

AN ADAPTIVE SEGMENTATION-BASED REGULARIZATION TERM FOR IMAGE RESTORATION

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ABSTRACT

This paper proposes an original inhomogeneous restoration (deconvolution) model under the Bayesian framework. In this model, regularization is achieved, during the iterative restoration process, with an adaptive segmentation-based regularization term whose goal is to apply local smoothness constraints on estimated constant areas of the image to be recovered. To this end, the parameters of this restoration *a priori* model relies on an unsupervised Markovian over-segmentation. To compute the MAP estimate associated to the restoration, we use a simple steepest descent procedure resulting in an efficient iterative process converging to a globally optimal restoration. The experiments reported in this paper demonstrate that the discussed method performs competitively and sometimes better than the best existing state-of-the-art methods in benchmark tests.

1. INTRODUCTION

The problem of recovering an original image x from a degraded observed version y , is usually known as a *restoration* problem. In many situations, where the imaging system is assumed to be linear and shift invariant, the transformation from x to y is well described by the following additive linear degradation model, $y = h*x+n$, where y , x and n represent respectively, the degraded or noisy and blurred observed image (of size N pixels), the undistorted true image and the corrupting additive and white Gaussian noise with variance σ^2 . h is the Point Spread Function (PSF) of the imaging system and $*$ is the linear convolution operator. We shall assume throughout this paper that the degradation model (PSF and variance of the white Gaussian noise) is known.

The simplest way to approach the restoration problem is to find x in the Maximum Likelihood ML sense ($\arg \max_x P_{Y|X}(y|x)$) or equivalently to find the least squares estimation of x , defined by $\arg \min_x \|y-h*x\|^2$. This so-called inverse-filtering solution, which best matches the probabilistic behavior of the data leads to unacceptable restoration solutions. In fact, the problem of recovering an original image from its degraded version is typical of ill-posed inverse problems in the sense of Hadamard [1] : the perfect knowledge of the degradation model is not sufficient to determine a restoration result with acceptable accuracy. To circumvent this difficulty, a conventional method is to regularize the solution by introducing *a priori* constraints [2]. Expressed as a prior distribution $P_X(x)$ of the unknown image x or analytically encoded through an energy function $\Omega(x)$ added to the likelihood term, this prior or energy term aims at reflecting the knowledge or beliefs

concerning the types of images acceptable as estimates and regularizes the optimization problem so that a unique solution always exists [3].

Probably the simplest prior model used to regularize the solution is $\Omega(x) = \|x\|^2$ which yields to the so-called Tikhonov regularization with the identity [4]. By using this prior, we penalize solutions exhibiting large variance in its grey level distribution. Another commonly used choice is the quadratic functional $\Omega(x) = \|C*x\|^2$, where C is a PSF associated to a high-pass filter (gradient or Laplacian). This penalty function penalizes solutions with large high-frequency fluctuations and thus enforces smoothness on the solution. Similar strategies, reinterpreted in the wavelet-domain have been proposed in [5] and [6]. Nevertheless, these models are homogeneous and assume that the original image to be recovered is smooth. However, this is a global requirement and therefore not very effective in terms of local smoothness. A more efficient image model assumes that only homogeneous regions are smooth, and that edges must remain sharp. In other words, a good prior model should adapt to the local characteristics and structure of the image to enable the solution to be less noisy in constant area and to exhibit sharper details in other regions.

In this way, an alternative approach to apply smoothness while preserving discontinuities, is to apply a smoothness constraint only on constant areas of the image to be recovered. This problem of identifying constant regions in a given image is a low-level task in image processing and is called a segmentation problem. A simple and adaptive local quadratic smoothness term would then consist in penalizing solutions exhibiting a luminance distribution with large variance within each regions (i.e., the set of connected pixels belonging to the same class). This regularization strategy, which assume as prior model that the image is piecewise smooth over pre-estimated regions is the one proposed in this paper. In order to extract a reliable over-segmentation map (robust to the noise), we have adopted a statistical framework allowing to estimate the solution in the MAP sense on the restoration result given by an iterative Wiener filter [7].

The paper is organized as follows. In Sections 2 and 3, we detail the proposed adaptive restoration prior model with its estimation procedure. Finally, section 4 presents experimental results and comparisons with existing techniques.

2. ADAPTIVE PRIOR MODEL

In order to impose local smoothness on constant areas of the image to be restored, we propose the following quadratic regularization term, based on a partition into regions, and whose goal is to

penalize solutions exhibiting a luminance distribution with large variance within each region

$$\Omega(x) = \|\rho(x)\|^2 = \|x - \Gamma(x)\|^2 \quad (1)$$

$\Gamma(x_s)$ designates the operator that gives the mean of grey level values of the region of belonging to the pixel at location s . In this context, the restoration problem is thus defined as the search of the global *minima* of the following energy function :

$$E(x) = \left\{ \|y - h * x\|^2 + \frac{\alpha}{2} \|x - \Gamma(x)\|^2 \right\}. \quad (2)$$

This search of the global *minima* is simply performed, in our application, by a steepest descent procedure which moves the estimates iteratively in the negative gradient direction, as follows :

$$\hat{x}^{[n+1]} = \hat{x}^{[n]} - \gamma \nabla E(x) \quad (3)$$

where γ is the step size. A large step size γ is needed for fast convergence, but a too large value may destabilize the iterative algorithm. $\nabla E(x)$ with $E(x)$ defined by Eq. (2) can be easily defined and allows to obtain the following iterative procedure of restoration

$$\hat{x}^{[n+1]} = \hat{x}^{[n]} + \gamma \left(h^{\#} * (y - h * x^{[n]}) - \alpha \rho'(\hat{x}^{[n]}) \rho(\hat{x}^{[n]}) \right) \quad (4)$$

where $h^{\#}(i, j) = h(-i, -j)$ (the coordinates (i, j) represents the discrete pixel locations and for h symmetric, we have $h^{\#} = h$), $*$ is the linear convolution operator, and, in this form of notation, the multiplication (between ρ' and ρ) is done point-by-point (or pixel-by-pixel). Let us now find an analytical expression for ρ' . If μ denotes the mean of the grey level values of the region to which the pixel at location s belongs, we have,

$$\frac{\partial \rho(x)}{\partial x_s} = \lim_{\epsilon \rightarrow 0} \frac{\rho(x_s + \epsilon) - \rho(x_s)}{\epsilon} = 1 - \frac{1}{\mathcal{N}} \quad (5)$$

where \mathcal{N} is the number of pixels of the region of belonging to pixel at location s . The successive approximation of the solution according to the minimization of the cost function expressed in (2) results in initializing $\hat{x}^{[0]}$ with the restoration result given by the Iterative Wiener filter, and by using the iterative process defined in Eq. (4) (with the use of Eq. (5)) until some convergence criterion is met.

3. PARAMETER ESTIMATION STEP OF PRIOR MODEL

Iterative Wiener Filter

The proposed restoration algorithm assumes knowledge of an oversegmentation of the original image x into homogeneous regions. Since this image (before degradation) is unknown, the first step of our algorithm consists of obtaining an approximation of the true image x , with an unsupervised iterative Wiener filtering [7]. Depending on the degradation model, this iterative Wiener filter yields restorations in which discontinuities are smoothed out and which suffer from spurious oscillations or ripples. This so-called Gibbs phenomenon is, in fact, due to the underlying Fourier basis elements whose support is over the entire spatial domain. Fig. 1c displays an example of restoration obtained with this procedure on a noisy and blurred cameraman image (uniform blur of size 9×9 , and white Gaussian noise variance of $\sigma^2 = 0.308$ ensuring a BSNR=40 dB). Nevertheless, the quality of the restored image by this procedure is sufficient to automatically estimate a Markovian segmentation that will be used in our restoration model.

Unsupervised Markovian Segmentation

To this end, we have adopted the monoscale version of the Markovian segmentation model described in [8], and already successfully applied to noisy sonar images, with Gaussian law, as degradation model to describe the luminance distribution within each class. In this approach, the unsupervised MRF-based segmentation problem consists in having a two-step process. First, a parameter estimation step is conducted to infer the noise model parameters, thanks to an iterative method called Iterative Conditional Estimation (ICE) [9], [10], [11], [12] which gives the best estimation of the likelihood distributions in the least-squares sense. This ICE procedure is initialized by a K-means clustering procedure [13] as proposed in [8]. Then, a second step is devoted to the segmentation itself based on the values of estimated parameters. To this end, we use a classical iterative local update strategy called Iterated Conditional Modes (ICM) [14] algorithm. For the initialization of this iterative algorithm, we exploit the segmentation map obtained in a ML sense. In our application, we take $K = 20$ classes. Fig. 1d displays an example of unsupervised twenty-class segmentation, exploiting MRF parameters estimated with the ICE procedure.

Partition Into Regions

We now exploit this over-segmentation in order to get a reliable partition \mathcal{R} of the image into homogeneous regions. To this end, we simply search the set of disjoint regions (i.e., the set of connected pixels belonging to the same class). In order to limit regions with a large number of pixels, which could produce an undesirable "staircase" or quantization effects by our regularization/prior term which tends to favor piecewise smooth restorations, we subdivide all the regions with more than 100 pixel size. Fig. 1e display an example of region partition of the cameraman image given by our procedure (in this example, 4834 regions have been found with variable sizes, 1 to 99 pixels, and an average size of 13.6 pixels size).

4. EXPERIMENTAL RESULTS

In all the experiments, we have considered the gradient descent iterative procedure defined in Eq. (4) (with the use of Eq. (5) and with $\gamma = 1$). $\hat{x}^{[0]}$ is given by the iterative Wiener filtering. The number of iteration of the ICE procedure is set to 20 (ensuring the stability of the ICE algorithm). The convergence criterion of the proposed restoration procedure is the stability of the MAP energy, i.e.,

$$\frac{E(\hat{x}^{[n]}) - E(\hat{x}^{[n-1]})}{E(\hat{x}^{[n]})} \leq \delta \quad (6)$$

with δ is a threshold, typically set, in our application, to $10^{-5} \sigma^2$. The regularization parameter α , that controls the contribution of the likelihood and prior terms is given by

$$\alpha = \zeta \cdot \frac{\|y - h * \hat{x}^{[0]}\|^2}{\Gamma(\hat{x}^{[0]})} \quad (7)$$

where $\hat{x}^{[0]}$ is the first restoration result given by the iterative Wiener Filter.

We now present experimental results and comparisons illustrating the performance of the proposed approach. We have taken for

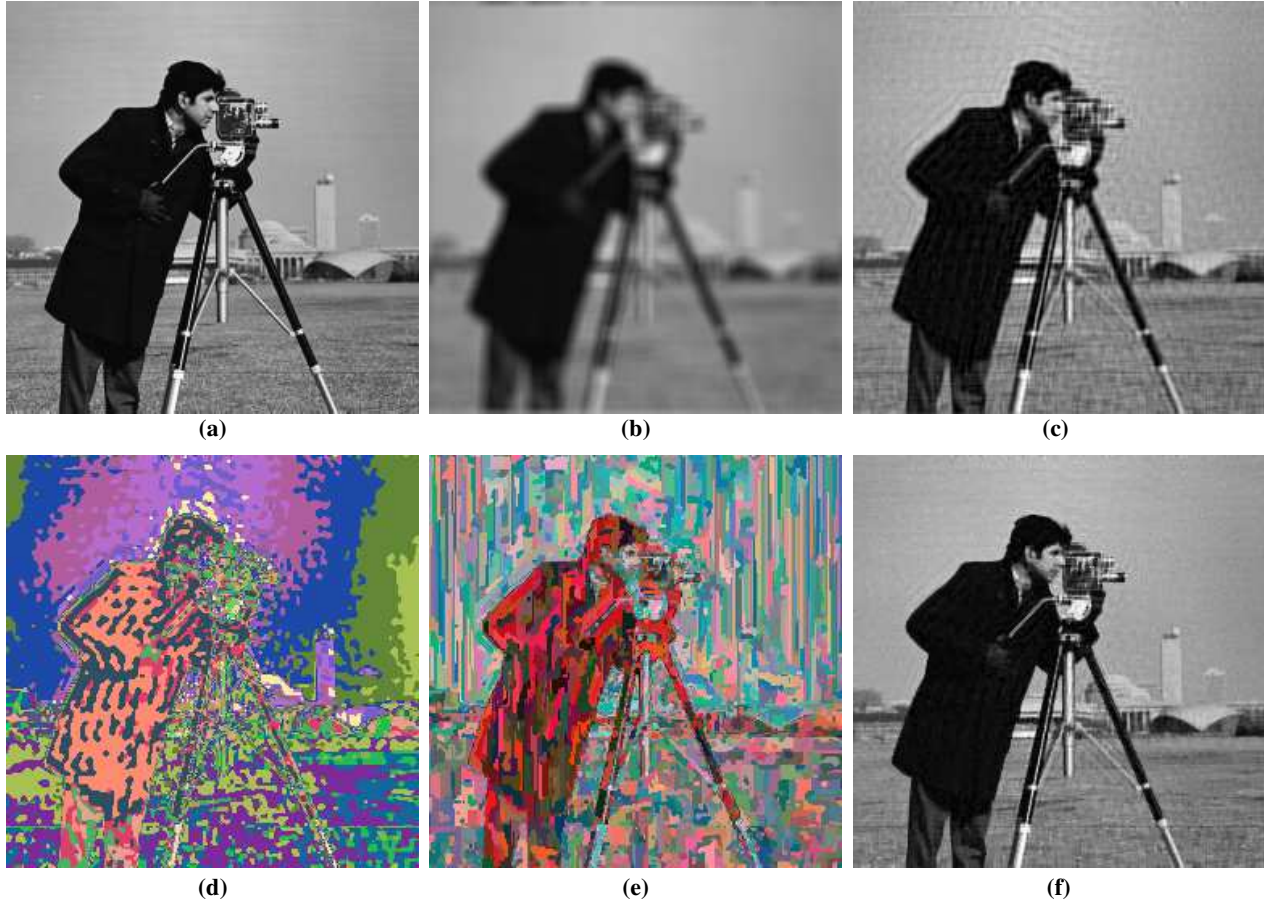


FIG. 1 – (a) Original image, (b) Noisy-blurred image with uniform blur of size 9×9 , and white Gaussian noise variance of $\sigma^2 = 0.308$ (BSNR=40 dB), (c) Iterative Wiener filter with additive correction term (ISNR=5.7 dB), (d) Unsupervised twenty-class segmentation, exploiting parameters $\hat{\Phi}$ estimated with the ICE procedure, (e) Partition into regions of the segmentation presented in (d), (f) Restored image using the proposed approach (ISNR=8.04 dB).

	Blur	σ^2	BSNR
Exp1	9×9 uniform	.308	40
Exp2	$h_{ij} = (1 + i^2 + j^2)$ $i, j = -7, \dots, 7$	2	32
Exp3	$[1, 4, 6, 4, 1]^t [1, 4, 6, 4, 1]/256$	49	18

TAB. 1 – Blur, noise variance and BSNR (dB) for each experiment

all the following experiments $K = 20$ classes for the segmentation step and $\zeta = 0.2$ for the restoration step.

For the first three experiments, we have replicated the scenarios used in the evaluation of state-of-the-art methods described in [15], [16], [17], [18], [19], with which we compare the proposed approach. In these experiments, original images are cameraman (Exps. 1, 2) and Lena (Exp. 3) both of size 256×256 . Table 1 displays the blur, the noise and the resulting blurred signal to noise ratio (BSNR) (i.e., the ratio between the variance of the noise and the variance of blurred image without noise) for each of the four experiments.

Table 2 shows the obtained ISNR (signal-to-noise improve-

ments) obtained by our approach compared to the algorithms described in [15], [16], [17], [18], [19] for the three experiments. Figs. 1a and 2 show respectively the original image, the blurred noisy image and the restored image using the proposed approach.

Our approach performs competitively, in several cases better than the best existing methods in benchmark tests. Except for Exp3 which is not far from a pure denoising problem for which Wavelet-based restoration procedure such as proposed in [15] give better ISNR results.

5. CONCLUSION

In this paper, we have proposed an adaptive edge-preserving Tikhonov regularization term for image restoration. This regularization term, whose goal is to promote piecewise smooth over pre-estimated homogeneous regions are based on an unsupervised Markovian segmentation. Let us note that the proposed regularization strategy is in fact the generalization (for K (number of classes) $\neq 1$) of the so-called Tikhonov regularization with the identity [4]. In this procedure this constraint is locally applied to pre-estimated homogeneous regions. The proposed technique efficiently adapts to local characteristics of the data and allows to obtain results

Method	ISNR (dB)		
	Exp1	Exp2	Exp3
Proposed algorithm	8.04	7.23	1.34
Figueiredo & Nowak [15]	7.59	6.93	2.94
Neelamani <i>et al.</i> [16]	7.30	-	-
Banham & Katsaggelos [17]	6.70	-	-
Jalobeau <i>et al.</i> [18]	-	6.75	-
Liu & Moulin [19]	-	-	1.08

TAB. 2 – ISNR (dB) of the proposed algorithm and of the methods [15], [16], [17], [18], [19] for the three experiments

which simultaneously exhibit no ringing or blocky artifacts, sharp edges, correctly restored textures and very low noise in homogeneous areas. In comparison with state-of-the-art, the experiments reported in this paper demonstrate that the discussed method performs competitively, and sometimes better, than the best existing state-of-the-art method in benchmark tests.

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FIG. 2 – From top to bottom : Noisy-blurred image Exp2 (see Table 1) and restored image using the proposed approach (ISNR=7.23 dB) (see Table 2).

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