ESTIMATION OF NOISE PARAMETERS ON SONAR IMAGES

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ABSTRACT

We use the Markov Random Field (**MRF**) model in order to segment sonar images, i.e. to localize the sea bottom areas and the projected shadow areas corresponding to objects lying on seafloor. This model requires on one hand knowledges about the statistical distributions relative to the different zones and on the other hand the estimation of the law parameters. The Kolmogorov criterion or the χ^2 criterion allow to estimate the distribution laws. The Estimation Maximization (**EM**) algorithm or the Stochastic Estimation Maximization (**SEM**) algorithm are used to determine the maximum likelihood estimate of the law parameters. Those algorithms are initialized with the **Kmean** algorithm. Results are showing on real sonar pictures.

keywords: sonar images, Markov Random Field, parameter estimation, Estimation Maximization algorithm, Stochastic Estimation Maximization algorithm.

1 INTRODUCTION

The subject of this study arises from the exploitation of sonar images to detect manufactured objects on the sea bottom. The detection and then the classification is based on the extraction and the analysis of their projected shadow shapes in sonar pictures. The extraction is obtained by a segmentation of sonar pictures using a Markov Random Field (**MRF**) model. The use of this model in this context affords an innovative viewpoint. As it will be showing in section 3, this method allows us to introduce prior knowledges about the shapes to be detected as well as to keep a link with the observations field.
The **MRF** model requires accurate knowledges of the statistical repartition of sonar picture pixels. Under some assumptions presented in section 2, on one hand the statistical distribution of the pixels relative to the bottom follows a Rayleigh law and on the other hand, the pixels relative to the shadow are statistically described with a Gaussian law.
To determine the maximum likelihood estimate of all statistical law parameters, we compare two algorithms presented in section 4: the Estimation Maximization (**EM**) algorithm and the Stochastic Estimation Maximization (**SEM**) algorithm. We have applied these processes on numerous real high resolution sonar pictures. The results are shown in section 5 with some conclusions in section 6.

2 PRINCIPLE OF SONAR IMAGERY

Sonar imagery is based on the sampling in elementary surfaces, called resolution cells, of an observed area. The dimensions of the resolution cell depend on sonar aperture, duration of the emitted signal and inclination of the sonar. It is necessary that the elementary surface dimensions are smaller than the dimensions of the object to be detected. We suppose this condition satisfied. If the sonar beam intercept the object with a grazing angle θ_g so it exists an area behind it which does not receive any acoustical energy. The projected shadow on sonar picture corresponds to a lack of signal (figure 1).

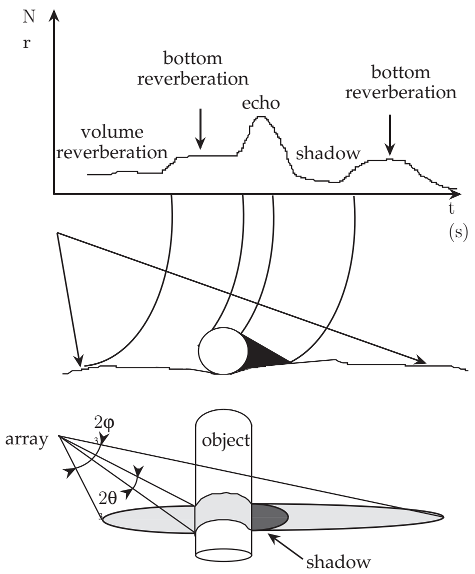

FIG. 1 – *evolution of the reverberated signal.*

If the minor lobes of the acoustical array are sufficiently weak, then the signal relative to the shadow area is essentially electronic noise, emanating from the processing chain, and the signal amplitude A is noised with a Gaussian distribution function:

$$G(A) = \frac{1}{\sqrt{2\pi}\sigma} \exp \frac{(A-\mu)^2}{2\sigma^2}$$

Where μ is the mean of the amplitude and σ the standard deviation. If the minor lobes of the acoustical array are not weak, the signal can be considered as coming from the sea bottom reverberation. Now we will see how to statistically describe the signal reverberating by the seafloor.

When a surface constituted by a large number of elementary scatterers is illuminated by a monochromatic and coherent source, then the amplitude A of the reflected wave is the sum of all scatterers contributions. The constructive and the destructive interferences of those reflected waves produce a noise corruption called speckle noise. If the reflectors are statistically independent and if the roughness is larger than the scale of the wave length, then for a large number of scatterers in the resolution cell, the amplitude A of the reflected wave follows a Rayleigh probability function[10,11,8] :

$$R(A) = \frac{A}{\alpha^2} \exp -\frac{A^2}{2\alpha^2}$$

Where $\alpha = \sqrt{\frac{2}{\pi}}\mu$ is the Rayleigh's law specific parameter.

We have a set of sonar images for which the amplitude of the reflected wave, function of the covered distance and the observation direction, is represented by a scale of grey level varying from 0 to 255. The histogram of merely bottom images shows that the grey level distribution does not begin at the value zero but at a quantity *min* which differs from a picture to another. The possible reasons to this shifted histogram are the different processes to establish the final sonar image: automatic control of gain, coding, reduction of the dynamic, ...
To take those phenomena into account, we propose a modified model of the Rayleigh law. We introduce the parameter *min* to shift the law. So the new expression of the probability function relative to the bottom reverberation is:

$$R(A, min) = \begin{cases} \frac{A-min}{\alpha^2} \exp -\frac{(A-min)^2}{2\alpha^2} & \text{when} \quad A > min \\ 0 & \text{elsewhere} \end{cases}$$

The Markovian modeling presented in the next section requires the knowledge of the statistical distribution law of the shadow area pixels. For that, we use the Kolmogorov criterion or the χ^2 criterion.[13]The Markovian modeling presented in the next section requires the knowledge of the statistical distribution law of the shadow area pixels. For that, we use the Kolmogorov criterion or the χ^2 criterion.

3 SONAR IMAGERY AND MARKOVIAN MODELING

The originality of our work is to develop an unsupervised hierarchical **MRF** modeling in order to solve efficiently this specific problem of shadow segmentation on sonar pictures. The segmentation of sonar images in two classes (label 0 for *shadow* and label 1 for *sea-bottom reverberation*) is stated as a statistical labeling problem according to a global Bayesian formulation. The aim of the **MRF** model is:

- to specify interactions between the sets of *observations* y (sonar picture) to be considered and *labels* x (binary image) to be determined ;
- to ensure the regularization of the set of labels by introducing generic *a priori* knowledge on objects of interest and on the shape of their shadows.

The main interest of **MRF** modeling consists in an explicit link between observations field y and label field x. We have to maximize, according to x, the following expression:

$$P_{X/Y}(x/y) = \frac{P_{X,Y}(x,y)}{P_Y(y)} \propto P_{Y/X}(y/x)P_X(x)$$

Under Markovian assumption, on one hand $P_X(x)$ is a Gibbs distribution (Hammersley-Clifford theorem):

$$P_X(x) = \frac{1}{Z} e^{-U_1(x)} \text{ with } Z = \sum_x e^{-U_1(x)}$$

and on the other hand, $P_{Y/X}(y/x)$ describes explicitly the link between observation field and label field. With noises described by exponential probability distributions (Gaussian or Rayleigh laws), the maximization of the preceding formula corresponds to the minimization of an energy function $U(x,y)$, which is composed of two terms:

$$U(x,y) = U_1(x,y) + U_2(x)$$

- The first one expresses the adequacy between observation and model of noise. In the case of sonar images, Rayleigh's law is a good degradation model to relate the observations to the label process.[2,5,18]
- The second energy term characterizes the fact that the expected label field is geometrically speaking rather regular. It describes *a priori* information.

We adopt an 8-connexity neighborhood and we consider the corresponding set $\mathcal{C}$ of two-site and four-site cliques (figure2).

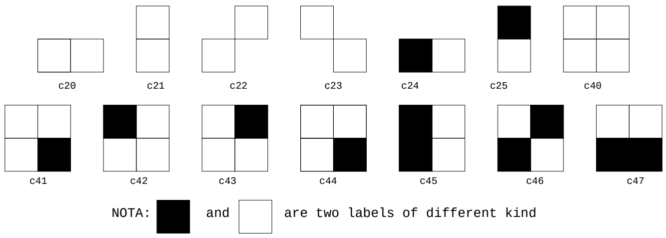

FIG. 2 – *Particular configurations c_{ij} of labels on two-site and four-site cliques adopted to express specific geometric shapes. The set of cliques is noted C*

With two-site and four-site clique, we can express specific geometric patterns well-fitted to the objects of interest and their shadows, by means of particular local configurations denoted c_{ij} (where $i \in \{2,4\}$ is the order of the cliques and j designates the type of local configuration lying on this clique). Through potential functions V_c, we either favor, or discourage particular or are neutral with clique configurations. The second energy term is defined as follows:

$$U_2(x) = \sum_{c \in C} V_c(x)$$

with $V_c(x) = -\beta_{ij}$ if x is of type c_{ij} on clique c.

The sign and value of β_{ij} parameters depend on the influence we want to give to configuration c_{ij}. This energy term requests up to twelve parameters. In order to have an unsupervised estimation, we use the "Qualitative Box" method[1] which enables us to calibrate β_{ij} parameters.[17]

Some studies[7,9,14] proposed a new approach (Markovian multiscale modeling), and a synthetic overview recently[6] tried to classify the different hierarchical **MRF**-based approaches. In our approach, the pyramidal structure of the label field is associated to a single observation level. The energy function is re-written at each scale as a coarser **MRF** model (derived from the one defined at full resolution). Structure and parameters are deduced from the original one.[16] In this way, the multigrid model developed will allow us to speed up the convergence rate and to improve the quality of the segmentation. The hierarchical modelization is only used for the label field, the observations (sonar picture) remain at the finest resolution. Then, the parameters associated with the observations have thus to be estimated automatically. The next part proposes some new results to do these noise parameter estimations, with unsupervised methods, in the context of MRF modeling for sonar picture segmentation.

4 NOISE MODEL PARAMETERS ESTIMATION

In this section, we show how **EM** *(Expectation Maximization)* method or **SEM** (*Stochastic Estimation Maximization*) algorithm can be used to determine a Maximum Likelihood (**ML**) estimate of the noise model parameters without any user interaction. The main interest of this section is to show how to use these algorithms for a specific application to sonar imagery where we take the variety of laws in the distribution mixture into account.
Firstly, we recall the mixture estimation problem in sonar imagery and the Maximum Likelihood estimators when the correct segmentation is known (X observable). The problem to estimate the model parameters directly from the image is then described. The **EM** method and the recent **SEM** algorithm are briefly described. Then, we propose a modification in order to estimate the noise model parameters of a mixture of different laws for a specific application to sonar imagery. Experimental results will be presented and discussed in section 5.

4.1 Mixture estimation problem in sonar imagery

We consider a couple of random fields $Z = (X, Y)$, with $Y = \{Y_s, s \in S\}$ is the field of observations located on a lattice grid S of N sites s. Each of the Y_s take its value in $\Lambda_{obs} = \{0, \dots, 255\}$ (256 grey levels) and each $X = \{X_s, s \in S\}$ in a finite set $\Omega = \{e_0 = \mathit{shadow}, e_1 = \mathit{sea\ bottom\ reverberation}\}$. We assume that the distribution of (X, Y) is defined by, firstly, $P_{X_i}(e_m) = \Pi_m$, $0 \leq m < K$, the proportion of the class e_m and, secondly, the distributions family $P_{Y/X_i}(y/e_m)$. The observable Y is called the *incomplete data* and Z the *complete data*. We suppose that the Y_i's are independent given X and the X_i's are also independent for $i = 1, \dots, N$. The superscript denote the iteration number and K, the number of classes in the image (=2 for this study).
Assuming that a sonar image corresponds to a finite mixture, we observe a sample $y = \{y_1, \cdots, y_N\}$, realization of Y which distribution has for density:

$$P_Y(y) = \sum_{p=0}^{K-1} \Pi_p \cdot P_{Y/X_i}(y/e_p) \qquad (1)$$

We have to estimate the parameters $\Phi_x = \Pi_m$ and Φ_y which define $P_X(e_m)$ and $P_{Y/X_i}(y/e_m)$ $(0 \leq m < K)$ respectively. In our case, these distributions vary with the class. More precisely we have shown that Gaussian law, $\mathcal{N}(\mu, \sigma^2)$, is a good degradation model to describe the noise added on the *shadow* class (essentially due to the electronic noise). In order to take the speckle noise phenomenon into account, we model the conditional density function of the *sea-bottom reverberation* class by a shifted Rayleigh's law, namely $\mathcal{R}(min, \alpha^2)$.[16]

These are a number of iterative methods to deal with this problem. Mostly (Fourier, polynomials and cumulate histogram methods) are inefficient in the case of an important distribution mixture or with a mixture of different laws.[15]

4.2 Maximum likelihood estimator

Suppose that X is observable and let $P_{Y/\Phi_y}(y/\phi_y)$ be the probability density function (**pdf**) of the incomplete data $Y = \{Y_1, \ldots, Y_n\}$ where Φ_y is a set of parameters that characterizes the **pdf**. A **ML** approach consists in finding an estimate $\hat{\Phi}_y$, such that:

$$\hat{\Phi}_y = \arg\max_{\Phi_y} \ \ln P_{Y/\Phi_y}(y/\phi_y) \tag{2}$$

If $P_{Y_i/\Phi_y}(y_i/\phi_y)$ is a Gaussian law, $(\mathcal{N}(\mu, \sigma^2))$, we easily obtain the empirical mean $\hat{\mu}_{ML}$ and variance $\hat{\sigma}^2_{ML}$:

$$\hat{\mu}_{ML} = \frac{1}{N} \cdot \sum_{i=1}^{N} y_i \tag{3}$$

$$\hat{\sigma}^2_{ML} = \frac{1}{N-1} \cdot \sum_{i=1}^{N} (y_i - \hat{\mu}_{ML})^2 \tag{4}$$

If each pixel Y_i follows a shifted Rayleigh's law, the log-likelihood function can be written as

$$\begin{aligned} \ln \mathcal{L}(\Phi_y) &= \ln P_{Y/\Phi_y}(y/\phi_y) \\ &= \ln \left\{ \prod_{i=1}^{N} \frac{(y_i - min)}{\alpha^2} \cdot \exp\left[-\frac{(y_i - min)^2}{2\alpha^2}\right] \right\} \end{aligned}$$

with $(min < y_i)\ \forall i$, and under the independence assumption. The **ML** estimates for the unknown parameter $\Phi_y = (min, \alpha^2)$ are given by the maximum value of the log-likelihood function. Setting the partial derivatives equal to zero, then we have to solve the system $\frac{\partial \ln \mathcal{L}(\Phi_y)}{\partial \Phi_y} = 0$, which gives the **ML** estimators of the *complete data*. ($\hat{y}_{min}$ is the minimum grey level of the sample y). We obtain the following results[12] :

$$\hat{\alpha}^2_{ML} = \frac{1}{2N} \cdot \sum_{i=1}^{N} (y_i - \widehat{min}_{ML})^2 \tag{5}$$

$$\widehat{min}_{ML} \approx \hat{y}_{min} - 1 \tag{6}$$

Nevertheless, X is not observable (we don't know what is the label associated with the pixel Y_i) and we have to estimate Φ_y only with $Y = y$. In order to find these **ML** estimates from the *incomplete data*, one way consist in using the iterative method called **EM** method[4] or the **SEM** algorithm.[3]

4.3 EM algorithm

In this section, we describe briefly the **EM** algorithm. The **EM** algorithm[4] is an iterative algorithm, introduced by *Dempster*, *Laird*, and *Rubin* to calculate the **ML** estimates when the observations can be viewed as *incomplete data*. This algorithm begins with an initial estimate $\hat{\Phi}^{[0]}$, and then consists in the two following steps at each iteration:

Estimation Step – Find the function

$$Q(\Phi, \hat{\Phi}^{[p]}) \stackrel{déf}{=} E[\ln P_{Z/\Phi}(z/\phi) \mid y, \hat{\phi}^{[p]}] \tag{7}$$

Maximization Step – Find $\hat{\Phi}^{[p+1]}$ that maximize Q.

$$\hat{\Phi}^{[p+1]} = \arg\max_{\Phi} \ Q(\Phi, \hat{\Phi}^{[p]}) \tag{8}$$

The **EM** algorithm can be interpreted as an alternative to the maximization of $P_{Z/\Phi}(z/\phi)$ over Φ, where the algorithm maximizes the expectation of $P_{Z/\Phi}(z/\phi)$ given the available information, namely the observed data Y, and the current estimate of the parameters $\Phi^{[p]}$. It has been shown that under some relatively general conditions, the estimates converge to the **ML** estimates, at least locally.
In what follows, we recall briefly the equations obtained to estimate the parameters of a mixture of Gaussian densities.[19] We examine in details the situation of a shifted Rayleigh's laws mixture, and the mixture of different laws.
The Y_i's are independent given X and X_i's are also independent for $i = 1, \ldots, N$. In this case, the log-likelihood function of the *complete data* can be written as

$$\begin{aligned} \ln P_{Z/\Phi}(z/\phi) &= \ln P_{X,Y/\Phi}(x, y/\phi) \\ &= \ln P_{Y/X,\Phi_y}(y/x, \phi_y) + \ln P_{X/\Phi_x}(x/\phi_x) \\ &= \sum_{i=1}^{N} \ln P_{Y_i/X_i,\Phi_y}(y_i/x_i, \phi_y) + \sum_{i=1}^{N} \ln P_{X_i/\Phi_x}(x_i/\phi_x) \end{aligned} \tag{9}$$

We now define an unobservable K dimensional vector $\mathbf{x}_i^t = [\mathbf{x}_{i,0}, \ldots, \mathbf{x}_{i,K-1}]$ associated with X_i which entries are all nil except for the k^{th} entry, which is one for which Y_i has actually been generated by the k^{th} density of the mixture. ($\mathbf{x}_i$ is substantially equivalent to the unobservable region process x_i), we have,

$$\ln P_{Y_i/X_i,\Phi_y}(y_i/x_i, \phi_y) = \mathbf{x}_i^t \cdot U(y_i/\phi_y) \tag{10}$$

$$\text{and similarly,} \quad \ln P_{X_i/\Phi_x}(x_i/\phi_x) = \mathbf{x}_i^t \cdot V(\phi_x) \tag{11}$$

$$\text{Where} \quad U(y_i/\phi_y) = [\ln P_{Y_i/X_i,\Phi_y}(y_i/e_0, \phi_y), \cdots, \ln P_{Y_i/X_i,\Phi_y}(y_i/e_{K-1}, \phi_y)]^t \tag{12}$$

$$\text{and} \quad V(\phi_x) = [\ln P_{X_i/\Phi_x}(e_0/\phi_x), \cdots, \ln P_{X_i/\Phi_x}(e_{K-1}/\phi_x)]^t \tag{13}$$

Therefore, the expression for $Q(\Phi, \hat{\Phi}^{[p]})$ becomes (using the equations (9)(10)(11)(12)(13)).

$$Q(\Phi, \hat{\Phi}^{[p]}) = \sum_{i=1}^{N} E[\mathbf{x}_i^t | y, \hat{\phi}^{[p]}] \cdot U(y_i/\phi_y) + \sum_{i=1}^{N} E[\mathbf{x}_i^t | y, \hat{\phi}^{[p]}] \cdot V(\phi_x) \tag{14}$$

Where the component of the conditional expectation are:

$$E[\mathbf{x}_{ik}|y] = P_{X_i/Y}(e_k/y) = \frac{P_{Y/X_i}(y/e_k) \cdot P_{X_i}(e_k)}{\sum_{j=0}^{K-1} P_{Y/X_i}(y/e_j) \cdot P_{X_i}(e_j)} \tag{15}$$

where $\mathbf{x}_{ik}$ is the k^{th} component of $\mathbf{x}_i$. We have dropped the conditioning on $\hat{\Phi}^{[p]}$ for notation convenience. If the x_i are independent of y_i for $j \neq i$, as assumed here, the preceding expression can be simplified:

$$E[\mathbf{x}_{ik}|y] = \frac{P_{Y_i/X_i}(y_i/e_k) \cdot P_{X_i}(e_k)}{\sum_{j=0}^{K-1} P_{Y_i/X_i}(y_i/e_j) \cdot P_{X_i}(e_j)} \tag{16}$$

Indeed, letting $\Pi_k^{[p]} = P_{X_i}(e_k)$, we have

$$\hat{\mathbf{x}}_{ik}^{[p]} = E[\mathbf{x}_{ik}|y] = \frac{\Pi_k^{[p]} \cdot P_{Y_i/X_i}(y_i/e_k)}{\sum_{j=0}^{K-1} \Pi_j^{[p]} \cdot P_{Y_i/X_i}(y_i/e_j)} \tag{17}$$

4.3.1 MIXTURE OF GAUSSIAN DISTRIBUTION

$P_{Y_i/X_i}(y_i/e_k)$ is Gaussian with mean μ_k and variance σ_k^2. Then it can be shown[19] that setting the partial derivatives of $Q(\Phi, \hat{\Phi}^{[p]})$ equal to zero and solving them for the unknown parameters, the following iterative scheme can be derived:

$$\mu_k^{[p+1]} = \frac{\sum_{i=1}^{N} \hat{\mathbf{x}}_{ik}^{[p]} \cdot y_i}{\sum_{i=1}^{N} \hat{\mathbf{x}}_{ik}^{[p]}} \tag{18}$$

$$(\sigma_k^{[p+1]})^2 = \frac{\sum_{i=1}^{N} \hat{\mathbf{x}}_{ik}^{[p]} \cdot (y_i - \hat{\mu}_k^{[p+1]})^2}{\sum_{i=1}^{N} \hat{\mathbf{x}}_{ik}^{[p]}} \tag{19}$$

$\hat{\mathbf{x}}_{ik}^{[p]}$ is given by (17) and

$$\hat{\Pi}_k^{[p+1]} = \frac{\sum_{i=1}^{N} \hat{\mathbf{x}}_{ik}^{[p]}}{N} \tag{20}$$

4.3.2 Mixture of Rayleigh distribution

$P_{Y_i/X_i}(y_i/e_k)$ is a shifted Rayleigh's law of parameters $\Phi_y = (min_k, \alpha_k^2)$. We search Φ_y that maximize the $Q(\Phi, \hat{\Phi}^{[p]})$ function (*Maximization Step*).

$$\hat{\Phi}_y^{[p+1]} = \arg\max_{\Phi_y} \; Q(\Phi, \hat{\Phi}^{[p]})$$

Setting the partial derivatives of $Q(\Phi, \hat{\Phi}^{[p]})$ equal to zero and using (10) (11), we obtain

$$\sum_{i=1}^{N} [\nabla_{\Phi_y} U^t(y_i/\phi_y)] \cdot E[\mathbf{x}_i | y, \hat{\phi}^{[p]}] + \sum_{i=1}^{N} [\nabla_{\Phi_y} V^t(\phi_x)] \cdot E[\mathbf{x}_i | y, \hat{\phi}^{[p]}] = 0 \tag{21}$$

$V^t(\phi_x)$ is not dependent of min and α^2, then we have,

$$\sum_{i=1}^{N} [\nabla_{\Phi_y} U^t(y_i/\phi_y)] \cdot E[\mathbf{x}_i | y, \hat{\phi}^{[p]}] = 0 \tag{22}$$

Solving this equation for the unknown parameters $\Phi_y = (min_k, \alpha_k^2)$, we obtain

$$\sum_{i=1}^{N} \mathbf{x}_{ik}^{[p]} \cdot \frac{\partial}{\partial \alpha_k^2} \left\{ \ln P_{Y_i/X_i}(y_i/e_k) \right\} \;=\; 0 \tag{23}$$

$$\sum_{i=1}^{N} \mathbf{x}_{ik}^{[p]} \cdot \frac{\partial}{\partial min_k} \left\{ \ln P_{Y_i/X_i}(y_i/e_k) \right\} \;=\; 0 \tag{24}$$

From (23) we have

$$(\hat{\alpha}_k^{[p+1]})^2 = \frac{\sum_{i=1}^{N} \hat{\mathbf{x}}_{ik}^{[p]} \cdot (y_i - \widehat{min}_k^{[p+1]})^2}{2 \cdot \sum_{i=1}^{N} \hat{\mathbf{x}}_{ik}^{[p]}} \tag{25}$$

$\widehat{min}$ given by (24) is not easily computable and we use another that estimates this parameter : **k-mean** algorithm (Cf. subsection 4.5).

4.3.3 Mixture of different distributions

In order to estimate the parameters of mixture of different laws (for example a Gaussian law for the first class and a shifted Rayleigh's law for the second class), the **EM** algorithm can be easily modified and works as follow.
During the *Estimation Step*, for each y_i, we define the next distribution $\mathbf{x}_i$=$[\hat{\mathbf{x}}_{i,k=0}, \ldots, \hat{\mathbf{x}}_{i,k=K-1}]$ with a Gaussian law for $P_{Y_i/X_i}(y_i/e_0)$ and $P_{Y_i/X_i}(y_i/e_1)$, a Rayleigh's law, from the current estimate of the parameters $\Phi^{[p]}$ of each class. During the *Maximization Step*, we use the expression (18)(19)(20) to estimate the parameters of the shadow class ($\hat{\mu}_0^{[p+1]}$, $(\hat{\sigma}_0^{[p+1]})^2$) and $\hat{\Pi}_0$ respectively, and the relation (25) (20) to re-estimate $(\hat{\alpha}^{[p+1]})^2$ and $\hat{\Pi}_1$ the parameters of the *sea bottom reverberation* class.

4.4 SEM algorithm

This subsection is devoted to the **SEM** algorithm, which is described in general terms. The **SEM**[3] is a recent density mixture estimator which is an improvement of the **EM** method obtained by the addition of a stochastic component. The **SEM** algorithm behaves as follows:

Initialization Step:
We take, for every observations y_i, a probability $P_{X_i/Y_i}(e_m/y_i)$ of its belonging to the class e_m, $(0 \leq m < K)$ and for every $p \geq 0$.

Stochastic Step:
For each y_i, we select from the set of classes $\{e_0, \dots, e_{K-1}\}$ an element according to the distribution $[P^{[p]}_{X_i/Y_i}(e_0/y_i), \dots, P^{[p]}_{X_i/Y_i}(e_{K-1}/y_i)]$. This selection defines a partition $[Q^{[p]}_0, \dots, Q^{[p]}_{K-1}]$ of the sample $y = \{y_1, \dots, y_N\}$.

Maximization Step:
The **SEM** algorithm supposes that every y_i belonging to $Q^{[p]}_m$ for each m $(0 \leq m < K)$ is realized according to the distribution defined by $P_{Y/X_i}(y/e_m)$, the density corresponding to the class e_m. By denoting $C^{[p]}_m = card(Q^{[p]}_m)$, $Q^{[p]}_m = (y^{[p]}_{1,m}, y^{[p]}_{2,m}, \dots, y^{[p]}_{C^{[p]}_m, m})$, we can estimate Φ_y, the parameters of the mixture with the Maximum Likelihood estimator of each class.

- In the case of mixture of two Gaussian distributions, we use the empirical mean and variance.
- In the case of mixture of two Rayleigh distributions, we use the **ML** estimators defined previously (4.3).

And in the two cases, Π_m is given by the empirical frequencies

$$\hat{\Pi}^{[p+1]}_m = \frac{C^{[p]}_m}{N} \tag{26}$$

Estimation Step:
For each y_i, we define the next distribution $[P^{[p+1]}_{X_i/Y}(e_0/y), \cdots, P^{[p+1]}_{X_i/Y}(e_{K-1}/y)]$ on the set of classes by the *a posteriori* distribution based on the current parameter $\Phi^{[p+1]}$:

$$P^{[p+1]}_m(y_i) = \frac{\Pi^{[p+1]}_m \cdot P_{Y_i/X_i}(y_i/e_m)}{\sum_{j=0}^{K-1} \Pi^{[p+1]}_j \cdot P_{Y_i/X_i}(y_i/e_j)} \tag{27}$$

Remember that $P^{[p+1]}_{Y/X_i}(y/e_m)$ designates the Gaussian distribution corresponding to $[\mu^{[p+1]}_m, (\sigma^{[p+1]}_m)^2]$ in the case of a mixture of two Gaussian distributions and the Rayleigh distribution corresponding to $[min^{[p+1]}_m, (\alpha^{[p+1]}_m)^2]$ in the case of a mixture of two Rayleigh distributions. If the sequence Φ becomes steady, the **SEM** algorithm is ended else we return to *Stochastic Step*. Numerous simulations show the correct behavior of the **SEM**. Its theoretical study is performed in the case of the mixture of two Gaussian distributions but this algorithm can be easily used to estimate the mixture of two Rayleigh's laws.

4.4.1 Mixture of different distributions

In the case of the mixture of two different laws, the general procedure remains the same. Nevertheless during the *Maximization Step*, we use the empirical mean, variance and frequency for the partition $Q^{[p]}$ associated to the *shadow* class defined by equation (3), (4), and (26) respectively and the **ML** estimator for the partition $Q^{[p]}$ associated to the *sea bottom reverberation* class (equation (5)(6) and (26)). During the *Estimation Step*, $P_{Y/X_i}(y/e_0)$ designates the Gaussian distribution and $P_{Y/X_i}(y/e_1)$ the Rayleigh distribution.

4.5 Initialization

Initial parameter estimates have a significant impact on the rapidity of the convergence of these algorithms and on the quality of the resulting final estimates. Remember that the **EM** method converges to a *locally* and not necessary *globally*

optimum estimate. Besides, the **EM** method can not estimate the *min* parameter of the *sea bottom reverberation* class. In our application, we use the following method:
The estimates of the initial parameters of the noise model are determined by running a $6 * 6$ non overlapping sliding window over the image and calculating the sample mean, variance and minimum grey level estimates. These estimates are then clustered into two classes using the **k-means** clustering algorithm. **ML** estimations are then used over the **k-means** segmentation to find $\Phi_y^{[0]}$, and $\Phi_x^{[0]}$.

5 RESULTS ON REAL SONAR PICTURES

The quality of the estimations is difficult to appreciate in absence of the real values. We can roughly perform such an evaluation with the comparison of the histograms with the probability densities mixture based on the estimated parameters. These iterative algorithms were applied to the image shown in the two Figures A_0. The first one represents a manufactured object shadow and the second one, several rock shadows and the shadow of a manufactured object. Figures C_0 show the resulting mixture solution in graphical form. The two dashed curves in the figures show the individual components $P_{Y/X_i}(y/e_m)(0 \leq m < K)$.In these cases, the **SEM** algorithm and the **EM** method seem to give good results. In both cases, the histogram is quite close to the mixture densities based on estimated parameters, and a segmentation with these estimates gives good results.
According to the kolmogorov distance and χ^2 criterion, the shadow class is better approximated by a Gaussian law rather than a Rayleigh law (Cf.table 1). For several numerous experimentations, the **SEM** algorithm is reliable in all the situations tested. On the other hand, the **EM** method has some difficulties to find a reasonable solution for a few images, in particular for poorly separated distributions and when the initialization is not very good.
The results obtained were foreseeable, remind that the use of the **EM** method is non trivial and its theoretical justification requires strong hypotheses.[19] Moreover the **SEM** algorithm is an improvement of the **EM** method obtained by the addition of a stochastic component. This is why the solution is less dependent of the initialization and the speed of convergence is appreciably improved.

sonar picture1	*distributions: bottom: Rayleigh shadow: Rayleigh*	*distributions: bottom: Rayleigh shadow: Gauss*
Kolmogorov distance	0.077	0.069
χ^2 *criterion*	4606	4144

sonar picture 2	*distributions: bottom: Rayleigh shadow: Rayleigh*	*distributions: bottom: Rayleigh shadow: Gauss*
Kolmogorov distance	0.127	0.120
χ^2 *criterion*	5647	5056

TAB. 1 – *SEM Algorithm: Kolmogorov distance and χ^2 criterion for a mixture of Rayleigh's distributions and a mixture of different distributions*

6 CONCLUSION

In sonar imagery, a low level step of a segmentation in two classes; *i.e. shadow* (due to the lack of acoustic reverberation) and *sea-bottom reverberation* is necessary and allows us the detection and then the classification of objects located on the sea floor. The use of Markov Random Field models within the framework of global Bayesian decision can give good results, but random field models are specified by a number of parameters (in particular the noise model parameters) which have to be estimated from the data to solve the problem of sonar image unsupervised segmentation.
In order to estimate these parameters, we have shown that the **EM** method and the **SEM** algorithm could be used and could be taken into account the variety of the laws in the distribution mixture. The two methods presented, lead to formula easy to calculate. The simplest one being the **EM** method in which an iterative procedure produces estimates of the mixture without *stochastic step*. The **EM** method gives satisfactory results provided that the distributions are not too poorly separated and when the initialization is not bad in which case the **SEM** method yields to better results. From this point of view, we can say that the **SEM** algorithm seems better suited in the context of sonar imagery. Numerous simulations have shown the correct behavior of the **SEM** in the case of a mixture of different laws. Results obtained are good and consistently better (according to the Kolmogorov distance and χ^2 criterion) than those obtained with a mixture

of two Rayleigh distributions.The parameters estimation of the hierarchical Markovian model (more precisely the spatial and inter-level cliques) will be the topic of our future research.

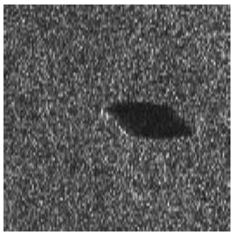

Fig A_0: sonar picture (object shadow)

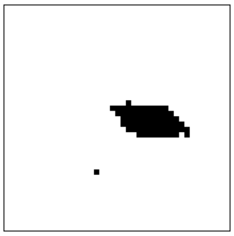

Fig B_0: result of the k-means clustering procedure

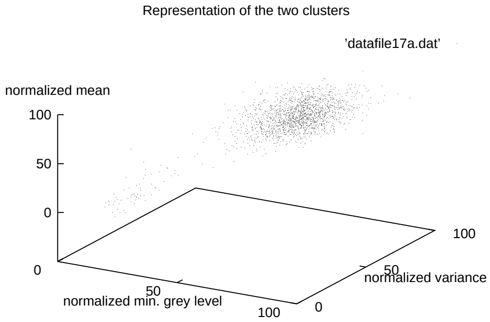

Fig B_1: representation of the two clusters for k-mean algorithm

	Initialization : **K-means**		
$\hat{\Phi}^{[0]}_{y_{(shadow)}}$	$0.04_{(\pi)}$	$36_{(moy)}$	$55_{(\sigma^2)}$
$\hat{\Phi}^{[0]}_{y_{(sea-bottom)}}$	$0.96_{(\pi)}$	$39_{(min)}$	$1061_{(\alpha^2)}$
	SEM		
$\hat{\Phi}^{[0]}_{y_{(shadow)}}$	$0.04_{(\pi)}$	$34_{(moy)}$	$30_{(\sigma^2)}$
$\hat{\Phi}^{[0]}_{y_{(sea-bottom)}}$	$0.96_{(\pi)}$	$39_{(min)}$	$1506_{(\alpha^2)}$
	EM		
$\hat{\Phi}^{[0]}_{y_{(shadow)}}$	$0.04_{(\pi)}$	$34_{(moy)}$	$28_{(\sigma^2)}$
$\hat{\Phi}^{[0]}_{y_{(sea-bottom)}}$	$0.96_{(\pi)}$	$39_{(min)}$	$1553_{(\alpha^2)}$

table II: estimated parameters

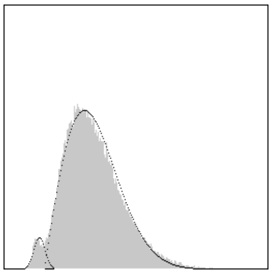

Fig C_0: Image histogram and estimated mixture (SEM)

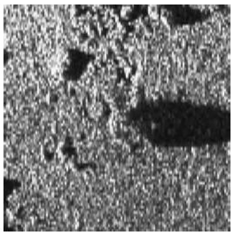

Fig A_0: sonar picture (object shadow and rock shadow)

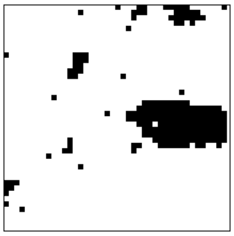

Fig B_0: result of the k-means clustering procedure

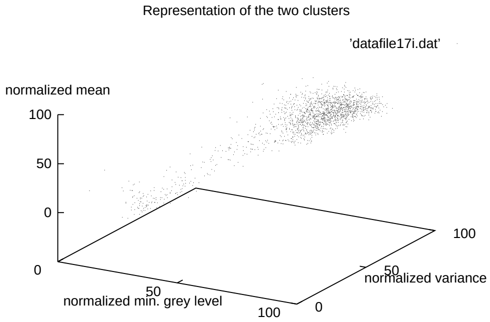

Fig B_1: representation of the two clusters for k-mean algorithm

	Initialization : **K-means**		
$\hat{\Phi}^{[0]}_{y_{(shadow)}}$	$0.11_{(\pi)}$	$34_{(moy)}$	$149_{(\sigma^2)}$
$\hat{\Phi}^{[0]}_{y_{(sea-bottom)}}$	$0.89_{(\pi)}$	$39_{(min)}$	$2830_{(\alpha^2)}$
	SEM		
$\hat{\Phi}^{[0]}_{y_{(shadow)}}$	$0.12_{(\pi)}$	$30_{(moy)}$	$56_{(\sigma^2)}$
$\hat{\Phi}^{[0]}_{y_{(sea-bottom)}}$	$0.88_{(\pi)}$	$39_{(min)}$	$4841_{(\alpha^2)}$
	EM		
$\hat{\Phi}^{[0]}_{y_{(shadow)}}$	$0.12_{(\pi)}$	$30_{(moy)}$	$55_{(\sigma^2)}$
$\hat{\Phi}^{[0]}_{y_{(sea-bottom)}}$	$0.88_{(\pi)}$	$39_{(min)}$	$4926_{(\alpha^2)}$

table II: estimated parameters

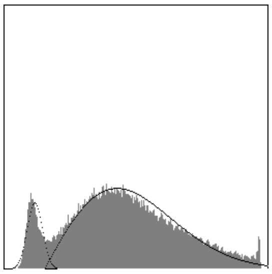

Fig C_0: Image histogram and estimated mixture (EM)

Figures A_0 are the original observations,figures B_0 the results of the **k-means** *clustering algorithm and Figures B_1,the two clusters associated to the shadow and sea bottom reverberation classes. The mixture of distribution is represented by Figure C_0. The result obtained are given in Table II.*

7 REFERENCES

[1] [GTS-464]. R. AZENCOTT. Image analysis and Markov fields. In *Proc. of the Int. Conf. on Industrial and Applied. Math, SIAM*, pages 53–61, Philadelphia, June 1988.

[2] [GTS-263]. W. S. BURDIC. *Underwater Acoustic System Analysis.* In Signal Processing Series. Prentice-Hall, New Jersey, 1984.

[3] [GTS-810]. G. CELEUX and J. DIEBOLT. Un algorithme d'apprentissage probabiliste pour la reconnaissance de mélange de densité. *Revue de statistiques appliquées*, 34(2):1988–1996, 1986.

[4] [GTS-705]. A.P. DEMPSTER, N.M. LAIRD, and D.B. RUBIN. Maximum likelihood from incomplète data via the em algorithm. *Royal Statistical Society*, pages 1–38, 1976.

[5] [GTS-281]. J. W. GOODMAN. Some fundamental properties of speckle. *Journal of Optical Society of America*, 66(11):1145–1150, November 1976.

[6] [GTS-365]. C. GRAFFIGNE, F. HEITZ, P. PEREZ, F. PRETEUX, M. SIGELLE, and J. ZERUBIA. Hierarchical Markov random field models applied to image analysis: a review. In *SPIE Neural Morphological and Stochastic Methods in Image and Signal Processing*, San Diego, 10-11 July 1995.

[7] [GTS-251]. F. HEITZ, P. PEREZ, and P. BOUTHEMY. Multiscale minimisation of global energy functions in some visual recovery problems. In *Computer Vision Graph. and Image Proces.: Image Understanding*, volume 59, January 1994.

[8] [GTS-825]. A. HILLION and J.M. BOUCHER. A unified approach to non-linear processing of multiplicative noise with applications to radar images. In *Signal Processing V. Theories and Applications. Proceedings of EUSIPCO-90, Fifth European Signal Processing Conference*, volume 3, pages 2027–30, Barcelona, Spain, Sept 1990.

[9] [GTS-622]. Z. KATO, M. BERTHOD, and J. ZERUBIA. A hierarchical Markov random field model and multitemperature annealing for parallel image classification. *Graphical Models and Image Processing*, 58(1), 1996.

[10] [GTS-812]. D. T. KUAN, A. A. SAWCHUK, T. C. STRAND, and P. CHAVEL. Adaptative restoration of images with speckle. *IEEE Transactions on Acoustic Speech and Signal Processing*, ASSP-35(3):373–383, march 1987.

[11] [GTS-816]. J.S. LEE. Speckle supression and analysis for synthetic aperture radar images. *OPTICAL INGINEERING*, 25(5):636–643, May 1986.

[12] [GTS-1046]. M. MIGNOTTE. Modèle markovien: estimation des paramètres. Technical Report 7, Ecole Navale, Mars 1996.

[13] [GTS-]. A. PAPOULIS. *Probability, Random Variables, and Stochastic Processes.* Mc Graw-Hill, 1991.

[14] [GTS-202]. P. PEREZ and F. HEITZ. Une approche multiéchelle à l'analyse d'images par champs Markoviens. *Revue Traitement du signal*, 9(6):459–472, 1992.

[15] [GTS-713]. J.G. POSTAIRE and P.A. VASSEUR. An approximate solution to normal mixture identification with application to unsupervised pattern classification. *IEEE trans. on PAMI*, (2):163–179, 1981.

[16] [GTS-641]. P. THOUREL. *Segmentation d'images sonar par modélisation markovienne hiérarchique et analyse multirésolution.* Phd (in french), Université de Bretagne Occidentale - Groupe de Traitement du Signal (Ecole Navale), July 1996.

[17] [GTS-384]. P. THOUREL, C. COLLET, P. BOUTHEMY, and P. PEREZ. Multiresolution analysis and MRF modeling applied to the segmentation of shadows in sonar pictures. In *Second Asian Conference on Computer Vision ACCV'95*, volume II, pages 81–85, Singapore, 1995.

[18] [GTS-264]. R. J. URICK. *Principles of Underwater Sound.* Mc Graw-Hill Book Company, 3d edition, 1983.

[19] [GTS-732]. T. YAMAZAKI, M.N. SHIRAZI, and H. NODA. Data-driven segmentation of degraded images based on markov random field models. *journal of the communications research laboratory*, pages 19–36, 1994.

The authors thank the Groupe d'Etude Sous-Marines de l'Atlantique for having provided us numerous real sonar pictures and the Direction des Recherches Etudes et Techniques for partial support of this work.