

## IFT-3245

## Démonstration 5, 19 novembre 2012.

1. Exercice 1, Chapitre 6.
2. Illustration des intervalles de confiance de type normal et student sur l'exemple `queueevent_ci.cpp`.
3. Illustration des techniques d'intervalles de confiance pour des variables discrètes avec SSJ.

Suppose that  $Y \sim \text{Binomiale}(n, p)$  with  $n = 10000$ . We seek  $p_1$  and  $p_2$  such that

$$\begin{aligned} 0.025 &= P[Y \geq 2 \mid p = p_1] = 1 - \sum_{i=0}^1 \binom{n}{i} p_1^i (1-p_1)^{n-i} \\ &= 1 - (1-p_1)^n - np_1(1-p_1)^{n-1} \end{aligned}$$

and

$$\begin{aligned} 0.025 &= P[Y \leq 2 \mid p = p_2] = \sum_{i=0}^2 \binom{n}{i} p_2^i (1-p_2)^{n-i} \\ &= (1-p_2)^n + np_2(1-p_2)^{n-1} + [n(n-1)/2]p_2^2(1-p_2)^{n-2}. \end{aligned}$$

Solving these equations, we find the following 95% confidence interval for  $p$  :

$$[p_1, p_2] = [2.422 \times 10^{-5}, 7.223 \times 10^{-4}].$$

(b) If we suppose that  $Y \sim \text{Poisson}(\lambda)$  with  $\lambda = np$ , we seek  $p_1$  and  $p_2$  such that

$$0.025 = P[Y \geq 2 \mid p = p_1] = 1 - \sum_{i=0}^1 \frac{e^{-np_1} (np_1)^i}{i!} = 1 - e^{-np_1} (1 + np_1)$$

and

$$0.025 = P[Y \leq 2 \mid p = p_2] = \sum_{i=0}^2 \frac{e^{-np_2} (np_2)^i}{i!} = e^{-np_2} (1 + np_2 + (np_2)^2/2).$$

Solving these equations, we find the 95% confidence interval :

$$[p_1, p_2] = [2.422 \times 10^{-5}, 7.225 \times 10^{-4}].$$

The Poisson approximation gives essentially the same interval as the binomial distribution.

4. Remarques sur le logit.