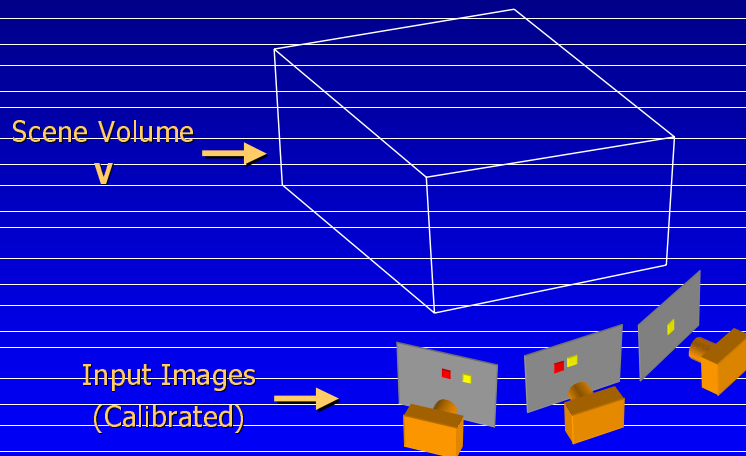


*SIGGRAPH 2000 Course on  
3D Photography*

**From Images to Voxels**

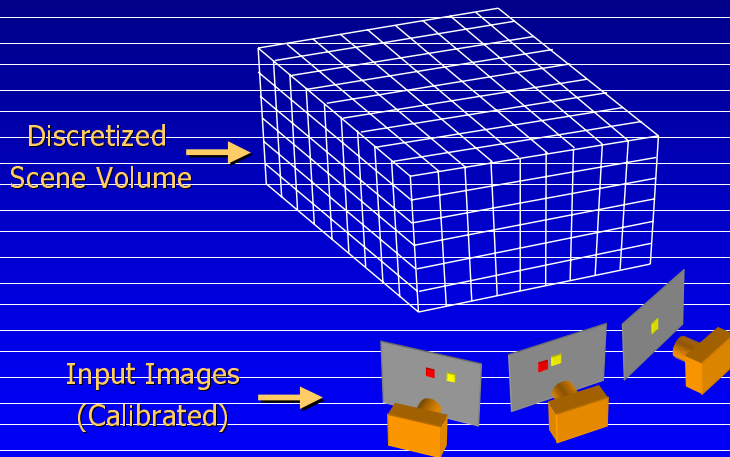
*Steve Seitz  
Carnegie Mellon University  
University of Washington  
<http://www.cs.cmu.edu/~seitz>*

**3D Reconstruction from Calibrated Images**



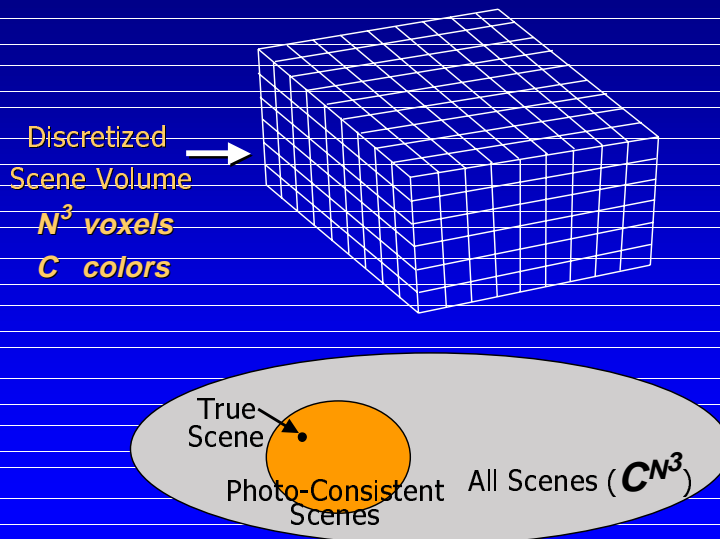
**Goal:** Determine transparency, radiance of points in  $V$

## Discrete Formulation: Voxel Coloring



**Goal:** Assign RGBA values to voxels in  $V$   
*photo-consistent* with images

## Complexity and Computability



## Issues

---

### *Theoretical Questions*

- Identify class of *all* photo-consistent scenes

### *Practical Questions*

- How do we compute photo-consistent models?

## Voxel Coloring Solutions

---

### *1. $C=2$ (silhouettes)*

- Volume intersection [Martin 81, Szeliski 93]

### *2. $C$ unconstrained, viewpoint constraints*

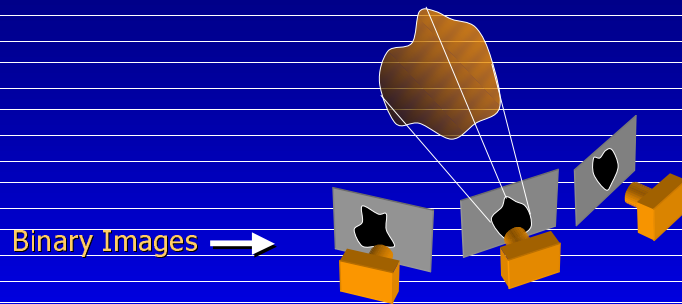
- Voxel coloring algorithm [Seitz & Dyer 97]

### *3. General Case*

- Space carving [Kutulakos & Seitz 98]

## Reconstruction from Silhouettes ( $C = 2$ )

---

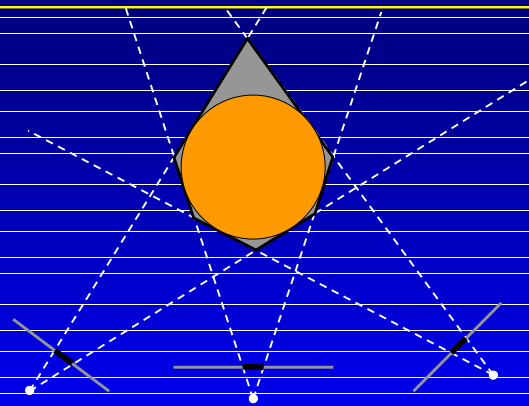


### Approach:

- *Backproject* each silhouette
- Intersect backprojected volumes

## Volume Intersection

---

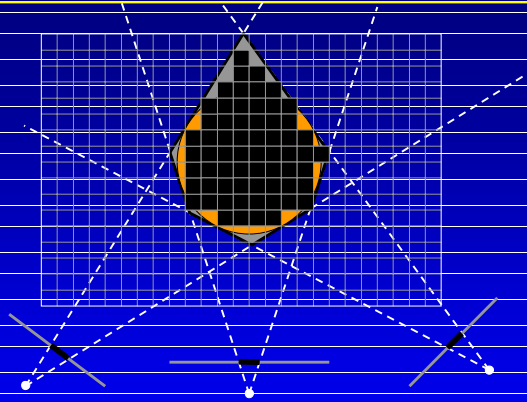


### Reconstruction Contains the True Scene

- But is generally not the same
- No concavities
- In the limit get *visual hull*

## Voxel Algorithm for Volume Intersection

---



*Color voxel black if on silhouette in every image*

- $O(MN^3)$ , for  $M$  images,  $N^3$  voxels
- Don't have to search  $2^{N^3}$  possible scenes!

## Properties of Volume Intersection

---

### *Pros*

- Easy to implement, fast
- Accelerated via octrees [Szeliski 1993]

### *Cons*

- No concavities
- Reconstruction is not photo-consistent
- Requires identification of silhouettes

## Voxel Coloring Solutions

---

### 1. $C=2$ (silhouettes)

- Volume intersection [Martin 81, Szeliski 93]

### 2. $C$ unconstrained, viewpoint constraints

- Voxel coloring algorithm [Seitz & Dyer 97]

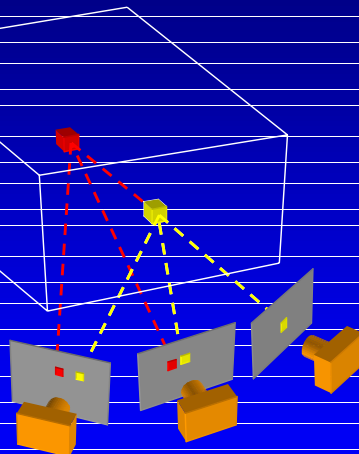
### 3. General Case

- Space carving [Kutulakos & Seitz 98]

## Voxel Coloring Approach

---

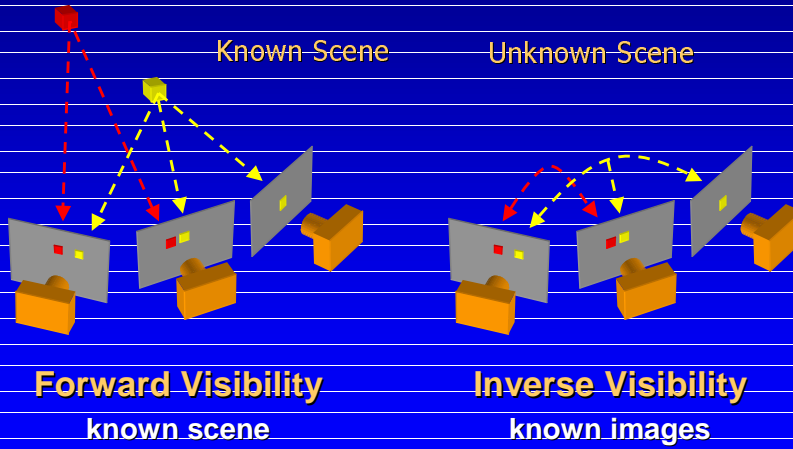
1. Choose voxel
2. Project and correlate
3. Color if consistent



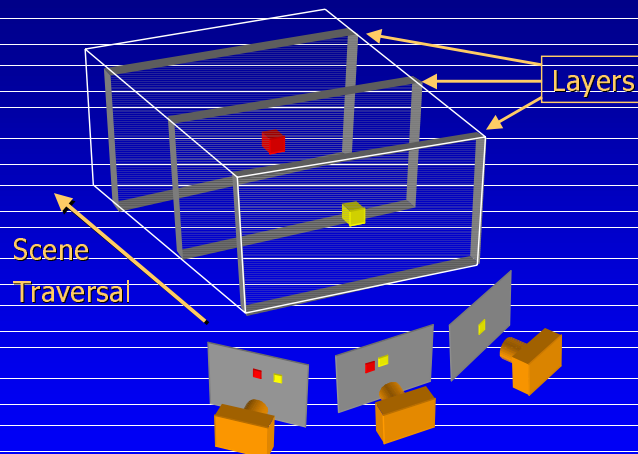
**Visibility Problem:** in which images is each voxel visible?

## The Global Visibility Problem

*Which Points are Visible in Which Images?*



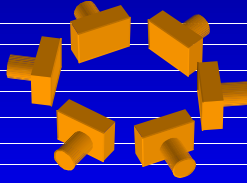
## Depth Ordering: Visit Occluders First!



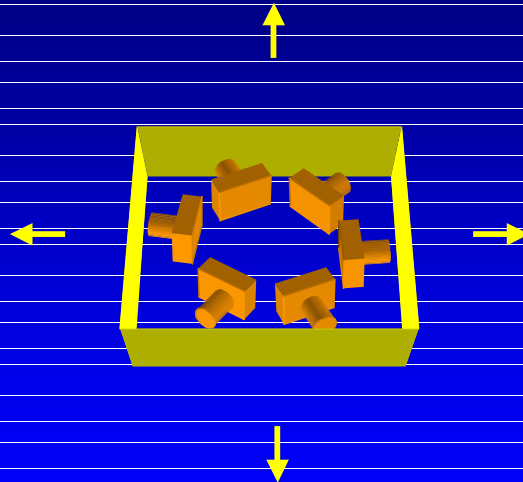
**Condition:** depth-order is *view-independent*

## Panoramic Depth Ordering

- Cameras oriented in many different directions
- Planar depth ordering does not apply



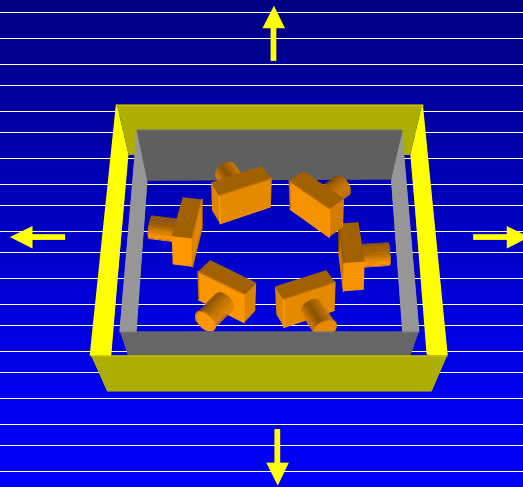
## Panoramic Depth Ordering



**Layers radiate outwards from cameras**

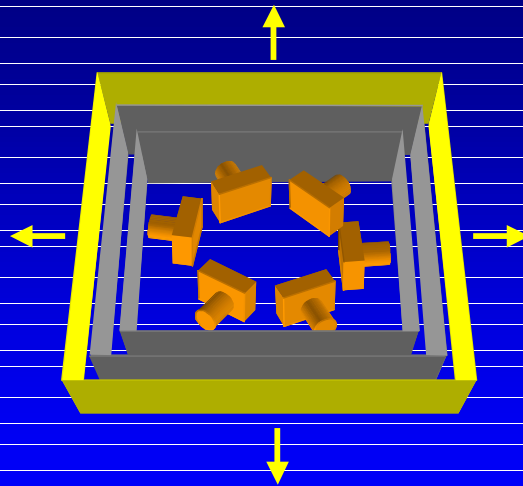


## Panoramic Layering



Layers radiate outwards from cameras

## Panoramic Layering

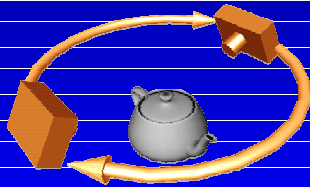


Layers radiate outwards from cameras

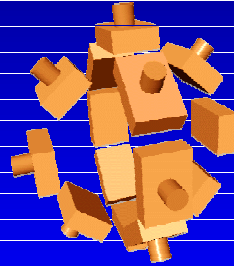
## Compatible Camera Configurations

### Depth-Order Constraint

- Scene outside convex hull of camera centers

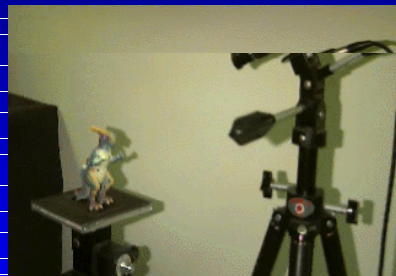


**Inward-Looking**  
cameras above scene



**Outward-Looking**  
cameras inside scene

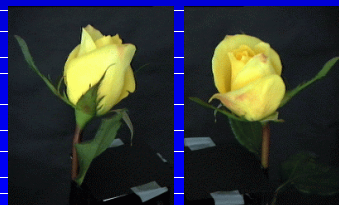
## Calibrated Image Acquisition



**Calibrated Turntable**  
360° rotation (21 images)



**Selected Dinosaur Images**



**Selected Flower Images**

## Voxel Coloring Results (Video)

---



### Dinosaur Reconstruction

72 K voxels colored  
7.6 M voxels tested  
7 min. to compute  
on a 250MHz SGI



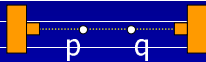
### Flower Reconstruction

70 K voxels colored  
7.6 M voxels tested  
7 min. to compute  
on a 250MHz SGI

## Limitations of Depth Ordering

---

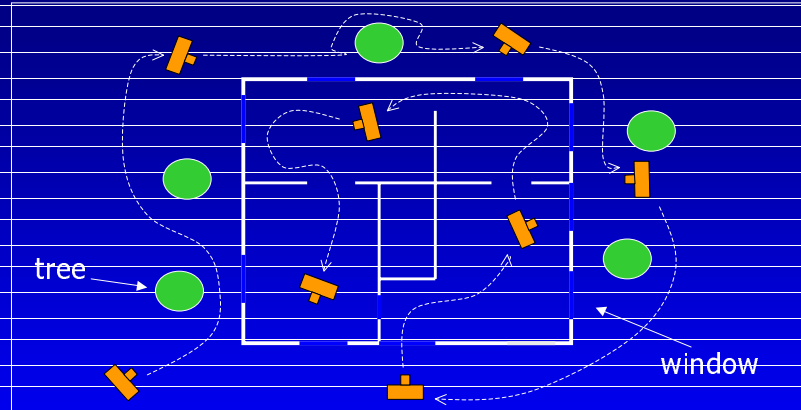
*A View-Independent Depth Order May Not Exist*



*Need More Powerful General-Case Algorithms*

- Unconstrained camera positions
- Unconstrained scene geometry/topology

## A More Difficult Problem: Walkthrough



*Input: calibrated images from arbitrary positions*

*Output: 3D model photo-consistent with all images*

## Voxel Coloring Solutions

### 1. $C=2$ (silhouettes)

- Volume intersection [Martin 81, Szeliski 93]

### 2. $C$ unconstrained, viewpoint constraints

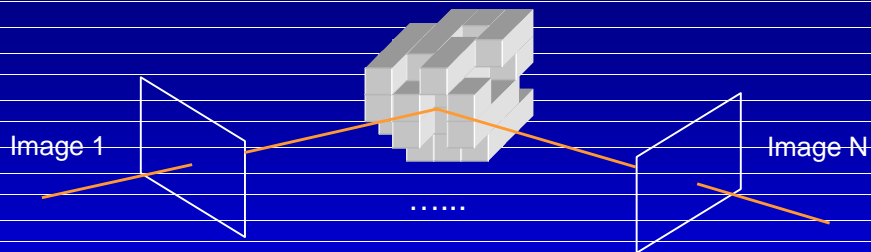
- Voxel coloring algorithm [Seitz & Dyer 97]

### 3. General Case

- Space carving [Kutulakos & Seitz 98]

## Space Carving Algorithm

---



### *Space Carving Algorithm*

- Initialize to a volume  $V$  containing the true scene
- Choose a voxel on the current surface
- Project to visible input images
- Carve if not photo-consistent
- Repeat until convergence

## Convergence

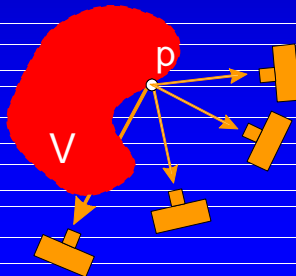
---

### *Consistency Property*

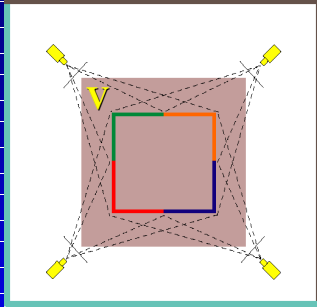
- The resulting shape is photo-consistent
  - > all inconsistent voxels are removed

### *Convergence Property*

- Carving converges to a non-empty shape
  - > a point on the true scene is *never* removed



## Output of Space Carving: Photo Hull



True Scene

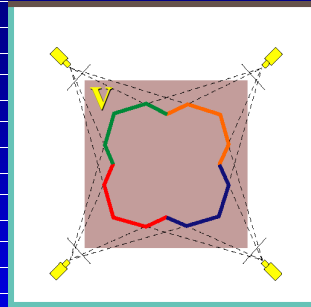


Photo Hull

The *Photo Hull* is the UNION of all photo-consistent scenes in  $V$

- Tightest possible bound on the true scene

## Space Carving Algorithm

*The Basic Algorithm is Unwieldy*

- Complex update procedure

*Alternative: Multi-Pass Plane Sweep*

- Efficient, can use texture-mapping hardware
- Converges quickly in practice
- Easy to implement



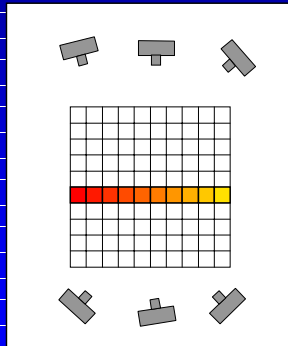
Results

Related Methods

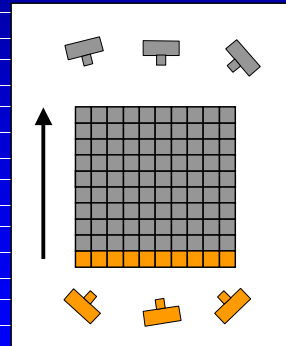


## Multi-Pass Plane Sweep

- Sweep plane in each of 6 principle directions
- Consider cameras on only one side of plane
- Repeat until convergence



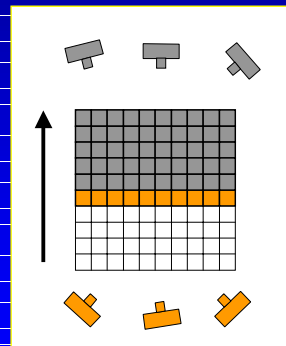
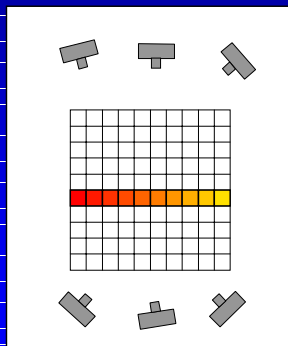
True Scene



Reconstruction

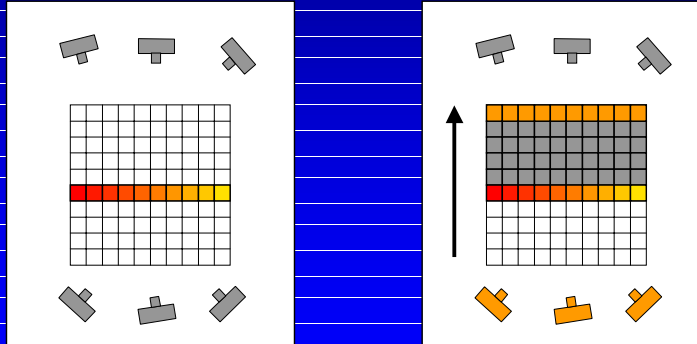
## Multi-Pass Plane Sweep

- Sweep plane in each of 6 principle directions
- Consider cameras on only one side of plane
- Repeat until convergence



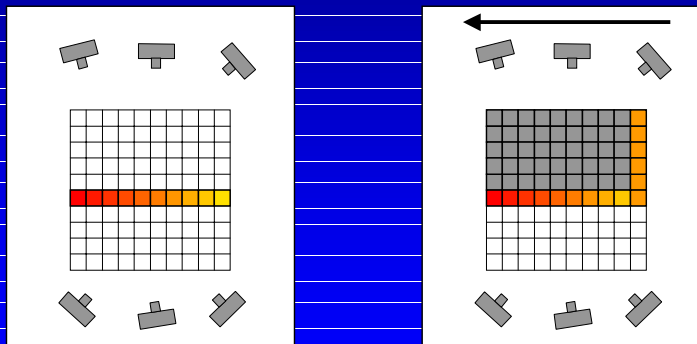
## Multi-Pass Plane Sweep

- Sweep plane in each of 6 principle directions
- Consider cameras on only one side of plane
- Repeat until convergence



## Multi-Pass Plane Sweep

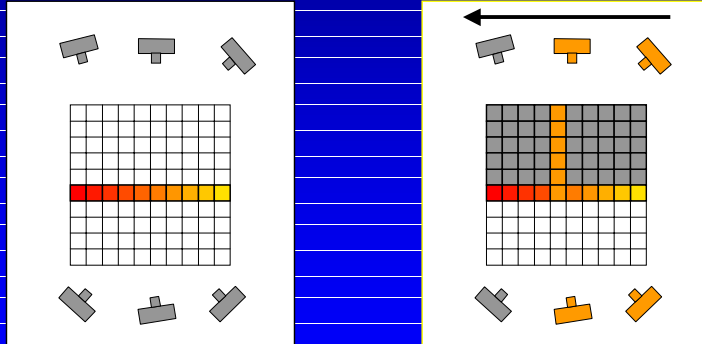
- Sweep plane in each of 6 principle directions
- Consider cameras on only one side of plane
- Repeat until convergence





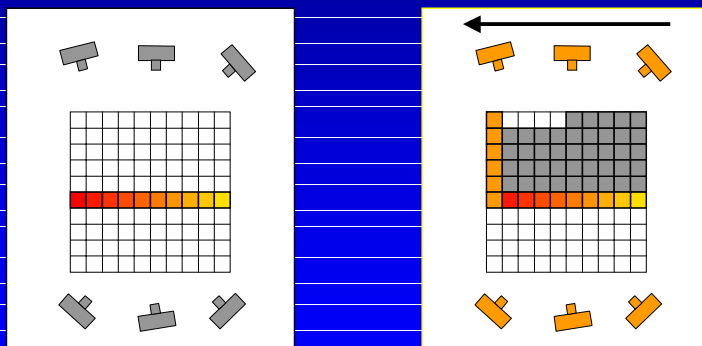
## Multi-Pass Plane Sweep

- Sweep plane in each of 6 principle directions
- Consider cameras on only one side of plane
- Repeat until convergence



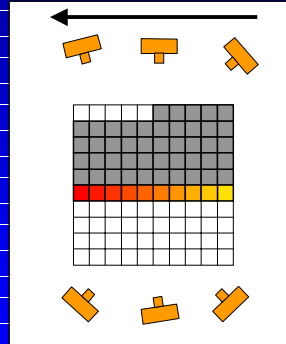
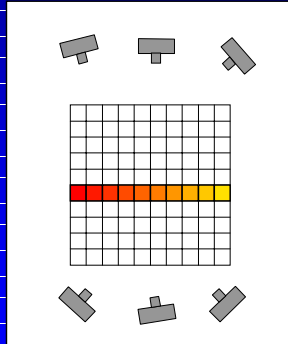
## Multi-Pass Plane Sweep

- Sweep plane in each of 6 principle directions
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## Multi-Pass Plane Sweep

- Sweep plane in each of 6 principle directions
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- Repeat until convergence



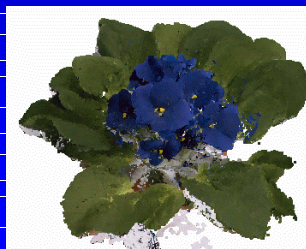
## Space Carving Results: African Violet



Input Image (1 of 45)



Reconstruction



Reconstruction



Reconstruction

## Space Carving Results: Hand

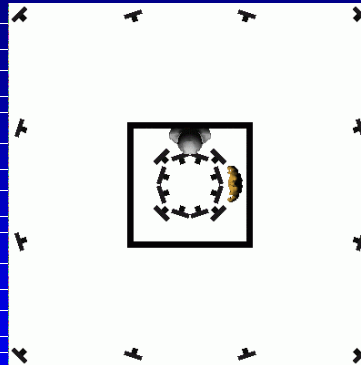
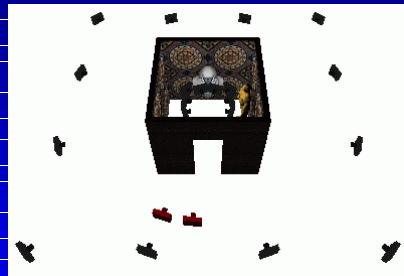


Input Image  
(1 of 100)



Views of Reconstruction

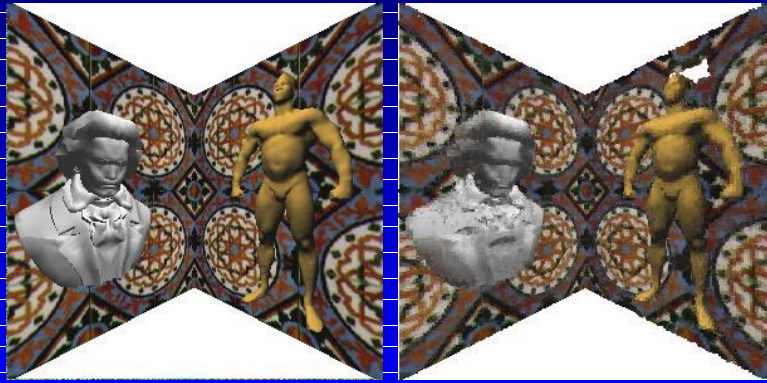
## House Walkthrough



*24 rendered input views from inside and outside*

## Space Carving Results: House

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**Input Image**  
(true scene)

**Reconstruction**  
370,000 voxels

## Space Carving Results: House

---



**Input Image**  
(true scene)

**Reconstruction**  
370,000 voxels

## Space Carving Results: House

---



New View (true scene)



Reconstruction



New View  
(true scene)



Reconstruction



Reconstruction  
(with new input view)

## Other Approaches

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**Level-Set Methods** [Faugeras & Keriven 1998]

- Evolve implicit function by solving PDE's

**Transparency and Matting** [Szeliski & Golland 1998]

- Compute voxels with alpha-channel

**Max Flow/Min Cut** [Roy & Cox 1998]

- Graph theoretic formulation

**Mesh-Based Stereo** [Fua & Leclerc 95]

- Mesh-based but similar consistency formulation

**Virtualized Reality** [Narayan, Rander, Kanade 1998]

- Perform stereo 3 images at a time, merge results

## Conclusions

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### *Advantages of Voxels*

- **Non-parametric**
  - > can model arbitrary geometry
  - > can model arbitrary topology
- **Good reconstruction algorithms**
- **Good rendering algorithms (splatting, LDI)**

### *Disadvantages*

- **Expensive to process hi-res voxel grids**
- **Large number of parameters**
  - > Simple scenes (e.g., planes) require lots of voxels

## Bibliography

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### *Volume Intersection*

- Martin & Aggarwal, "Volumetric description of objects from multiple views", *Trans. Pattern Analysis and Machine Intelligence*, 5(2), 1991, pp. 150-158.
- Szeliski, "Rapid Octree Construction from Image Sequences", *Computer Vision, Graphics, and Image Processing: Image Understanding*, 58(1), 1993, pp. 23-32.

### *Voxel Coloring and Space Carving*

- Seitz & Dyer, "Photorealistic Scene Reconstruction by Voxel Coloring", *Proc. Computer Vision and Pattern Recognition (CVPR)*, 1997, pp. 1067-1073.
- Seitz & Kutulakos, "Plenoptic Image Editing", *Proc. Int. Conf. on Computer Vision (ICCV)*, 1998, pp. 17-24.
- Kutulakos & Seitz, "A Theory of Shape by Space Carving", *Proc. ICCV*, 1998, pp. 307-314.

## Bibliography

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### *Related References*

- Faugeras & Keriven, "Variational principles, surface evolution, PDE's, level set methods and the stereo problem", IEEE Trans. on Image Processing, 7(3), 1998, pp. 336-344.
- Szeliski & Golland, "Stereo Matching with Transparency and Matting", Proc. Int. Conf. on Computer Vision (ICCV), 1998, 517-524.
- Roy & Cox, "A Maximum-Flow Formulation of the N-camera Stereo Correspondence Problem", Proc. ICCV, 1998, pp. 492-499.
- Fua & Leclerc, "Object-centered surface reconstruction: Combining multi-image stereo and shading", Int. Journal of Computer Vision, 16, 1995, pp. 35-56.
- Narayanan, Rander, & Kanade, "Constructing Virtual Worlds Using Dense Stereo", Proc. ICGV, 1998, pp. 3-10.