Practical characterization of quantum devices without tomography

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Quantum tomography is the main method used to assess the quality of quantum information processing devices, but its complexity presents a major obstacle for the characterization of even moderately large systems. However, tomography generates much more information than is often sought. Taking a more targeted approach, we develop schemes that enable (i) estimating the fidelity of an experiment to a theoretical ideal description, (ii) learning which description within a reduced subset best matches the experimental data. Both these approaches yield a significant reduction in resources compared to tomography. In particular, we show how to estimate the fidelity between a predicted pure state and an arbitrary experimental state using only a *constant* number of Pauli expectation values selected at random according to an importance-weighting rule. In addition, we propose methods for certifying quantum circuits and learning continuous-time quantum dynamics that are described by local Hamiltonians or Lindbladians. This extended abstract is a synthesis of Refs. [1] and [2], which the reader can consult for complete details on the results, methods and proofs outlined below.

The building blocks for quantum computers have been demonstrated in a number of different physical systems [3–8]. In order to quantify how closely these demonstrations come to the ideal operations, the experiments are fully characterized via either *quantum state tomography* [9] or *quantum process tomography* [10]. An important advantage of these methods is that they require only simple local measurements. The main drawback however is that tomography fundamentally requires both experimental and data post-processing resources that increase exponentially with the number of particles n [11].

The exponential cost of tomography is a practical problem even for systems with relatively small numbers of qubits. For example, recent ion trap experiments characterizing an 8 qubit state required 10 hours of measurements, despite collecting only 100 samples per observable [5]. Surprisingly, the post-processing of the data obtained from these experiments took approximately a week [12]. Under similar time scales, the characterization of a 16 qubit state would take years of measurements, and over a century of data post-processing. This is clearly a major obstacle in the demonstration of working quantum computers, even at sizes moderately larger than what has been demonstrated to date.

Moreover, despite the fact that universal quantum computation can be realized with one- and two-qubit elementary operations, it is not sufficient to characterize individual gates and then infer the behavior of the combined system. This is because larger systems may have significant noise contributions from correlated sources, as seen in recent experiments [8]. Controlling this kind of correlated noise is an important technical challenge. Indeed, one of the key assumptions in fault-tolerant quantum computation is that the noise on elementary components does not scale badly with the system size [13]. Characterizing multi-qubit states and operations is therefore necessary for the development of large quantum information processors.

In a nutshell, tomography is *currently* an obstacle to the experimental study of moderately large quantum devices. Thus, providing *scalable* characterization methods that remain experimentally feasible is of paramount importance to the quantum information processing community.

In our work [1, 2], we show how to efficiently characterize a large class of states and operations—including some that are universal resources for quantum computation—without resorting to tomography, and using only local measurements and the preparation of product states. We exploit the fact that what is really needed in many situations is *less* than full tomography. We often have an idea of what has been realized in the laboratory, so we are interested in asking for much less information—*e.g.*, we only want to know the distance (in some metric) to a particular theoretical target (*certification*), or to learn the identity of

the state or operation within a restricted set of possibilities (*learning*).

Certification

Certification is the task of estimating the fidelity $F(\hat{\sigma}, \hat{\rho})$ between an experimental state $\hat{\sigma}$ and a theoretical target $\hat{\rho}$. We only make two basic assumptions. First, we assume that the target state $\hat{\rho}$ is *pure*. However, we do not assume any additional structure or symmetry, so our method goes beyond previous work [14, 15] to encompass nearly all of the states of interest in experimental quantum information science (e.g., the GHZ and W states, stabilizer states, cluster states, matrix product states, PEPS, MERA or indeed any tensor network state, unique ground states of local Hamiltonians, etc.) in a unified framework. Second, we assume that we can measure n-qubit Pauli observables. Thus our method is applicable to any system that is capable of single-qubit gates and readout, *i.e,* with currently available technology, *without* needing to rely on 2-qubit gates or entangled measurements.

Our method works by measuring a random subset of Pauli observables chosen according to an "importance-weighting" rule. Roughly, we select Pauli operators that are most likely to detect deviations from the desired state $\hat{\rho}$. We use the resulting measurement statistics to estimate the fidelity $F(\hat{\rho}, \hat{\sigma})$, where $\hat{\sigma}$ is the actual state in the lab. This simple and direct method always requires drastically fewer resources than full tomography—and in some important cases (depending on $\hat{\rho}$), the required resources scale polynomially with the number of qubits n. Our scheme offers four significant advantages:

(1) For any state, the number of distinct Pauli operators to sample is constant—independent of the system size and depending only on the desired accuracy of the estimate—compared to the 4^n distinct experiments needed by standard tomography, or the $\mathcal{O}(n2^n)$ settings required by compressed sensing techniques [16].

(2) The data post-processing of our scheme is trivial, while the correct method of processing tomography data is a matter of current debates and different methods produce significantly different results [12].

(3) For many choices of the target state $\hat{\rho}$, such as stabilizer and matrix product states, and including the W state, sampling from the "importance-weighting" distribution has a classical processing cost polynomial in n. For instance, for stabilizer states, sampling amounts to generating an index uniformly and checking that it corresponds to a Pauli operator in the stabilizer group of the state.

(4) The sample complexity of our scheme (i.e., the number of copies of the state $\hat{\sigma}$ that are required, taking into account the need to repeat measurements in order to estimate expectation values) is quite low in many common cases, and is always much smaller than for tomography. The precise number depends on the target state $\hat{\rho}$. When $\hat{\rho}$ is a stabilizer state, the sample complexity is *constant*; when $\hat{\rho}$ is the W state, it is $\mathcal{O}(n^2)$. In the worst case, where $\hat{\rho}$ is an arbitrary pure state, the sample complexity is bounded by $\mathcal{O}(2^n)$, which is still a quadratic improvement over tomography. (We show that tomography for pure states using Pauli measurements requires at least $\Omega(2^{2n}/n)$ copies of the unknown state.)

This Monte Carlo certification method can be extended to continuous variable systems, such as a single optical mode in a cavity, by describing a state $\hat{\rho}$ by its Wigner function [17] rather than its decomposition of the Pauli group. It also extends naturally to quantum channels, where the goal is to estimate the fidelity between a quantum process $\mathcal{E}$ occurring in an experiment and a desired unitary evolution $\mathcal{U}$. This can be achieved using only the ability to prepare Pauli eigenstates and measure Pauli observables, without needing an ancilla. In particular, our method can be used to certify Clifford circuits, which are an important tool in fault-tolerant quantum computation, in constant time, i.e., while using the channel $\mathcal{E}$ only a constant number of times, independent of the number of qubits and gates in the circuit $\mathcal{U}$.

Learning

Learning consists of identifying the theoretical description from a restricted set of possibilities that best matches the experimental data. There exist many classes of "variational" states in physics that can be

specified with a small number of parameters. We provide examples where these parameters can be extracted directly from experiments, circumventing tomography and hence drastically reducing the complexity. These include stabilizer states, matrix product states (MPS), the corresponding processes—Clifford gates and MPS Choi matrices—and also local Hamiltonians and Lindbladians.

Stabilizer states can be *learned* efficiently, as pointed out by Aaronson and Gottesman [18], although the known method for efficient stabilizer learning requires entangling measurements. Aside from the direct generalizatin of the stabilizer approach, Clifford group operations can be learned efficiently [19] if one has access to Bell measurements and the inverse of the operation being learned. The problem of performing these tasks efficiently with strictly local measurements and without the need for the inverse remains open. Variational parameters for matrix product states can be extracted from local measurements, as shown in [20].

Models of universal quantum computation exist where the idea of discrete gates is not a natural fit. Instead, the system evolves in a continuous way, governed by some dynamical equation. Important examples include local Hamiltonians and Lindbladians that are universal for adiabatic quantum computation [21] and dissipation-driven quantum computation [22] respectively. The most direct way to determine how accurately these dynamics can be realized is to estimate the time evolution generator of the system, and explicitly check how it compares against the ideal target generator. A method has been proposed [23] to perform this task. It essentially relies on the infinitesimal evolution operator. However, there are in general two essential caveats to this approach: *1)* the evolution time t must be extremely short as t goes as the inverse of the norm of the Hamiltonian, going to 0 as the number of particles grows; *2)* the method requires resources, both experimental and computational, that grow exponentially with the number of particles. All these problems disappear when the Hamiltonian is *local*, which is nonetheless sufficient to achieve universal quantum computation [21, 22]. The Lieb-Robinson bound [24, 25] states that local Hamiltonians generate a causal evolution, with effects that propagate at a finite velocity v. Thus, only the part of the Hamiltonian acting on a region located at a distance $d \approx vt$ of the local observable to be measured contributes to its evolution. This fact solves all the problems associated to the proposal of [23]. Our method to learn the value of a local generators uses only *(i)* the preparation of initial product states, *(ii)* the simultaneous measurement of a constant number of single-qubit operators, *(iii)* a number of experimental settings linear in the system size, *(iv)* and classical post-processing of complexity $\mathcal{O}(n^3)$.

Outlook

We have demonstrated that by using a targeted approach to the extraction of information from experiments, the exponential overhead of tomography can be completely avoided for a number of physically interesting cases. In the cases where the exponential overhead cannot be avoided, our approach leads to significant practical advantages for verification experiments when compared to state and process tomography protocols. Looking beyond fidelity estimation, it would be interesting to directly estimate and bound an entanglement measure [12], which would obviate the need for an entanglement witness.

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