the energy surface and not too small to avoid high computational requirements.

We have validated our 3D reconstruction method on several pair of radiographic images of normal and scoliotic spines (postero-anterior and lateral views). For the experiments, we have chosen β=1 for the weighting factor penalizing the prior energy term with respect to the external energy. The reconstruction and optimization procedure takes about 82 seconds on a standard Pentium II 633 workstation. The proposed method allows a good reconstruction of the L2 vertebra. Figure 5 and Figure 6 present projections of the shape of a L2 and T06 vertebra on postero-anterior and lateral radiographic images for a scoliotic patient. Let us note that the estimated global deformation parameters after reconstruction (i.e. the parameter vector b, setting the amplitude of each deformation mode of the scoliotic deformations) can then be used to quantify the scoliosis, its nature or to analyze the improvement of orthopedic or surgical corrections.

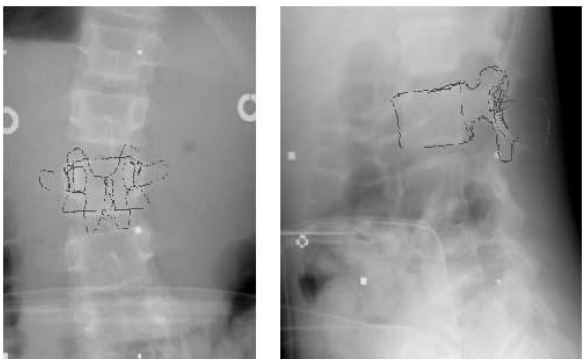


Figure 5. Projection of L2 on the two radiographic views (i.e. postero-anterior and lateral views).

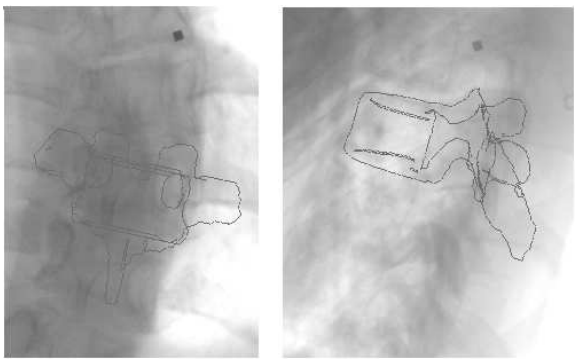


Figure 6. Projection of T06 on the two radiographic views (i.e. postero-anterior and lateral views).

5. Conclusion

We have presented an original statistical method of 3D reconstruction of scoliotic vertebrae using both the contours extracted from biplanar radiographic images and on *a prior* knowledge on the geometrical structure of each vertebra. This technique can also be viewed as an original 3D/2D registration or segmentation model. The 3D reconstruction problem is stated as the minimization of a cost function for each vertebra and solved with a gradient descent technique combined with a sampling strategy. This method has been validated on a number of pair of radiographic images demonstrating its efficiency and robustness. Besides, the proposed method remains sufficiently general to be applied to other medical reconstruction problems (with two or several radiographic views). We now intent to improve the energy function by integrating a region homogeneity term, to refine the statistical model by local deformations and to use global optimization technique.

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