Improving Robustness of Monte-Carlo Global Illumination with Directional Regularization

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Abstract

Directional regularization offers great potential to improve the convergence rates of Monte-Carlo-based global illumination algorithms. In this paper, we show how it can be applied successfully by combining unbiased bidirectional strategies, photon mapping, and biased directional regularization.

CR Categories: Computer Graphics [I.3.7]: Three-Dimensional Graphics and Realism—Color, shading, shadowing, and texture Computing Methodologies [I.6.8]: Simulation and modeling—Types of Simulation:Monte Carlo

Keywords: bidirectional path tracing, photon mapping, variance reduction, multiple importance sampling

1 Introduction and Related Work

While light transport is well described by the *rendering equation* [Kajiya 1986], simulating efficiently all global-illumination effects remains a challenging problem. Several solutions have been proposed, but the category of Monte-Carlo integration methods proves to be the most general and popular today.

An efficient method for generating path samples for the Monte-Carlo process is the *Bidirectional Path Tracing* (BDPT) [Veach and Guibas 1995b], where random sub-paths are scattered in the virtual scene, starting from light sources and/or camera. To form a full path, *explicit connections* between sub-paths are created and their contributions are weighted accordingly to their probability of existence. However, highly specular materials, directional lights, etc., introduce singularities in the light path. These singularities prevent explicit connections between camera and light paths, ultimately leading to higher variance in the image or worse, to missing features (e.g., caustics from a point light seen through a mirror).

The sampling strategy of *Photon Mapping* (PM) methods [Jensen 2001] connects sub-paths within a local *spatial* range query. Although PM introduces bias through *spatial regularization*, it is proved consistent [Knaus and Zwicker 2011], reduces noise in some complex scenarios (e.g., caustics), and allows the simulation of features not handled with unbiased methods. On the other hand, PM suffers from artifacts and noise in other scenarios (e.g., illumination on a glossy surface) where unbiased BDPT is more efficient.

Some researchers have explored the possibility of introducing bias using *directional regularization* to treat specular surfaces [Kniep et al. 2009; Kaplanyan and Dachsbacher 2013] but restrain its application only to scenarios not handled by previous methods.

In this paper we generalize the application of *directional regularization* as a complement of standard Monte-Carlo methods in order to improve their robustness, not only in particular cases, but also to reduce noise in early stages of rendering. Our approach is similar to *Vertex Connection and Merging* (VCM) [Georgiev et al. 2012] and *Unified Path Space* UPS [Hachisuka et al. 2012], where *spatial regularization* forms an additional sampling strategy. Unlike them, we use *directional regularization* as an additional sampling strategy weighted with respect to other strategies in the context of *multiple importance sampling* (MIS) [Veach and Guibas 1995a].

2 Background and Notations

2.1 Light Transport Estimation

Veach [1995a] expresses the rendering equation as a combination of path samples drawn by several strategies j:

$$\hat{I} = \frac{1}{N}\sum_{i=1}^{N} w_j(\bar{x}_{i,j})\frac{F(\bar{x}_{i,j})}{P_j(\bar{x}_{i,j})}, \tag{1}$$

where $\bar{x}_{i,j}$ represents a path composed of k vertices $[v_1, \ldots, v_k]$ sampled with probability $P_j(\bar{x}_{i,j})$ using the j-th strategy. $\hat{I}$ is a measurement on the sensor (i.e., the amount of energy gathered by a pixel) consisting of N Monte-Carlo samples, and $F(\bar{x}_{i,j})$ is the energy associated with the path. $F(\bar{x}_{i,j})$ is mainly composed of BRDF terms $f_m(\omega_i^m, \omega_o^m)$ representing the energy scattered at path vertex v_m between the direction of vertices v_{m-1} and v_{m+1} represented as ω_i^m and ω_o^m.

To reduce variance, Veach and Guibas [1995a] propose a MIS scheme where the final contribution of a random sample $\bar{x}_{i,j}$ is weighted by (using the *balance heuristic*):

$$w_j(\bar{x}_{i,j}) = \frac{1}{N_j}\frac{P_j(\bar{x}_{i,j})}{\sum_{s'=1}^{s} P_{s'}(\bar{x}_{i,s'})}, \tag{2}$$

for s sampling strategies and N_j samples from the j-th strategy.

2.2 Singularities and Regularization

Specular BRDFs (e.g., due to mirror, glass, point light) introduce singularities. E.g., a mirror BRDF can be modeled at vertex v_m as:

$$f_m(\omega_i^m, \omega_o^m) = F_r\delta(R(\omega_i^m) - \omega_o^m), \tag{3}$$

where δ is the *Dirac delta* function, R is the reflection operator for incident direction, and F_r includes a geometry term and a reflection coefficient.

To create an explicit connection in BDPT between two random sub-paths, $[v_1, \ldots, v_p]$ and $[v_{p+1}, \ldots, v_k]$, we need to evaluate the sub-terms of $F(\bar{x}_{i,j})$: $f_p(\omega_i^p, \omega_o^p) f_{p+1}(\omega_i^{p+1}, \omega_o^{p+1})$. To ensure that

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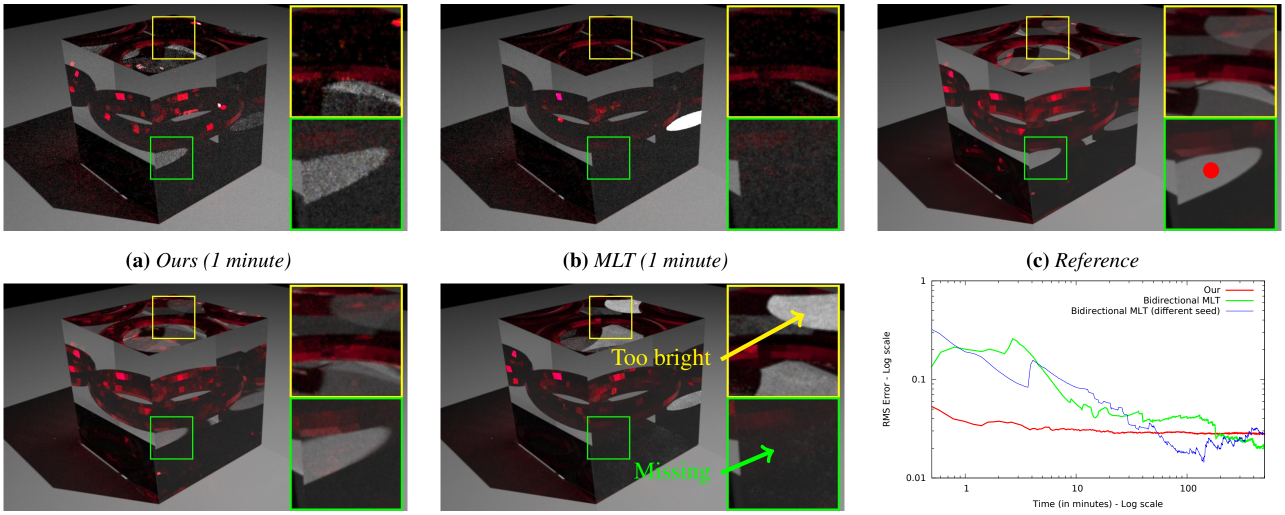


(a) *Ours (1 minute)* **(b)** *MLT (1 minute)* **(c)** *Reference*

(d) *Ours (25 minutes)* **(e)** *MLT (25 minutes)* **(f)** *RMS Error*

Figure 1: *A highly glossy torus enclosed in a glass cube and lit by a small area light. The caustics seen through the torus are especially difficult to reproduce. We render this scene using BDPT with a MLT sampling strategy with and without directional regularization. After one minute of computation, our strategy (a) reproduces most of the features seen in the reference image (c) while the rendering without regularization (b) misses most of them. After 25 minutes, our method (d) closely matches the final image while the rendering without regularization still misses important features (e) or overbrightens them. The bias introduced by our method leads to slighly blurred caustics (d) (bottom inset) that get sharper in the final image. An associated video illustrates this behavior and supplemental materials gives additional pixel convergency results for the red dot in the inset of (c). The plot (f) represents the log-based RMS error using high dynamic range images, as compared to the reference image. We compared our method as well to two runs of BDPT without regularization. Observe that our method leads to a better and more stable behavior in early stages of rendering. At the end of the rendering, differences between our method and the method without regularization is mainly stochastic noise barely discernable in low dynamic range images.*

the path scatters energy, this term should be non-null. However, because v_p and v_{p+1} are generated by two independent random processes (i.e., generating sub-paths from lights and camera), $F(\bar{x}_{i,j})$ is null if the BRDF of v_p or v_{p+1} is singular. A similar constraint holds in PM but only relies on one BRDF term. As a consequence, paths with many singularities will have very few or even no bidirectional sampling strategies available, and will therefore miss these phenomena, or result in noise in rendered images.

Directional regularization [Kaplanyan and Dachsbacher 2013] replaces $\delta(R(\omega_i)-\omega_o)$ in singular models by a finite support function

$$\hat{\delta}(R(\omega_i)-\omega_o) = \begin{cases} \frac{1}{2\pi(1-\cos\epsilon)} & \text{if } R(\omega_i)\cdot\omega_o > \cos\epsilon \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

where ϵ is the half aperture of a small cone. They only use this directional regularization to sample specific light paths.

In the following, we introduce this regularization in any sampling scenario to increase robustness. Our discussion focuses on reducing noise while keeping a small amount of introduced bias.

3 MIS Directional Regularization

Our formulation is based on the *Unified Path Space* from Hachisuka et al. [2012]. When a ray hits a specular surface, the outgoing direction is normally constrained by Snell's law. By introducing a directional perturbation, we allow a scattering direction to be uniformly sampled within a thin cone aligned with the mirror direction.

Consider a path composed of two unbiased sub-paths, $[v_1, \ldots, v_p]$ and $[v_{p+1}, \ldots, v_k]$, connected through a segment between vertices v_p and v_{p+1}. The probabilities of sampling both sub-paths are $\overrightarrow{P_j}$ and $\overleftarrow{P_j}$. We define the probability of sampling the full path as:

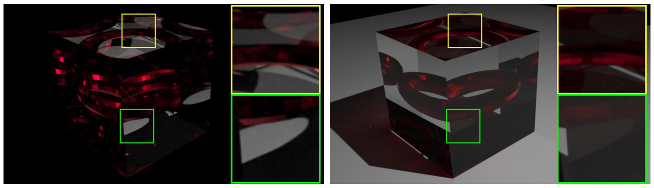
$$P_j(\bar{x}_{i,j}) = \frac{\overrightarrow{P_j}\overleftarrow{P_j}}{[2\pi(1-\cos\epsilon)]^{\min(N_s,2)-N_r}}, \quad (5)$$

where N_r is the number of specular surfaces involved in the connection (i.e., 2 if both v_p and v_{p+1} are specular, 1 if only one is, 0 if both are non-specular). N_s is the number of specular surfaces in the entire path. The "min" term ensures that no more than two regularizations can be introduced (i.e., when connecting two specular surfaces) and also treats the case where there are less than two specular surfaces in the path.

If a singularity appears in the evaluation of the connection, directional regularization (Equation 4) is used to compute the amount of scattered energy. To reduce the introduced bias, we apply virtual perturbations where the sampling of a regularized surface always returns the correct specular scattering direction.

Multiple Importance Sampling Once a path has been sampled using one strategy (possibly introducing regularization), its *probability density function* (PDF) must be evaluated with respect to other strategies using Equation 5 and weighted using MIS (Equation 2).

Consistency In order to make our method consistent, we must progressively reduce ϵ. Kaplanyan and Dachsbacher [2013] provide the details of this reduction and show that the directional regularization behaves exactly as a spatial regularization (in both cases it is a density estimation over a 2D surface). [Hachisuka et al. 2012;



(a) *Biased contributions* **(b)** *Unbiased contributions*

Figure 2: *Biased (a) and unbiased (b) contributions for the torus scene. Except for complex caustics paths, the image is mainly unbiased. Both strategies contribute to the caustics (bottom insets).*

Georgiev et al. 2012] give all the details about the consistency of MIS between biased 2D regularizations and unbiased samples.

Introduction of Bias Equation 5 shows that the PDF value is linked to two variable terms: $\overrightarrow{P_j}\overleftarrow{P_j}$ and N_r. When the path does not include regularization (i.e., $N_r = 0$), this results in a larger PDF value than when the path includes regularizations (i.e., $N_r \in \{1, 2\}$). Therefore paths created with regularizations have a lower weight, thanks to MIS (Equation 2). Therefore our method usually features low bias, except in complex sampling cases where the term $\overrightarrow{P_j}\overleftarrow{P_j}$ shows wide variation between the biased and unbiased strategies. This is where our contribution makes a difference.

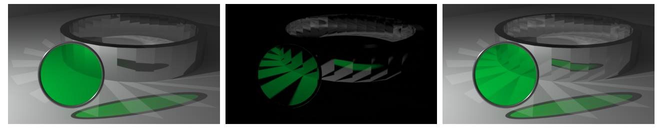
Implementation Our method can be easily integrated in the material model of a standard bidirectional path tracer without any change to the MIS computation. This keeps the MIS weight computation simple and hence eases the implementation process. We detail this in our supplementary material, and show that this approach takes into account unbiased bidirectional connections, vertex merging (i.e., spatial regularization or photon density estimation), and directional regularization in a simpler way than what was proposed by [Hachisuka et al. 2012; Georgiev et al. 2012].

4 Results

In order to illustrate well our method, we designed four scenes with simple geometry, but featuring complex light transport scenarios, such as small area light sources, mirror reflections, perfect refractions, etc. Our algorithm is implemented in approximately 50 lines of C++ as a special material inside [LuxRender 2008] without any other changes in the rendering algorithms. The code is available and released under the GPL licence. All results are generated using LuxRender on an Intel i7 950 @ 3 GHz using 8 CPU threads. The initial half aperture angle is of $\epsilon = 0.04$ radians.

In Figure 1 a highly glossy torus is enclosed in a glass cube and lit by a small area light. Although this scene can be rendered using unbiased BDPT, the small size of the light source makes it really difficult to sample, even with *Metropolis Light Transport* (MLT) [Kelemen et al. 2002]. The directional regularization (used with BDPT and MLT) shows a better convergence behavior during the early stages of rendering. However, we observe that after a few hours of rendering, our method appears to converge a little slower. This is due to the asymptotic convergence behavior of regularization, which is equivalent to PM and less efficient than unbiased BDPT. However, at this rendering stage, the differences between methods correspond to a small amount of bias and noise that is hardly visible in low dynamic range images. Figure 2 shows the weighting between biased and unbiased strategies for the torus scene.

Our method does not affect simple light paths and results in no additional bias in such configurations. Similarly to [Kaplanyan and Dachsbacher 2013], the directional regularization allows BDPT to sample a new category of paths, previously impossible to sample with unbiased BDPT, as depicted in Figure 3.



(a) *Weighted unbiased* **(b)** *Weighted biased* **(c)** *All strategies*

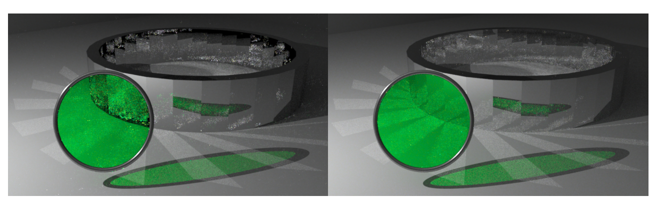
Figure 3: *A perfect mirror ring lit by a point light source rendered by BDPT and our MIS regularization. The scene contains a transparent green lens that prevents standard BDPT explicit connections to the camera. (a) The contribution of unbiased strategies where features are missing. (b) The weighted contribution of biased strategies, which only contribute to the missing parts and do not add bias in other parts of the image. (c) The sum of the weighted strategies in the final image, exhibits no bias in simple features and depicts complex features that BDPT cannot usually sample.*

Note that the torus scene can be rendered with unbiased methods only, although with slow convergence as depicted in Figure 1. This scene shows the differences between our approach and the single directional regularization [Kaplanyan and Dachsbacher 2013]. Because they only focus on paths that cannot be sampled with unbiased methods, they do not improve the converging behavior of this scene. We show that using directional regularization even on paths that can be sampled by unbiased methods improves the converging behavior. In Figure 4 we show that using many biased strategies instead of one leads to a better variance reduction.

Finally we compare our strategy to PM. In many complex cases, the density estimation of PM is really efficient to reduce variance. Even though, we show in Figure 5 that in some scenes directional regularization is a better sampling scheme than PM.

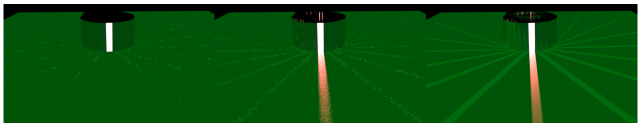
5 Discussion

Directional regularization is an efficient variance reduction scheme with BDPT, MIS, and MLT. Our goal is not to show that it is better than unbiased BDPT or PM, but that it is an efficient sampling strategy worth using in combination with these other strategies.



(a) *Single regularization* **(b)** *MIS regularization (ours)*

Figure 4: *Comparison between single regularization [Kaplanyan and Dachsbacher 2013] (a) and our MIS regularization (b). They select only the bidirectional strategy that introduces the minimal amount of bias. For this scene it is the strategy that connects a light path to the camera, but it exhibits high variance. In our case, we exploit all available strategies, including connecting a camera path to the light, which is a more efficient sampling scheme here and results in less noise in the image.*



(a) *VCM* **(b)** *Regularized BDPT* **(c)** *Reference*

Figure 5: *A mirror ring lit by a distant directional light. The floor is a mix between two materials: a green diffuse and a red glossy. The scene extent is* $100\times$ *larger than the cylinder, hence paths sampled from the light have low probability of reaching the camera and camera paths cannot sample the caustics. VCM is then inefficient because neither PM or bidirectional strategies are efficient. The directional regularization is able to create a connection between the mirror and the distant light. Both methods ran for one hour.*

Rendering Engine	Vertex size	Memory footprint
LuxRender [2008]	$\approx$ 250 bytes	1900 MB
SmallVCM [2012]	120 bytes	920 MB

Table 1: *Approximate memory overhead of PM (VCM) at a* 1920×1080 *resolution. This approximation only takes into account the photon data (position, direction, BSDF, probabilities) and not the data structure overhead for storage. This memory footprint is not compatible with constrained renders (such as GPU) and increases with resolution. On the other hand, directional regularization in BDPT does not have this memory requirement and therefore can be used on GPU and high resolution renders.*

Recently, [Georgiev et al. 2012; Hachisuka et al. 2012] have shown that merging different sampling methods, such as PM and unbiased BDPT, leads to a robust rendering algorithm. However, some light phenomena can only be rendered with directional regularization [Kaplanyan and Dachsbacher 2013]. Following their work, we show that directional regularization can outperform these other strategies in some complex cases. Directional regularization can be easily implemented inside any BDPT, and therefore inside VCM/UPS. This leads to a robust and holistic hybrid algorithm combining the sampling strengths of BDPT, PM, and directional regularization, able to handle all kinds of light phenomena.

Directional regularization without PM is even useful in some cases. First the engineering task of implementing BDPT is easier than implementing VCM (which needs to combine BDPT and PM). Moreover there are cases where the additional memory usage of PM (detailed in Table 1) is problematic, for example in GPU rendering where the amount of memory is limited. Additionally, motion blur and light dispersion can also suffer from correlated sampling, as demonstrated in Figure 6. This leads to the conclusion that directional regularization can be used by itself inside BDPT.

Conclusion We proposed a simple yet powerful extension to path space regularization. The originality of our approach consists in combining multiple importance sampling with directional regularization. We show that directional regularization is an efficient sampling strategy for complex lighting scenarios and, when coupled with BDPT and/or PM, that it leads to more robust rendering methods. In future work, we will focus on building a better heuristic for setting an initial value for the aperture ϵ, which is currently set by the user. We would also like to find a new weighting term w_i designed to precisely control the amount of introduced bias.

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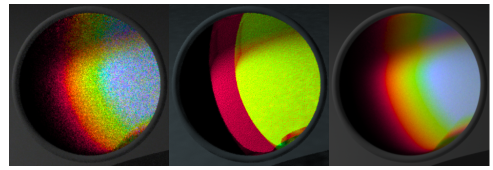

(a) *BDPT* **(b)** *VCM* **(c)** *Reference*

Figure 6: *A highly dispersive lens without directional regularization. After a few seconds of rendering (two photon passes), PM (VCM) has only sampled two wavelengths, because photons and gathering rays must sample the same wavelength to allow connections. Although less noisy, the VCM image suffers from important banding artifacts, but not BDPT. Depending on the rendering context, avoiding banding in early stages of rendering is an important property of algorithms without caches, such as BDPT.*

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