Neural Networks: promises of current research

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des Systèmes Adaptatifs http://www.iro.umontreal.ca/~lisa



www.apstat.com

Current research on deep architectures

A few labs are currently researching deep neural network training:

- Geoffrey Hinton's lab at U.Toronto
- Yann LeCun's lab at NYU
- Our LISA lab at U.Montreal
- ... many others to come! (we hope)

I will talk about my latest work (with Hugo Larochelle, Yoshua Bengio, Pierre-Antoine Manzagol): Extracting and Composing Robust Features with Denoising Autoencoders \rightarrow upcoming ICML 2008

- Building good predictors on complex domains means learning complicated functions.
- These are best represented by multiple levels of non-linear operations i.e. deep architectures.
- Learning the parameters of deep architectures proved to be challenging!

- Solution 1: initialize at random, and do gradient descent (Rumelhart et al., 1986).
 → disappointing performance. Stuck in poor solutions.
- Solution 2: Deep Belief Nets (Hinton et al., 2006): initialize by stacking Restricted Boltzmann Machines, fine-tune with Up-Down.
 → impressive performance.

Key seems to be good unsupervised layer-by-layer initialization...

- Solution 3: initialize by stacking autoencoders, fine-tune with gradient descent. (Bengio et al., 2007; Ranzato et al., 2007)
 → Simple generic procedure, no sampling required. Performance almost as good as Solution 2
- ... but not quite. Can we do better?

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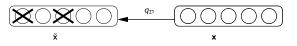
Open question: what would make a good unsupervised criterion for finding good initial intermediate representations?

- Inspiration: our ability to "fill-in-the-blanks" in sensory input. missing pixels, small occlusions, image from sound, ...
- Good fill-in-the-blanks performance \leftrightarrow distribution is well captured.
- \rightarrow old notion of associative memory (motivated Hopfield models (Hopfield, 1982))

What we propose: unsupervised initialization by explicit fill-in-the-blanks training.

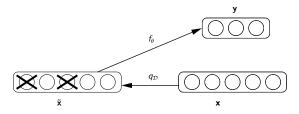


- Clean input x ∈ [0, 1]^d is partially destroyed, yielding corrupted input: x̃ ~ q_D(x̃|x).
- $\tilde{\mathbf{x}}$ is mapped to hidden representation $\mathbf{y} = f_{\theta}(\tilde{\mathbf{x}})$.
- From **y** we reconstruct a $\mathbf{z} = g_{\theta'}(\mathbf{y})$.
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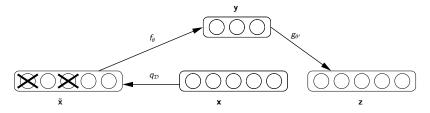
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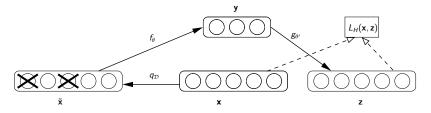
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The input corruption process $q_{\mathcal{D}}(\tilde{\mathbf{x}}|\mathbf{x})$



- Choose a fixed proportion ν of components of **x** at random.
- Reset their values to 0.
- Can be viewed as replacing a component considered missing by a default value.

Other corruption processes could be considered.

Form of parameterized mappings

We use standard sigmoid network layers:

•
$$\mathbf{y} = f_{\theta}(\mathbf{\tilde{x}}) = \operatorname{sigmoid}(\underbrace{\mathbf{W}}_{d' \times d} \mathbf{\tilde{x}} + \underbrace{\mathbf{b}}_{d' \times 1})$$

• $g_{\theta'}(\mathbf{y}) = \operatorname{sigmoid}(\underbrace{\mathbf{W}}_{d \times d'} \mathbf{y} + \underbrace{\mathbf{b}}_{d' \times 1}).$

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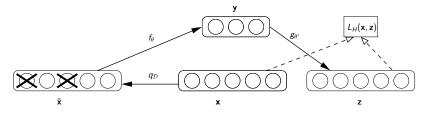
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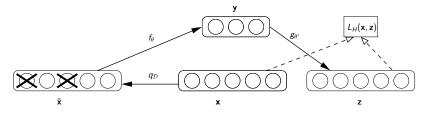
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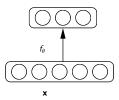


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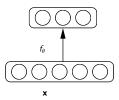
- **(a)** Remove scaffolding. Use f_{θ} directly on input yielding higher level representation.
- Learn next level mapping $f_{\theta}^{(2)}$ by training denoising autoencoder on current level representation.
- Iterate to initialize subsequent layers.



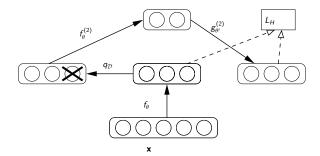
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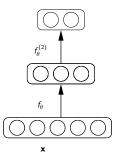
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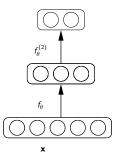
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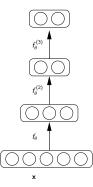
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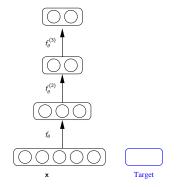
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- Initial deep mapping was learnt in an unsupervised way.
- \rightarrow initialization for a supervised task.
- Output layer gets added.
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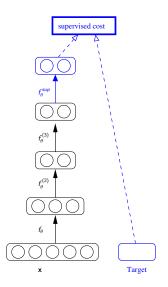
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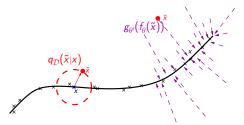
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Perspectives on denoising autoencoders

Manifold learning perspective



Denoising autoencoder can be seen as a way to learn a manifold:

- Suppose training data (x) concentrate near a low-dimensional manifold.
- Corrupted examples (•) are obtained by applying corruption process $q_{\mathcal{D}}(\widetilde{X}|X)$ and will lie farther from the manifold.
- The model learns with $p(X|\tilde{X})$ to "project them back" onto the manifold.
- Intermediate representation Y can be interpreted as a coordinate system for points on the manifold.

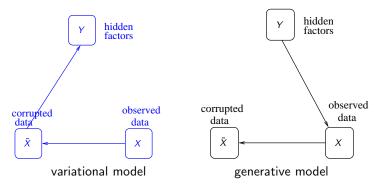
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Information theoretic perspective

- Consider $X \sim q(X)$, q unknown. $\widetilde{X} \sim q_{\mathcal{D}}(\widetilde{X}|X)$. $Y = f_{\theta}(\widetilde{X})$.
- It can be shown that minimizing the expected reconstruction error amounts to maximizing a lower bound on mutual information I(X; Y).
- Denoising autoencoder training can thus be justified by the objective that hidden representation Y captures as much information as possible about X even as Y is a function of corrupted input.

Perspectives on denoising autoencoders Generative model perspective

• Denoising autoencoder training can be shown to be equivalent to maximizing a variational bound on the likelihood of a generative model for the corrupted data.



- We used a benchmark of challenging classification problems (Larochelle et al., 2007) available at http://www.iro.umontreal.ca/~lisa/icml2007
- 8 classification problems on 28×28 grayscale pixel images (d = 784).
- First 5 are harder variations on the MNIST digit classification problem. Last 3 are challenging binary classification tasks.
- All problems have separate training, validation and test sets (test set size: 50 000).

basic: subset of original MNIST digits: 10 000 training samples, 2 000 validation samples, 50 000 test samples.



rot: applied random rotation (angle between 0 and 2π radians)



bg-img: background is random patch from one of 20 images



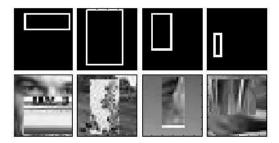
bg-rand: background made of random pixels (value in 0...255)



rot-bg-img: combination of rotation and background image

Benchmark problems

• rect: discriminate between tall and wide rectangles on black background.



- rect-img: borderless rectangle filled with random image patch. Background is a different image patch.
- convex: discriminate between convex and non-convex shapes.



We compared the following algorithms on the benchmark problems:

- **SVM**_{*rbf*}: suport Vector Machines with Gaussian Kernel.
- **DBN-3**: Deep Belief Nets with 3 hidden layers (stacked Restricted Boltzmann Machines trained with contrastive divergence).
- SAA-3: Stacked Autoassociators with 3 hidden layers (no denoising).
- SdA-3: Stacked Denoising Autoassociators with 3 hidden layers.

Hyper-parameters for all algorithms were tuned based on classificaiton performance on validation set. (In particular hidden-layer sizes, and ν for **SdA-3**).

Dataset	SVM _{rbf}	DBN-3	SAA-3	SdA-3 (ν)
basic	3.03±0.15			$2.80_{\pm0.14}~(10\%)$
rot	$11.11_{\pm 0.28}$			$10.29_{\pm 0.27} \ (10\%)$
bg-rand	$14.58 \scriptstyle \pm 0.31$	6.73±0.22	11.28±0.28	$10.38_{\pm 0.27}$ (40%)
bg-img	22.61±0.37			$16.68_{\pm 0.33}$ (25%)
rot-bg-img	55.18±0.44	47.39±0.44	51.93±0.44	44.49 _{±0.44} (25%)
rect	2.15±0.13	$2.60{\scriptstyle \pm 0.14}$	$2.41{\scriptstyle \pm 0.13}$	$1.99_{\pm 0.12} \; (10\%)$
rect-img	24.04±0.37	22.50±0.37	24.05±0.37	21.59 _{±0.36} (25%)
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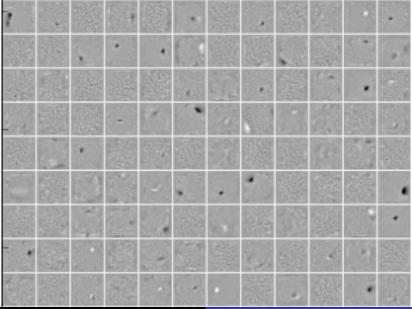
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- Pretraining as denoising autoencoders appears to be a very effective initialization technique.
- SdA-3 outperformed SVMs on all tasks, the difference being statistically significan on 6 of the 8 problems.
- Compared to Deep Belief Nets, SdA-3 was significantly superior in 4 of the problems, while being significantly worse in only 1 case.
- More striking is the large gain from using noise (SdA-3) compared to the noise free case SAA-3.
- Note that best results on most MNIST tasks were obtained with first hidden layer size of 2000, i.e. with over-complete representations (since d = 784).

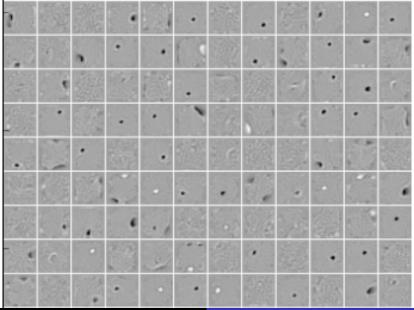
- To view the effect of noise, we look at filters learnt by the first layer on mnist basic for different destruction levels ν.
- Each filter corresponds to the weights linking input **x** and one of the first hidden layer units.
- Filters at the same position in the image for different destruction levels are related only by the fact that the autoencoders were started from the same random initialization point.

$\underset{0\ \%\ destroyed}{\text{Learnt filters}}$



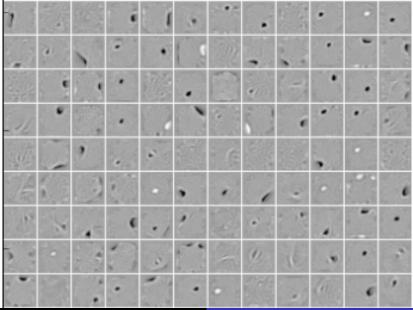
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Learnt filters 10 % destroyed



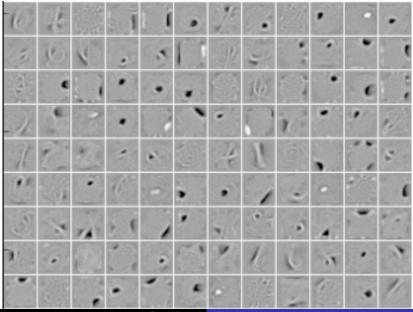
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Learnt filters 25 % destroyed



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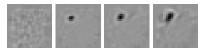
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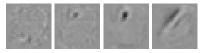
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- with no noise, many filters appear similarly uninteresting (undistinctive almost uniform grey patches).
- as we increase the noise level, denoising training forces the filters to differentiate more, and capture more distinctive features.
- higher noise levels tend to induce less local filters (as we expected).
- one can distinguish different kinds of filters, from local blob detectors, to stroke detectors, and some full character detectors at the higher noise levels.



Neuron A (0%, 10%, 20%, 50% destruction)



Neuron B (0%, 10%, 20%, 50% destruction)

- Unsupervised initialization of layers with an explicit denoising criterion appears to help capture interesting structure in the input distribution.
- This leads to intermediate representations much better suited for subsequent learning tasks such as supervised classification.
- Resulting algorithm for learning deep networks is simple and improves on state-of-the-art on benchmark problems.
- Future work will investigate the effect of different types of corruption process.

THANK YOU!

Performance comparison

Dataset	SVM _{rbf}	SVM _{poly}	DBN-1	DBN-3	SAA-3	SdA-3 (ν)
basic	3.03±0.15	3.69±0.17	3.94±0.17	$3.11 {\pm} 0.15$	3.46±0.16	2.80±0.14 (10%)
rot	$11.11 {\pm} 0.28$	15.42±0.32	14.69±0.31	10.30±0.27	10.30±0.27	10.29±0.27 (10%)
bg-rand	$14.58{\scriptstyle\pm0.31}$	16.62±0.33	9.80±0.26	6.73±0.22	11.28±0.28	10.38±0.27 (40%)
bg-img	22.61±0.37	24.01±0.37	16.15±0.32	16.31±0.32	23.00±0.37	16.68±0.33 (25%)
rot-bg-img	55.18±0.44	56.41±0.43	52.21±0.44	47.39±0.44	51.93±0.44	44.49±0.44 (25%)
rect	2.15±0.13	2.15±0.13	4.71±0.19	2.60±0.14	2.41±0.13	1.99±0.12 (10%)
rect-img	24.04±0.37	24.05±0.37	23.69±0.37	22.50±0.37	24.05±0.37	21.59±0.36 (25%)
convex	19.13±0.34	19.82±0.35	19.92±0.35	18.63±0.34	18.41±0.34	19.06±0.34 (10%)

red when confidence intervals overlap.

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