3D Path Recovery of an IVUS Transducer With Single-Plane Angiography

Denis Sherknies  
Université de Montréal  
dsherknie@umontreal.ca

Jean Meunier  
Université de Montréal  
meunier@umontreal.ca

Jean-Claude Tardif  
Montreal Heart Institute  
tardifjc@icm.umontreal.ca

Abstract
The recovery of the 3D path of the transducer used during an IntraVascular UltraSound (IVUS) examination is of primary importance to assess the exact 3D shape of the vessel under study. Traditionally, the reconstruction is done using biplane angiography. In this paper we explain, with three projection models, how single-plane angiography can be used to perform this task.

Three types of projection geometry are analyzed: orthographic, weak perspective and full perspective. In orthographic and weak perspective projection geometries, the catheter path can be reconstructed without prior transducer depth informations. With full perspective projection geometry, precise depth location of reference points are needed in order to minimize the error of the recovered transducer’s angle of incidence.

We present the mathematical foundation and some simulations of the catheter path reconstruction. While reconstructing the 3D catheter path from a single-view projection is shown to be feasible, some heuristics are needed in order to obtain a path simulating the curvature of the heart peri-cardium vessels.

Keywords: IVUS, Angiography, 3D-Reconstruction.

1. Introduction
In recent years, the combination of two well-known medical image modalities, X-ray angiography and intravascular ultrasound (IVUS), has given spectacular results. This is mainly due to the fact that the two imaging techniques complement themselves well: IVUS gives accurate information on the vessels cross-section and angiography provides a mean to reconstruct the vessels 3D curvature.

The 3D reconstruction of the IVUS images gives a more accurate assessment of the vessel lumen and plaque morphology. The first 3D reconstructions were done by straight stacking the images one on top of the other [1]. This method creates incorrect volumetric representation because the images are not parallel from one another but they are rather perpendicular to the catheter centerline, which follows the curvature of the vessels. The use of biplane angiography enables more precise IVUS image orientations and 3D localization [2], [3], [4], but these apparatus are complicated to operate and are not always available in many clinical centers.

In this paper, we present a method for recovering the IVUS transducer orientation from a single-plane projection (monocular vision), thus permitting to reconstruct the catheter path curvature. The whole process of the incidence angle recovery is based on the foreshortening effect of the IVUS transducer as seen on the angiogram. By comparing the known length of the transducer with the measured length of its projection, it is possible to obtain the incidence angle of the transducer. By combining a sequence of images, we can reconstruct the 3D catheter (transducer) path.

2. Projection Models

We analyze the recovery of the IVUS transducer from a full perspective projection, an orthographic projection and a weak perspective projection. The conventions used in this article are as follows (see figure 1): the origin of the coordinate system axes is placed at the X-ray emission source of the imaging system; the axes are oriented so that the projection plane is perpendicular centered along the z axis; the projection plane is coincident with the angiogram; the segment $S = \{P_1, P_2\}$ represents the transducer location during a pull-back and the segment $s = \{p_1, p_2\}$ is the projection of the transducer on the angiogram. The angulation of the transducer is define in relation to the projection plane, a transducer parallel to the projection plane is said to be at $0^\circ$, and at $90^\circ$ when perpendicular to the projection plane.

2.1 Full perspective projection

If we define $P_i = \{X_i, Y_i, Z_i\}$ as a 3D point and $p_i = \{u_i, v_i\}$ a 2D point, the relation between $P_i$ and its projection $p_i$, for a pin-hole camera model, is given by

$$u_i = f \frac{X_i}{Z_i} \quad \text{and} \quad v_i = f \frac{Y_i}{Z_i} \quad (1)$$

where $f$ is the focal distance. We assume here that the camera is placed at the origin in the 3D space, that the projection plane is perpendicular to the $z_i$ axis and centered on it.
(see figure 1). The Euclidean distance between two points $P_1 = [X_1, Y_1, Z_1]$ and $P_2 = [X_2, Y_2, Z_2]$ can be found with

$$d = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (Z_2 - Z_1)^2}. \quad (2)$$

Expressing equation 1a as a function of $Z_2$, we obtain

$$Z_2 = f \frac{X_2}{u_2}, \quad (3)$$

using equation 3 to substitute $Z_2$ in equation 1b and expressing it as $Y_2$ gives

$$Y_2 = \frac{v_2 X_2}{u_2}. \quad (4)$$

Substituting equations 3 and 4 in 2 gives

$$d = \sqrt{(X_2 - X_1)^2 + (u_2 \frac{X_2}{u_2} - Y_1)^2 + (f \frac{X_2}{u_2} - Z_1)^2}, \quad (5)$$

which solved for $X_2$ gives

$$X_2 = \frac{u_2 a \pm \sqrt{u_2^2 [a^2 + b(d^2 - X_1^2 - Y_1^2 - Z_1^2)]}}{b}, \quad (6)$$

where $a = u_2 X_1 + v_2 Y_1 + f Z_1$ and $b = f^2 + u_2^2 + v_2^2$.

By substituting equation 6 in equations 3 and 4 we are able to find the 3D coordinates of $P_2$ knowing the location of two projected points $p_1, p_2$, the location of a 3D point $P_1$ and the distance between two 3D points $d$.

\[ \text{Figure 1. Full perspective projection geometry.} \]

\[ \text{Figure 2. Orthographic projection geometry.} \]

2.2 Orthographic projection

In an orthographic projection, the relation between a 3D point $P_i = [X_i, Y_i, Z_i]$ and its projection $p_i = [u_i, v_i]$, is given by

$$u_i = X_i \quad \text{and} \quad v_i = Y_i. \quad (7)$$

As in the case of full perspective projection, we assume that the projection plane is perpendicular to the $z_i$ axis and centered on it (see figure 2). By using the equivalences found in equations 7, we are able to rewrite equation 2 as

$$Z_2 = Z_1 \pm \sqrt{d^2 - \left[ (u_2 - u_1)^2 + (v_2 - v_1)^2 \right] f'^2}, \quad (8)$$

which gives the depth coordinate of point $P_2$.

2.3 Weak perspective projection

A weak perspective projection is realized by having an orthogonal projection followed by an isotropic scaling factor [5]. The scaling factor is in fact, the ratio of the focal distance on the average depth of the scene, defined by $f' = \frac{f}{\overline{Z}}$, where $\overline{Z} = \frac{1}{n} \sum_{i=1}^{n} Z_i$. The relation between the 3D points and their projections are $u_i = f' X_i$ and $v_i = f' Y_i$. Equation 8 can thus be rewritten as

$$Z_2 = Z_1 \pm \sqrt{d^2 - \left[ (u_2 - u_1)^2 + (v_2 - v_1)^2 \right] f'^2}. \quad (9)$$

3. 3D Path reconstruction method

The steps involved in our method of 3D path transducer recovery are as follows. First, in order to simulate a projection, we created a 3D path, the 3D spiral in figure 3, which
was divided in segments of 0.5 cm, corresponding to the transducer length. In figure 3 only segments $S_1$ and $S_i$ are shown. Each of the segments were projected using one of the projection geometry previously described. The resulting data simulate the cine-angiographic images of the IVUS transducer during a pull-back.

From this point on, the real reconstruction procedure begins. For the first frame, we take the projection plane coordinates $[p_1^1, p_2^1]$ and an estimated depth coordinate $P_{11}^1$ in order to derive the depth coordinate for point $P_{12}^1$. For frame $i + 1$, the procedure is repeated with $P_{i+1}^1 = P_{i2}^2$. By using the recovered depth coordinates at once, we are able to represent the path in 3D. Although there exist two solutions for each frame, since a square root is involved in the depth recovery, selecting the correct solution is facilitated by the fact that for two adjacent frames, the path cannot reverse direction. An ambiguity still exist when the path becomes parallel to the projection plane, at which point it is numerically impossible to know if the transducer, on the next frame, will go away or toward the projection plane. Using heuristics like the heart maximum size or the general shape of the vessels can help alleviate such ambiguities.

4. ERROR ANALYSIS

The precision in the transducer path reconstruction is dependent on the estimated or measured values of many parameters. Here we briefly discuss the effects that have an error in the measured projection foreshortening length or in the initial depth localisation in full and weak perspective geometry.

Because of the trigonometric nature of the projection geometry, for a constant foreshortening effect, the estimated angular variation of the transducer will depends on its initial angle. In figure 4, we can see that a variation of 10% on the projected length of the transducer placed parallel to the projection plane, gives an angular variation of $25.8^\circ$ compared to an angular variation of only $5.7^\circ$ when the transducer is placed at an angle of $84.3^\circ$ from the projection plane. This expresses the fact that whatever technique is used to recover the 3D orientation from a projection, the error margin will be greater as the transducer tends to be parallel to the projection plane. This effect can be found on the angiogram where the continuous nature of the projected length of the transducer is quantized by the pixel representation. If we use typical values for field of view of 10 cm, transducer length of 0.5 cm and an angiography image resolution of 1024 x 1024 pixels, the variation of the projected length would be less than 2% the transducer length. At worst, for a transducer oriented at $0^\circ$, this would produce an angular variation of $11^\circ$.

In the full perspective geometry (see figure 5), when we do a back-projection of two fixed points $[p_1, p_2]$, for a constant length $d$ and for different fixed 3D points $P_i^1$, we get different solutions for points $P_{i2}^2$. Each of these segments has a different angulation. The relation between the 3D points $P_i^1$ and the resulting segment’s angulation is that, if we over estimate the 3D location of a point, the angulation will be underestimated. By using typical values for the location of the projection plane and object plane of 100 cm and 75 cm respectively, a depth of view of 10 cm (the normal heart size), and a transducer length of 0.5 cm, it is possible to determine the worst case error in angulation estimation. An error of $\pm 5$ cm in the estimated depth coordinate, with
a transducer placed at 0° results in a variation of estimated angulation of ±20°, decreasing rapidly to ±3° when the transducer is placed at 45°.

In the case of weak perspective or orthographic projections (figure 6), the angulation of the 3D segments \( [P^1_1, P^2_1] \), found by back-projection of segment \( [p_1, p_2] \), is irrelevant on the initial 3D point estimation \( P^3_1 \). This is an interesting aspect of the weak perspective geometry, besides simplified calculations. In the worst case, if we compare the full perspective to the weak perspective geometry, we get the same angular variation than when an initial 3D point location is miss estimated, in the full perspective geometry alone.

5. CONCLUSION

We have sketch a solution to get 3D information about the angle of the transducer of an IVUS catheter from a single-plane angiographic image. Analytical formulations were derived. Three projection geometries were examined, and we found that the weak perspective geometry being a simplified approximation, shows worst case value errors similar to the full perspective geometry, despite the fact that no depth information is necessary. Depth recoveries were done on simulated projections and found to be conclusive. The validation of the method using a phantom is underway and will be presented in a future article.

References


