

Name:

Lab/Affiliation:

Level (MSc/PhD - which year):

Registered? (if not, briefly explain, e.g. auditing):

Relevant courses you took (e.g. ML class, linear algebra):

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Answer each of the following questions as shortly as possible. If you don't know an answer, put a question mark. *The quiz will not be scored and will not count towards your class grade.*

- (1) How familiar are you with the following notions/techniques (write a score between 0 and 10, 0 meaning you never heard of it, 10 meaning you're very familiar):
- Linear independence
  - rank of a matrix
  - Singular value decomposition
  - Jordan canonical form
  - Pseudo-inverse
  - Orthogonal projection
  - Algebraic/geometric multiplicity of an eigenvalue
  
  - ridge regression
  - generalization bound
  - bias-variance tradeoff
  - maximum likelihood estimator
  - method of moments
  - gradient
  - probability density function
  
  - Big O notation
  - NP-hard problem
- (2) Give the definitions of the expectation  $\mathbb{E}[X]$  and variance  $\mathbb{V}[X]$  of a discrete random variable  $X$  taking its values in  $\{1, \dots, n\}$ .
- (3) Write down a sufficient condition for a function  $f$  to be convex.

(4) Give a short proof of Bayes rule:  $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$ .

(5) What is the dimension of the linear space spanned by the following sets of vectors:

$$S_1 = \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} : a + b = c, a, b, c \in \mathbb{R} \right\} \qquad S_2 = \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} : a^2 + b^2 = c, a, b, c \in \mathbb{R} \right\}$$

(6) Are the following matrices invertible?

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 2 \\ 3 & -1 & 0 \\ 4 & 0 & 2 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

(7) What are the eigenvalues and corresponding eigenvectors of the matrix  $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ ?

(8) Give a sufficient condition for a matrix to be invertible.

(9) Write down an example of a defective matrix, i.e. a matrix that is not diagonalizable.